A Simple Learning Variable Structure Control Law for Rigid Robot Manipulators

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Abstract: In this paper, we consider the problem of designing a simple learning variable structure system for repeatable tracking control of robot manipulators. We combine a variable structure control law as the robust part for stabilization and a feedforward learning law as the intelligent part for nonlinearity compensation. We show that the tracking error asymptotically converges to zero. Finally, we give computer simulation results in order to show the effectiveness of our method.

Keywords: Learning control, robot, sliding mode control, switching surface, variable structure system

1. Introduction

The variable structure system is well known to be robust to system uncertainties and numerous variable structure control algorithms have been proposed for the robust stabilization of incompletely modeled or uncertain systems. In the variable structure control system the controller structure around the plant is intentionally changed by using a viable high speed switching feedback control to obtain a desired plant behavior or response. Using the switching feedback control law the variable structure control system drives the trajectory of the system onto a specified and user-chosen surface, which is termed the sliding surface or the switching surface, and maintain the trajectory on this sliding surface for all subsequent time. The central feature of the variable structure system is the so-called sliding mode on the switching surface within which the system remains insensitive to internal parameter variations and extraneous disturbance(DeCarlo *et al.* 1988; Utkin 1977).

Recently, based on the variable structure system theory many researchers have proposed variable structure control laws for tracking control of robot manipulators(Abdallah *et al.* 1991; Chen *et al.* 1990; Lewis *et al.* 1993; Slotine 1985; Slotine and Li 1991; Yeung and Chen 1988; Young 1978 and references therein). In order to compensate for the nonlinear coupling terms and the gravitational plus frictional force terms, the conventional variable structure control laws usually use feedforward terms requiring on-line computation. Inaccurate feedforward compensation results in very large control efforts to guarantee sliding mode on a switching surface. In order to get good tracking performance while reducing the large switching feedback control gain, the conventional variable structure control laws require accurate feedforward compensation leading to heavy computational burdens. To overcome this problem, we propose a simple learning variable structure control law for the rigid robot manipulator under the assumption that the robot is used to perform a repetitive task. This assumption is not so restrictive because an industrial robot in assembly line is usually called upon to execute repetitive operations. A feedback switching control law is used as usual to drive the tracking error to a switching surface despite of uncertainties, and a very simple learning scheme is incorporated to learn the feedforward nonlinear compensation term. Our feedforward compensation term is different from those of the conventional variable structure control methods in that our feedforward term only uses the past control sequence. Our feedforward compensation term does not require accurate information about the robot, it does not have to be computed in real time and the learning algorithm is very simple, thus in our method we can very efficiently compensate for the uncertain nonlinear terms without accurate information about the robot dynamics as opposed to the previous variable structure tracking control methods for the robot and the magnitude of the switching feedback control gain can be set not so very large despite of an inaccurate robot dynamics model.

2. Problem Formulation

Consider a rigid robot manipulator system governed by the following dynamics

$$M[q(t)]\ddot{q}(t)+C[q(t),\dot{q}(t)]\dot{q}(t)+g[q(t),\dot{q}(t)]=\tau(t)+d(t) \quad (1)$$

where $q(t)\in\Re^n$ is the generalized joint variables, $M[q(t)]\in\Re^{n\times n}$ is the inertia matrix, $C[q(t),\dot{q}(t)]\dot{q}(t)\in\Re^n$ is the centripetal plus Coriolis force vector, $g[q(t),\dot{q}(t)]\in\Re^n$ represents the gravitational plus frictional force vector, $\tau(t)\in\Re^n$ is the joint control input vector, and $d(t)\in R^n$ is the unknown disturbance input vector which is assumed to be T-periodic, $d(t)=d(t+T)$.

When the robot is used to perform the same task repeatedly, the desired trajectories and the unknown feedforward compensation inputs can be specified using T-periodic functions as follows:

$$\Big(q_d(t),\dot{q}_d(t),\ddot{q}_d(t),\tau_d(t)\Big) = \Big(q_d(t+T),\dot{q}_d(t+T),\ddot{q}_d(t+T),\tau_d(t+T)\Big) \quad (2)$$

where the subscript d represents the desired trajectories and inputs, the desired input vector $\tau_d(t)$ is given by

$$\tau_d(t)=M[q_d(t)]\ddot{q}_d(t)+C[q_d(t),\dot{q}_d(t)]\dot{q}_d(t)+g[q_d(t),\dot{q}_d(t)]-d(t)$$

Now, let the control consist of a variable structure control input as the robust part and a feedforward input as the intelligent part

$$\tau(t) = \tau_{ff}(t) + \tau_{vs}(t) \tag{3}$$

where τ_{ff} is the feedforward learning control input and τ_{vs} is the variable structure feedback control input. The feedforward learning control input τ_{ff} compensates for the nonlinear coupling terms and the gravitational plus frictional force terms. The variable structure feedback control input τ_{vs} drives the tracking error on a switching surface and stabilizes the error dynamics given below. Following the similar line of the previous results (Kuc *et al.* 2000), one can get the following error dynamics of robotic system by linearizing the uncertain robotic system along the T-periodic desired trajectories (2) for $t \in [0, T]$:

$$M_d\ddot{e} + N_d\dot{e} + V_d e = \tau_{vs} + \tilde{\tau}_{ff} \tag{4}$$

where $\tilde{\tau}_{ff} = \tau_{ff} - \tau_d$, $e = q(t) - q_d(t)$, and

$$M_d = M[q_d(t)],\quad N_d = C_d + F_d,\quad V_d = D_d + C_d + G_d,$$

$$C_d = C[q_d(t), \dot{q}_d(t)],\quad F_d = \frac{\partial g}{\partial \dot{q}}\Big|_{(q_d,\dot{q}_d)}$$

$$D_d = \frac{\partial M}{\partial q}\Big|_{(q_d)}\ddot{q}_d,\qquad G_d = \frac{\partial g}{\partial q}\Big|_{(q_d,\dot{q}_d)}$$

Thus, our problem can be formulated as for the error dynamics (4) deriving a variable structure feedback control law τ_{vs} and a feedforward learning law τ_{ff} guaranteeing that the tracking error asymptotically converges to zero.

The following technical lemmas will be used to derive our main results:

Lemma 1: :(Desoer and Vidyasagar, 1975) Consider the following linear system

$$\dot{x}(t) = Ax(t) + Bu(t),\quad y(t) = Cx(t)$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^p$, and A, B and C are constant matrices with appropriate dimensions. Assume that $u(t)$ belongs to L^2 and the transfer function $C(sI-A)^{-1}B$ is stable and strictly proper. Then $y(t)$ is continuous and $y(t)$ converges to zero.

Lemma 2: : (Narendra and Annaswamy, 1989) If $y(t) \in L^2 \cap L^\infty$, and $\dot{y}(t)$ is bounded, then $y(t)$ converges to zero.

3. Main Results

Let the switching surface be

$$\sigma = e + \dot{e} = 0 \tag{5}$$

and let τ_{vs} be of the form

$$\tau_{vs}(t) = -\alpha\sigma(t) - \rho(t)\mathrm{Sgn}[\sigma(t)]$$

$$= -\begin{bmatrix} \alpha\sigma_1(t) + \rho(t)\mathrm{sign}(\sigma_1(t)) \\ \vdots \\ \alpha\sigma_n(t) + \rho(t)\mathrm{sign}(\sigma_n(t)) \end{bmatrix} \tag{6}$$

where $\mathrm{sign}(\cdot)$ is the signum function, α is a positive scalar,

$$\rho(t) = \lambda\|\dot{e}\| + \delta\|e\| + \epsilon \tag{7}$$

ϵ is a nonnegative scalar, and

$$\lambda = \max_{t\in[0,T]} \|M_d\|$$

$$\delta = \max_{t\in[0,T]} \Big(\|D_d\| + \|F_d\| + \|G_d\|\Big)$$

By introducing the switching feedback control law (6) to the error dynamics (4), one can get the following equation:

$$M_d\dot{\sigma} = -(C_d+F_d)\sigma + M_d\dot{e} - (D_d - F_d + G_d)e - \alpha\sigma + \tilde{\tau}_{ff} - \rho(t)\mathrm{Sgn}(\sigma) \tag{8}$$

By using the properties that $\dot{M}_d - 2C_d$ is skew-symmetric and the friction coefficient matrix is positive definite, i.e. $F_d = \partial g/\partial \dot{q}\,\big|_{(q_d,\dot{q}_d)} > 0$, one can show that if $\tilde{\tau}_{ff} = 0$ then the reachability condition is satisfied for all nonzero σ:

$$\begin{aligned} \frac{d}{dt}(\sigma^T M_d \sigma) &= 2\sigma^T M_d\dot{\sigma} + \sigma^T \dot{M}_d \sigma \\ &\leq -2\alpha\|\sigma\|^2 + 2\sigma^T\Big(M_d\dot{e} - (D_d - F_d + G_d)e\Big) - \rho(t)\|\sigma\| + 2\sigma^T\tilde{\tau}_{ff} \\ &\leq -2\alpha\|\sigma\|^2 - 2\epsilon\|\sigma\| + 2\sigma^T\tilde{\tau}_{ff} \;\leq\; -2\alpha\|\sigma\|^2 - 2\epsilon\|\sigma\| \;<\; 0 \end{aligned} \tag{9}$$

Remark 1: : The above inequality implies that for the case of inaccurate feedforward compensation a very large switching feedback control gain is required to guarantee sliding mode on the switching surface. Especially, if the feedforward compensation input is zero, then $\tilde{\tau}_{ff} = -\tau_d$ and to guarantee the reachability condition the switching feedback control gain $\rho(t)$ is required to be sufficiently large so that $\rho(t) \geq \|M_d\dot{e} - (D_d - F_d + G_d)e - \tau_d\|$. This demands incorporating an effective nonlinearity compensation algorithm into the variable structure feedback system in order to get good tracking performance while reducing the control efforts.

Now, let the feedforward learning control input τ_{ff} be updated by the following repetitive learning rule:

$$\tau_{ff}(t) = \tau_{ff}(t-T) - \beta\sigma(t-T) \tag{10}$$

where β is the positive learning gain such that $2\alpha > \beta > 0$, then one can establish the following theorem:

Theorem 1: *:* Considering the error dynamics (4) with the control law (3) which consists of the variable structure control law (6) and the feedforward learning law (10), if $2\alpha > \beta > 0$ then the tracking error e asymptotically converges to zero.

Proof : Define a Lyapunov functional as

$$V(t) = \sigma^T M_d \sigma + \frac{1}{\beta}\int_t^{t+T} \tilde{\tau}_{ff}^T \tilde{\tau}_{ff}\, d\nu \tag{11}$$

Then the time derivative of V along (4) is given by

$$\dot{V}(t) = 2\sigma^T M_d\dot{\sigma} + \sigma^T\dot{M}_d\sigma + \frac{1}{\beta}\tilde{\tau}_{ff}^T(t+T)\tilde{\tau}_{ff}(t+T) - \frac{1}{\beta}\tilde{\tau}_{ff}^T(t)\tilde{\tau}_{ff}(t) \tag{12}$$

By referring to the inequality (9) the above equation (12) can be reduced to

$$\dot{V}(t) \leq -2\alpha\|\sigma\|^2 - 2\epsilon\|\sigma\| + 2\sigma^T\tilde{\tau}_{ff}$$

$$+\frac{1}{\beta}\tilde{\tau}_{ff}^T(t+T)\tilde{\tau}_{ff}(t+T) - \frac{1}{\beta}\tilde{\tau}_{ff}^T(t)\tilde{\tau}_{ff}(t) \quad (13)$$

The learning rule (10) implies $\tilde{\tau}_{ff}(t+T) = \tilde{\tau}_{ff}(t) - \beta\sigma(t)$, thus (13) can be rewritten as

$$\dot{V}(t) \leq -(2\alpha - \beta)\|\sigma\|^2 - 2\epsilon\|\sigma\|$$

This implies that $\sigma \in L^2$ if $2\alpha > \beta$. After all, by using Lemma 1 we can conclude that the tracking error e converges to zero.

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Remark 2: : If we replace the control law (3) and the repetitive learning rule (10) with

$$\tau(t) = \mathrm{Sat}[\tau_{ff}(t)] + \tau_{vs}(t) \quad (14)$$

$$\tau_{ff}(t) = \mathrm{Sat}[\tau_{ff}(t-T)] - \beta\sigma(t-T) \quad (15)$$

where

$$\mathrm{Sat}(x) = \begin{bmatrix} \mathrm{Sat}(x_1) \\ \vdots \\ \mathrm{Sat}(x_n) \end{bmatrix}, \ \mathrm{Sat}(x_i) = \begin{cases} \tau^* & x_i > \tau^* \\ x_i & -\tau^* \leq x_i \leq \tau^* \\ -\tau^* & x_i < -\tau^* \end{cases}$$

and τ^* is a sufficiently large constant satisfying $\tau^* \geq \max_{t\in[0,T]} \|\tau_d(t)\|$, then by using Lemma 2 and the fact that

$$\Big(\mathrm{Sat}[\tau_{ff}(t)] - \tau_d(t)\Big)^T \Big(\mathrm{Sat}[\tau_{ff}(t)] - \tau_d(t)\Big)$$

$$\leq \Big(\tau_{ff}(t) - \tau_d(t)\Big)^T \Big(\tau_{ff}(t) - \tau_d(t)\Big)$$

one can show that $\sigma(t) \in L^2 \cap L^\infty$, $\tau_{ff}(t) \in L^\infty$, $\dot{\sigma}(t) \in L^\infty$, thus $\lim_{t\to\infty} \sigma(t) = 0$, and therefore e as well as $\dot{e}$ converges to zero.

Remark 3: : Instead of (10) the following repetitive learning rule can be used without losing the stability property

$$\tau_{ff}(t) = \tau_{ff}(t-T) - \beta\sigma(t) \quad (16)$$

For this case, β does not have to satisfy the constraint $2\alpha > \beta > 0$ and β has only to be positive.

Remark 4: : One may use the following bounded variable structure feedback control law instead of (6)

$$\tau_{vs}(t) = -\begin{bmatrix} \bar{\rho}_1 \mathrm{sign}(\sigma_1(t)) \\ \vdots \\ \bar{\rho}_n \mathrm{sign}(\sigma_n(t)) \end{bmatrix} \quad (17)$$

where $\bar{\rho}_i$ is a positive constant design parameter. The above bounded variable structure feedback control law (17) and the feedforward learning control input τ_{ff} (10) will at least locally stabilize the error dynamics (4).

Remark 5: : It should be noted that the initial resetting condition $e(kT) = \dot{e}(kT) = 0$ for $k = 0, 1, \cdots$ is not necessary in the proposed learning variable structure control laws.

4. Numerical Example

Consider a PUMA-type 2-Link manipulator whose dynamics can be written explicitly as (MARK W.Spong, M.Vidysagar 1989)

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (18)$$

where

$$l_1 = 0.5m, \ m_1 = 3Kg, \ l_2 = 0.5m, \ m_2 = 3Kg, \quad g = 9.8m/sec^2$$

$$M_{11} = (m_1 + m_2){l_1}^2 + m_2{l_2}^2 + 2m_2l_1l_2cos(q_2)$$

$$M_{12} = M_{21} = m_2{l_2}^2 + m_2l_1l_2cos(q_2)$$

$$M_{22} = m_2{l_2}^2$$

$$h = m_2l_1l_2sin(q_2)$$

$$C_{11} = -h\dot{q}_2, \quad C_{12} = -h(\dot{q}_1 + \dot{q}_2), \quad C_{21} = h\dot{q}_1, \quad C_{22} = 0$$

$$G_1 = g(m_1l_{c1} + m_2l_1)cos(q_1), \quad G_2 = gm_2l_{c2}cos(q_2)$$

For simplicity, we assume that $T = 1$, $q_1(k) = q_2(k) = \dot{q}_1(k) = \dot{q}_2(k)$ for $k = 0, 1, \cdots$, samplerate $= 0.001$ msec and

$$q_{d1} = q_{d2} = \frac{\pi}{4}\sin(2\pi t)$$

By referring to Theorem 1 one can obtain the following learning variable structure control law

$$\tau_1 = -5\sigma(t) - (\lambda\|\dot{e}\| + \delta\|e\| + \epsilon)\mathrm{Sgn}(\sigma) + \tau_{ff}$$

$$\tau_{ff}(t) = \tau_{ff}(t-1) - 5\sigma(t-1)$$

where $\sigma = q - q_d + \dot{q} - \dot{q}_d$. The initial feedforward input is given by $\tau_{ff}(t) = 0$ for all $t \in [0 \ \ 1]$. Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 show the simulation results with $\lambda = 50$, $\delta = 50$ and $\epsilon = 0.01$.

As shown in Figures 1,2,3,4,5,6 and 7 one can see that our method effectively handle the problem of controlling a rigid robot manipulator performing the same task repeatedly.

5. Conclusions

In this paper, we considered the problem of designing a simple learning variable structure control law for a rigid robot manipulator performing the same task repeatedly. In the proposed learning variable structure control law, a switching feedback control law is used as usual to drive the tracking error to a switching surface despite of uncertainties, and a very simple feedforward learning rule is incorporated to compensate for the nonlinear coupling terms and the gravitational plus frictional force terms while reducing the control efforts. Since our learning rule for feedforward compensation is very simple and it only uses the past control sequence, the proposed learning variable structure control law has advantages in the computational aspect and it does not require accurate information about the robot. By using numerical simulations for controlling a SCARA-type robot manipulator, we showed the effectiveness of our method.

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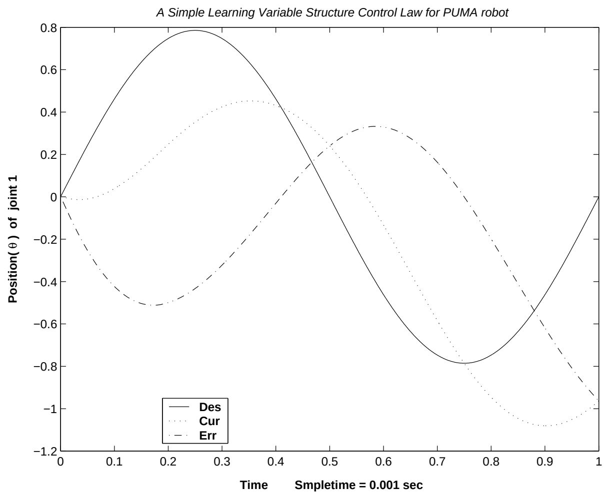

Fig. 1. Desired and actual output trajectories of Joint1 (Solid:Desired; Dotted:Actual; Dashdot:Error)

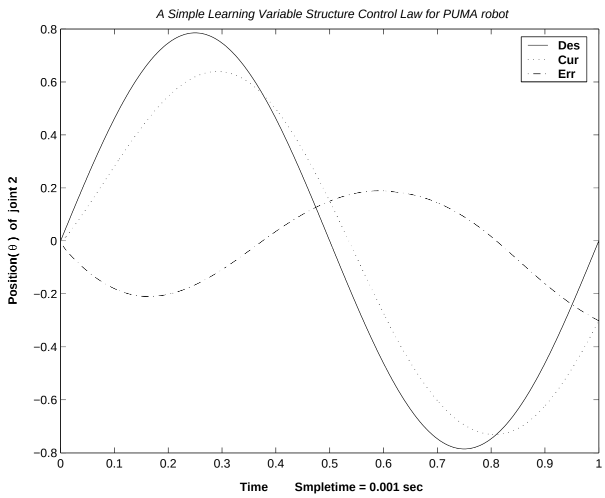

Fig. 2. Desired and actual output trajectories of Joint2 (Solid:Desired; Dotted:Actual; Dashdot:Error)

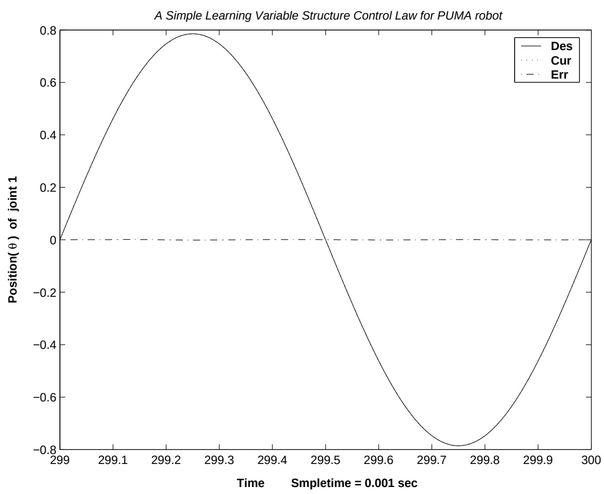

Fig. 3. Desired and actual output trajectories of Joint1 (Solid:Desired; Dotted:Actual; Dashdot:Error)

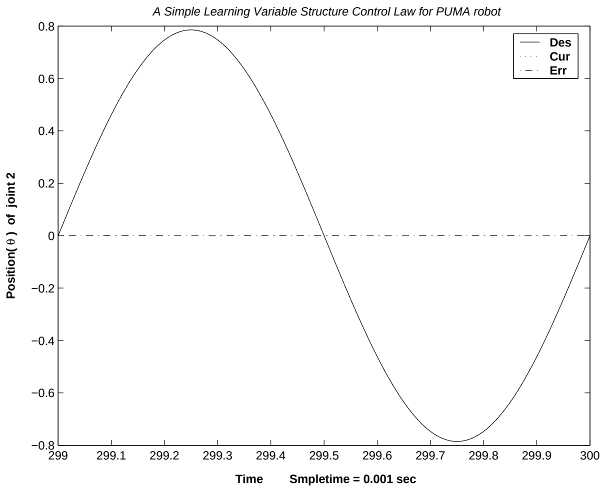

Fig. 4. Desired and actual output trajectories of Joint2 (Solid:Desired; Dotted:Actual; Dashdot:Error)

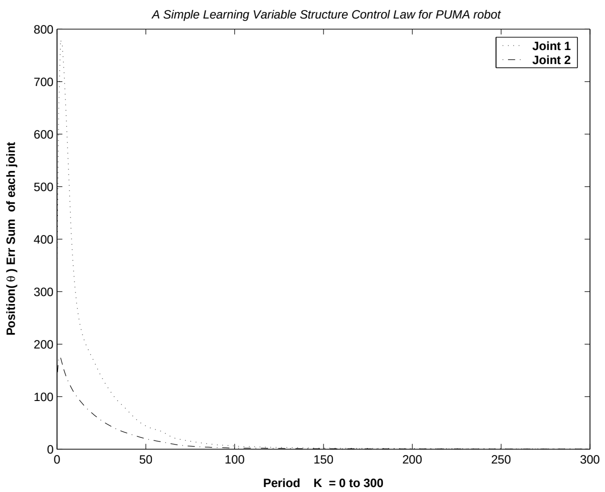

Fig. 5. The sum of position error per period of each joint (Dotted:Joint1; Dashdot:Joint2)

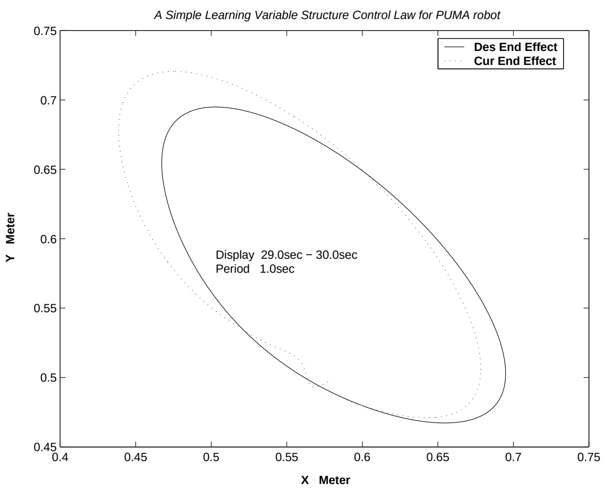

Fig. 6. Desired and actual output trajectories of end effect (Solid:Desired; Dotted:Actual; Dashdot:Error)

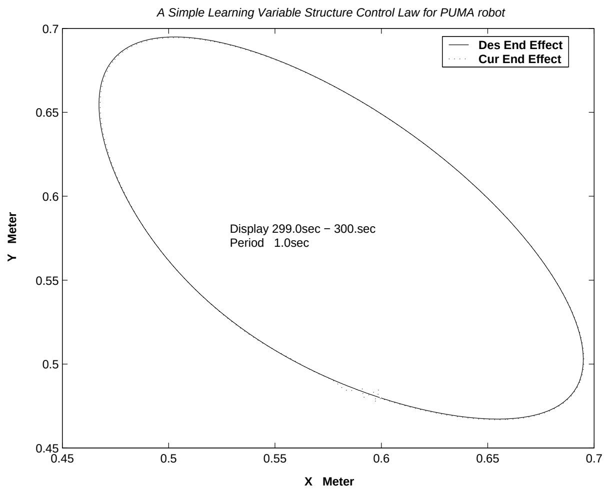

Fig. 7. Desired and actual output trajectories of end effect (Solid:Desired; Dotted:Actual; Dashdot:Error)

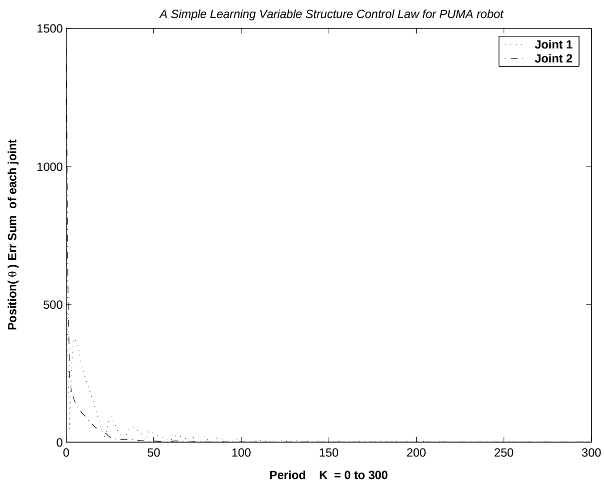

Fig. 8. The sum of position error per period of each joint (Dotted:Joint1; Dashdot:Joint2)

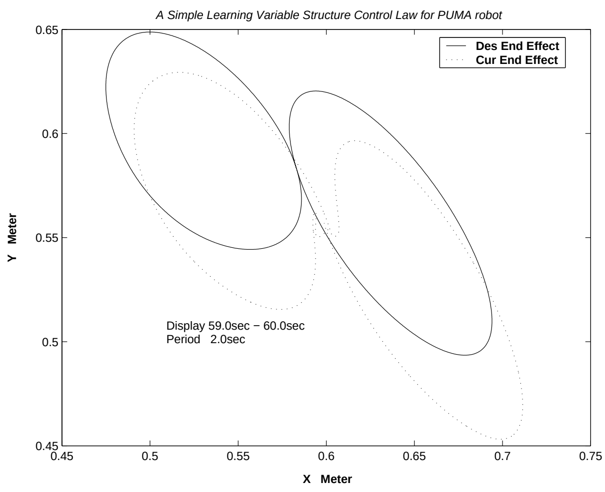

Fig. 9. Desired and actual output trajectories of end effect (Solid:Desired; Dotted:Actual; Dashdot:Error)

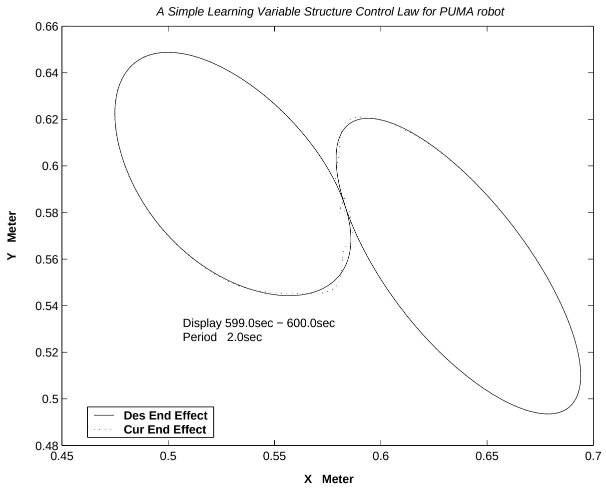

Fig. 10. Desired and actual output trajectories of end effect (Solid:Desired; Dotted:Actual; Dashdot:Error)

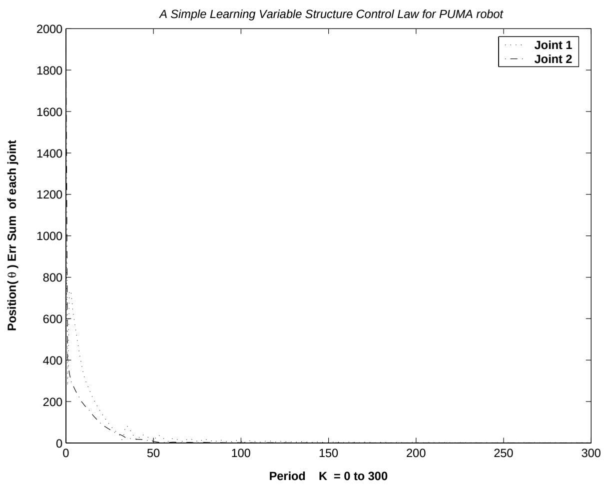

Fig. 11. The sum of position error per period of each joint (Dotted:Joint1; Dashdot:Joint2)