Control of Real-Time Systems with Random Time-Delays

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Abstract: This paper considers the optimal control problem in real-time control systems with random time-delays. It proposes an algorithm which uses the linear quadratic (LQ) control method and a dedicated technique to compensate for the time-delay effects. Since it is assumed that the time-delays are unknown but the probability distribution of the delays are known *a priori*, the algorithm considers the mean value of the time-delays as a nominal value for random delay compensation. An example is given to show the performance of the proposed algorithm, where an inverted pendulum system is controlled over a controller-area network (CAN). Simulation results show that the proposed algorithm provides good performance results. It is shown that our algorithm is comparable to existing algorithms in both computation cost and performance.

Keywords: Random time-delay, optimal control, digital control, inverted pendulum, CAN

1. INTRODUCTION

As today's control systems become more complex, the use of an operating system for scheduling of given tasks becomes common in control applications. Multiplexing of tasks by scheduling is necessary particularly in a distributed control system where multiple control tasks may require the CPU time at the same time. However, such scheduling inevitably brings about undesirable time-delays which are time-varying and random in general. Moreover, when the control loops are closed over a communication network as is common in distributed control systems, network-induced time-delays occur during the transmission of information in the control loops. These delays are also time-varying and random.

In digital control systems, as long as the time-delays are much smaller than the sampling period, they may be disregarded in the controller design phase. However, in a system with a short sampling period, small time-delays can cause performance degradation and even instability when appropriate compensation is not given to the system [1]. The relationship between the sampling period and time-delays and its effect on system performance is well illustrated in [2]. Therefore, time-delays should be accounted for when the system requires fast real-time control.

A large number of works for time-delay systems have been reported in the literature. Mostly, constant time-delay cases have been dealt with [3]. As distributed systems controlled over a communication network become common in control application areas, random time-delay cases become an important problem since the network induces random time-delays.

The control of systems with random time-delays is relatively new. Several groups have carried out works relevant to these areas. Zhang and Branicky [4] analyzed the stability of networked control systems where random time-delays arise in the network during information transmission. They made a criterion for the system to be stable under mild conditions. Luck and Ray [5] considered a control problem posed in systems with random time-delays. Their method uses an observer to predict foregoing states. The method is equivalent to those that use a buffer to make random time-delays constant. If the worst-case delay is identified, sufficient amount of buffer can be used to eliminate the randomness of the delays.

Nilsson *et al.* [6] proposed an optimal controller to compensate for random time-delays. They assumed that some delays are known *a priori* but others are not. Using the dynamic programming approach, they derived an optimal control law which accounts for both types of delays. Their controller uses the mean value of unknown time-delays to compensate for the effects. It is shown that their optimal controller using the mean value can outperform other control schemes such as Luck and Ray's [5] and those neglecting time-delay effects. However, as indicated by themselves, their scheme requires complicated calculation for a state feedback gain.

Robust control theory also gives a solution to the control problem of systems with random time-delays. Wang *et al.* [7] regarded time-delays as structured multiplicative uncertainties. Since the time-delays can be parameterized, well-known robust control algorithms such as the μ-analysis method can be used for a system with random time-delays.

Those methods mentioned above, however, have limitations in either computation cost or performance. There arise the need for an efficient method in both computation cost and performance. This paper proposes an algorithm to compensate for the time-delay effects. In particular, random time-delays are considered, which are common in distributed real-time systems.

The paper is organized as follows. Section 2 formulates the problem of the paper. It presents a linear model of a discrete-time system with time-delays. Section 3 reviews discrete-time linear quadratic (LQ) control theory. In Section 4, we propose the control algorithm. The algorithm uses the LQ control method and a dedicated technique to compensate for the time-delay effects. An example is given in Section 5 to demonstrate the performance of the proposed algorithm. The example uses an inverted pendulum control system where the control loop is closed over a controller-area network (CAN). With simulation results, this section illustrates that the proposed algorithm provides good performance results. It is shown that our algorithm is comparable to existing methods in both computation cost and performance. Conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

To formulate the problem to be considered, some constraints are indispensable. Thus, throughout this paper, we make the following assumptions.

- The states and output of the system is sampled periodically.
- The time-delays are randomly varying. The probability distribution is known *a priori.*
- The time-delays are less than one sampling period.
- Past time-delay values are memorized so that the controller can use the stored values. However, the value of current time-delay is not known at the current sample time.

Consider a time-delay linear system

$$\dot{x}(t) = Ax(t) + Bu(t - \tau(t)) \tag{1}$$

where x is the system state, u is the input and $\tau(t)$ is the time-varying delay, and all matrices and vectors are assumed to have appropriate dimension. If we use ZOH (Zero Order Hold) for discretization of (1), we obtain the following discrete-time system model [8].

$$x(k+1) = \Phi x(k) + \Gamma_0 u(k) + \Gamma_1 u(k-1) \tag{2}$$

where
$$\Phi = e^{Ah}$$
$$\Gamma_0 = \int_0^{h-\tau_k} e^{As} ds\, B$$
$$\Gamma_1 = \int_{h-\tau_k}^{h} e^{As} ds\, B = e^{A(h-\tau_k)} \cdot \int_0^{\tau_k} e^{As} ds\, B$$

Here, h is the sampling period and τ_k is the time-delay at the k-th sample time, which is random, time varying, and less than one sampling period. The system configuration of (2) is shown in Fig. 1. Since the sensor-to-controller delays are usually known to the controller at current sample time, these delays are embedded in the controller-to-actuator delays which are unknown at the current time in general.

In this paper, only the time-delays less than one sampling period are treated. The case of time-delays longer than one sampling period can be modeled in a similar way; the model with the longer delays is also given in [8] in detail.

With this model and the aforementioned assumptions, we define the problem of the paper.

Problem Statement: Given the time-system model (2) and a performance index, find a control law to minimize the performance index.

3. DISCRETE-TIME LINEAR QUADRATIC CONTROL

The LQ design methodology is well established in both continuous-time and discrete-time linear systems. One can easily find this theory from textbooks regarding optimal

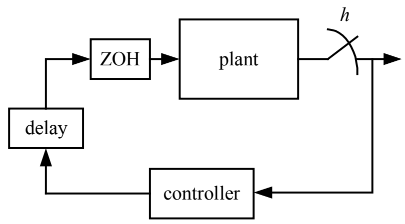

Fig. 1. System configuration.

control. Here, the discrete-time LQ design method presented in [8] is briefly reviewed.

Recall the discrete-time system model in (2) without the delay term as

$$x(k+1) = \Phi x(k) + \Gamma u(k) \tag{3}$$

where $\Gamma = \int_0^h e^{As} ds\, B$, and other parameters are same as in (2). If the initial state value $x(0)$ is given, the LQ problem is to determine the control sequence

$$u(0), u(1), \cdots, u(N-1)$$

that minimizes the cost function

$$\begin{aligned} J &= x^T(N) Q_0 x(N) \\ &+ \sum_{k=0}^{N-1} (x^T(k) Q x(k) + 2x^T(k) W u(k) + u^T(k) R u(k)) \end{aligned} \tag{4}$$

where Q, R are positive definite weighting matrices and Q_0 is a positive semi-definite matrix.

The solution of this optimal control problem is

$$u(k) = -L(k)x(k) \tag{5}$$

where $L(k) = (R + \Gamma^T S(k+1)\Gamma)^{-1}(\Gamma^T S(k+1)\Phi + W^T)$

and $S(k)=S^T(k) \geq 0$ is the solution of the Riccati equation

$$\begin{aligned} S(k) &= \Phi^T S(k+1)\Phi + Q - (\Phi^T S(k+1)\Gamma + W) \\ &\times (\Gamma^T S(k+1)\Gamma + R)^{-1}(\Gamma^T S(k+1)\Phi + W^T) \end{aligned} \tag{6}$$

which is solved iteratively backward from the final condition $S(N)=Q_0$. The condition for the existence of a unique solution is $S(N) \geq 0$ and $R + \Gamma^T S(k)\Gamma > 0$.

4. CONTROL WITH RANDOM TIME-DELAYS

4.1 Control Algorithm

Assume that the system (2) is controllable. Then, we get the following theorem.

Theorem 1: *If the system (2) is controllable, then* $\Gamma_0^T\Gamma_0$ *is nonsingular, i.e., its inverse exists.*

For the proof of this theorem, we need the following lemma.

Lemma 1 [9]: *Let A be an* $i \times j$ *matrix. Then*

$$\rho(AC) = \rho(A) \ \ \textit{and} \ \ \rho(DA) = \rho(A)$$

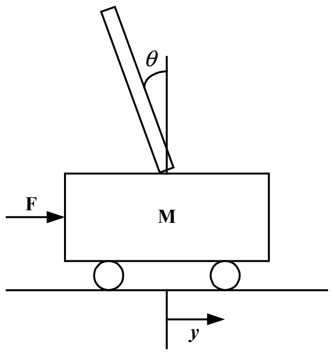

Fig. 2. Cart with an inverted pendulum.

for any $i\times i$ and $j\times j$ nonsingular matrices C and D, where $\rho(A)$ means the rank of A.

This lemma is derived from Sylvester's inequality theorem. Theorem 1 is proved using this lemma.

Proof of Theorem 1

If the system (2) is controllable, then there exists a controllability matrix

$$[\Gamma_0 \quad \Phi\Gamma_0 \quad \cdots \quad \Phi^{n-1}\Gamma_0]$$

which has a full rank. This implies that Γ_0 must have a full column or row rank. If Γ_0 has a full column or row rank, Γ_0^T has a full row or column rank, respectively. Therefore, from Lemma 1, $\Gamma_0^T\Gamma_0$ has a full rank and thus is nonsingular. **QED.**

With this theorem, we have derived a simple but useful time-delay compensation algorithm.

Let the non-delayed system use state feedback for control, i.e., $u'(k) = -Lx(k)$. Then, if the amount of time-delays is known *a priori*, the input term in (2) can be replaced by a state feedback term as

$$\Gamma_0 u(k) + \Gamma_1 u(k-1) = u'(k) = -\Gamma L x(k) \qquad (7)$$

where L is a state feedback gain for assigning the poles of the non-delayed system to desired locations. This equation indicates that the system (2) can be regarded as a non-delayed discrete-time system with state feedback.

When we adopt the concept of pseudo-inverse, we obtain the following control law.

$$u(k) = -L_1\, x(k) - L_2\, u(k-1) \qquad (8)$$

where $L_1 = (\Gamma_0^T\Gamma_0)^{-1}\Gamma_0^T\Gamma L$

$L_2 = (\Gamma_0^T\Gamma_0)^{-1}\Gamma_0^T\Gamma_1$

and Γ_0^T is multiplied to both sides to make the pseudo-inverse $(\Gamma_0^T\Gamma_0)^{-1}\Gamma_0^T$. Notice that according to Theorem 1, if the system is controllable, the pseudo-inverse always exists. Here, the notation for h and τ_k is omitted in the equations for the sake of simplicity.

This control algorithm compensates for the time-delay effects if the amount of time-delays is known. Therefore, if the control algorithm is used, a time-delay system modeled as in (2) can be considered as a conventional non-delayed system under the assumption that the amount of time-delays is known *a priori*. Instead of using pole-placement, one can use the LQ control method for the feedback gain L.

4.2 Unknown, Random Time-Delay Case

In the control algorithm in Section 4.1, it is assumed that the amount of time-delays is known *a priori*. But this is not the case when time-delays occur in communication networks in a distributed control system. In such a system, time-delays are random and only the probability distribution may be known. In particular, when the controller sends information to actuators, the controller does not know how long the time-delays are. This means that the controller must compensate for the time-delay effects by estimating the amount of time-delay occurring in the controller-to-actuator communication.

For an unknown, random time-delay case, using the mean value of time-delays for compensating for the time-delay effects is a viable solution under mild conditions. Nilsson *et al.* [6] proposed optimal controllers for systems with random time-delays, which uses the mean value of time-delays for compensation for the effects of unknown, random time-delays. Our approach is similar to that of Nilsson *et al.* in that it uses the mean value of time-delays for the compensation. Compared to the algorithm of Nilsson *et al.*, the computation of our algorithm is simple. Since the feedback gain matrix is calculated off-line for a non-delayed system, our algorithm can be implemented easily. The mean value is used as a nominal value for time-delay compensation and estimated from the probability distribution of time-delays obtained from experiments. Here, the nominal value means the time-delay value to be compensated for. Since the time-delays of concern are random and not known to the controller at current time, certain value must be selected as the representative (or nominal) value of those random time-delays.

In summary, our control law for unknown, random time-delays is given as

$$u(k) = -L_1(h, \mathrm{E}\{\tau_k\})\, x(k) - L_2(h, \mathrm{E}\{\tau_k\})\, u(k-1) \qquad (9)$$

where $\mathrm{E}\{x\}$ is the expected (or mean) value of x.

5. EXAMPLE

In this section, we demonstrate the performance of the proposed algorithm using an inverted-pendulum control example. Inverted pendulum systems are widely used in control theory for evaluation of designed controllers. Those systems require real-time control since balancing the unstable pendulum limits the control bandwidth. Thus, the effects of time-delays are significant in those systems. In this paper, we use an inverted pendulum system as an example of time-delay system control.

5.1 Inverted Pendulum

Consider a cart with a pendulum as shown in Fig. 2. The cart is controlled by the force F to balance the pendulum. The pendulum system is nonlinear, but we assume that the

Table 1. Parameters used in the inverted pendulum system

M	Mass of the cart	0.5 kg
m	Mass of the pendulum	0.5 kg
b	Friction of the cart	0.1 N/m/sec
l	Length to pendulum center of mass	0.3 m
I	Inertia of the pendulum	0.006 kg.m^2
g	Acceleration of gravity	9.8 m/sec^2

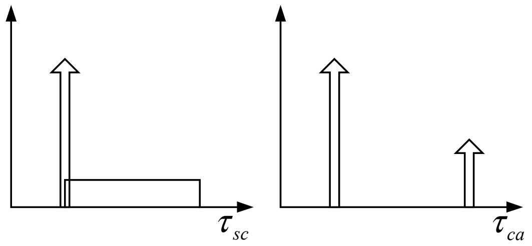

Fig. 3. Time-delay model of a CAN [11]. The left is the sensor-to-controller delay and the right is the controller-to-actuator delay.

movement of the pendulum lies in some bound so that the system can be modeled as a linear one.

The inverted pendulum system model being considered is given as

$$\dot{x}(t) = Ax(t) + Bu(t)$$

where the system matrices are [10]

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-(I+ml^2)b}{I(M+m)+Mml^2} & \dfrac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-mlb}{I(M+m)+Mml^2} & \dfrac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ \dfrac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \dfrac{ml}{I(M+m)+Mml^2} \end{bmatrix}$$

where M is the mass of the cart, m the mass of the pendulum, b the friction of the cart, l the length to the pendulum center of mass, g the acceleration of gravity, and I the inertia of the pendulum. The parameter values used in this example are listed in Table 1 [10]. The state of the system is chosen as

$$x(t) = [y(t),\ \dot{y}(t),\ \theta(t),\ \dot{\theta}(t)]^T$$

where y is the cart position and θ is the pendulum angle from the upright position.

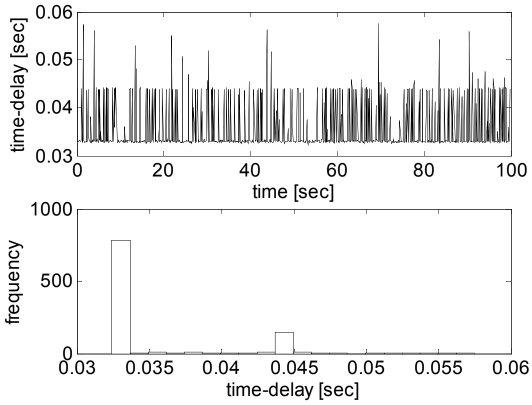

Fig. 4. Random time-delays in the example. The delays are generated 1000 times during 100 sec. The upper shows the time history of the delays and the lower shows the distribution of the delays, which are based on Nilsson's model [11].

5.2 Time-Delay Model

In this paper, we consider an inverted pendulum system where the control loop is closed over a CAN network. The network, of course, induces random time-delays in the loop. We use the time-delay model of a CAN network proposed by Nilsson [11]. He obtained from experiments the probability distribution of time-delays occurring in the CAN network.

The CAN protocol is a prioritized network protocol and uses the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CD) medium access method for resolving contention. Since the CAN protocol has data prioritization, the worst-case transmission time-delay can be estimated. This feature enables the CAN protocol to be used in control systems where deterministic latency in information transmission is needed. The worst-case delay time in transmission on the CAN protocol can be calculated by the algorithm proposed by Tindel *et al*. [12].

In the Nilsson model, the time-delays consist of sensor-to-controller and controller-to-actuator delays. He identified the model of each time-delay as shown in Fig. 3. The sensor-to-controller delay looks like a Dirac delta plus uniform distribution function in the probability distribution. The controller-to-actuator delay takes two Dirac delta functions under the assumption that only one load process is running. The reason for this delay model is well justified in [11], and other network-induced delay aspects are discussed therein.

5.3 Controller Evaluation

The proposed controller is evaluated in this example using the random time-delay model discussed in Section 5.3. We use the LQ method for determining the state feedback gain in (7). The parameters of the cost function (4) are chosen as

$$Q_0 = 0,\ \ R = 1,$$

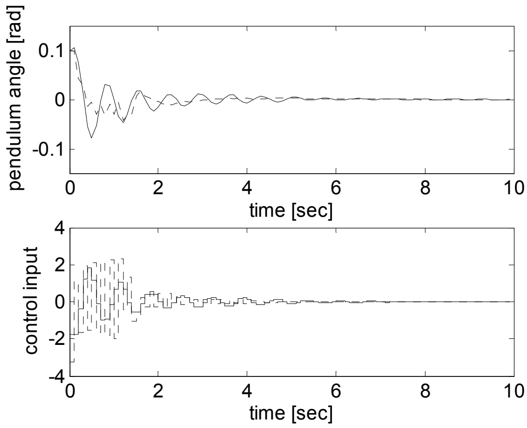

Fig. 5. Inappropriate selection of the nominal delay value. The solid lines are drawn for the no-delay case and the dotted lines are for the case where the nominal value is 43 msec.

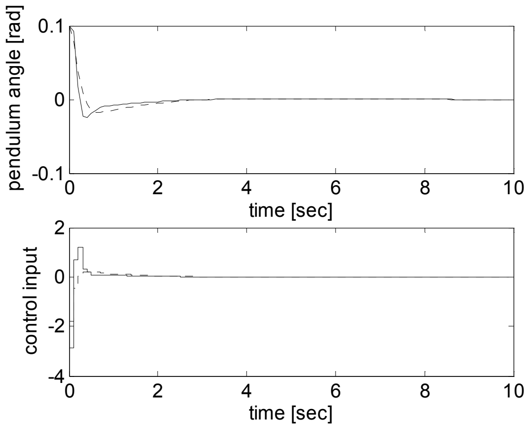

Fig. 6. Appropriate selection of the nominal delay value. The solid lines are drawn for an appropriate nominal value case. The value is 35 msec, the mean value of the time-delays. The dotted lines are for the case where the system is delay-free and the controller is a conventional LQ controller.

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Since the control purpose is to keep θ small (i.e., near zero), a large weighting factor is imposed on θ.

The cart-pendulum plant is discretized using the ZOH equivalent model in (2). The sampling period is chosen such that the time-delay effects are significant when the compensation is not given and still the rule of thumb for sampling period selection is preserved. According to [8], the rule of thumb is to choose the sampling period h such that $0.2 \le \omega h \le 0.6$, where ω is the natural frequency of the closed loop system. If the sampling period is too short, the time-delay effects are not significant. However, it is not practical in distributed control systems where many control tasks need the CPU time. If the sampling period is too long, the closed loop system will become unstable. Thus, an appropriate trade-off should be made. In this example, the sampling period is chosen as 100 msec.

The system configuration is as same as in Fig. 1. Thus, it is assumed that there arise random time-delays in the controller-to-actuator loop. The random time-delay model is based on the model of Nilsson [11]. The delays are emulated by a pseudo random generator in the MATLAB package. Fig. 4 shows the distribution of random time-delays in the control loop.

Using the discrete-time LQ control method described in Section 3 and the proposed algorithm, we simulate the inverted pendulum control system on several nominal delay values, using the MATLAB package. When the nominal value is not appropriately selected, the system performance will not be satisfactory as shown in Fig. 5. Two cases are simulated. In the first case, the nominal value is zero (solid lines) and in the second case, it is 43 msec (dotted lines). It is shown that either a no-delay or longer delay may not be a candidate for a nominal value for compensation. Therefore, with inappropriate nominal values, the proposed controller cannot provide satisfactory performance results.

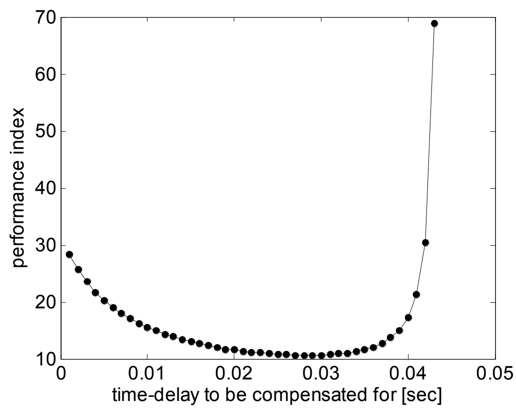

Fig. 7. Performance of the proposed controller. The mean value of performance results are obtained from 1000 simulations for each nominal delay.

Fig. 6 shows an appropriate selection case for the nominal value of random time-delays. With an appropriate nominal value (i.e., 35msec, the mean value of the time-delays), the proposed controller provides good performance results. The response of the proposed controller in the presence of time-delays in Fig. 4 (solid lines) is shown comparable to that of a conventional LQ controller with a delay-free system (dotted lines).

Performance results of the proposed controller is shown in Fig. 7, which are the mean values of the performance indices obtained from 1000 simulations for each nominal delay. This verifies that under a given time-delay distribution such as in Fig. 4, the mean value of the time-delays is a good nominal value for the proposed controller. However, an optimal value for the nominal value is less than the mean value by 5 msec in Fig. 7. The reason is not justified analytically yet.

6. CONCLUSIONS

This paper has proposed a control algorithm that compensates for random time-delay effects. The algorithm uses the LQ control method and a dedicated technique, which uses the concept of pseudo-inverse in the system dynamic model.

As an example, an inverted pendulum system is controlled over a CAN, where network-induced random time-delays occur in the control loop. Since it is assumed that the time-delays are unknown but the probability distribution of the delays are known *a priori*, the algorithm considers the mean value of the time-delays as a nominal value for random delay compensation.

Simulation results show that inappropriate selection of the nominal time-delay value (e.g., no delay or maximum delay) will not provide satisfactory performance. It may even destabilize the controlled system. It is also shown that the mean value (or those around the mean value) of the time-delays is acceptable for the nominal value for the time-delay compensation.

Our algorithm requires less computational power than others such as Nilsson *et al*. [6]. From the performance results, therefore, it is shown that the algorithm is comparable to others in both computational cost and performance. However, our algorithm assumes that the states are all available. This assumption is not practical or impossible in some cases. In addition to this drawback, the system parameters are assumed known, which may be a problem in practical situations. Future works include observer-based control and robust control to cope with the drawback, and analytical justification of the observation in Fig. 7 that an optimal value for the nominal value is less than the mean value.

REFERENCES

[1] A. Cervin and B. Lincoln, *Jitterbug Reference Manual*, Dep. Automatic Control, Lund Institute of Technology, Sweden, 2003; available at: http://www.control.lth.se/~lincoln/jitterbug/.

[2] F.-L. Lian, J. R. Moyne, and D. M. Tilbury, "Performance evaluation of control networks: Ethernet, ControlNet, and DeviceNet," *IEEE Contr. Syst. Mag*., vol. 21, no. 1, pp. 66-83, Feb. 2001.

[3] M. S. Mahmoud, *Robust Control and Filtering for Time-Delay Systems*, Marcel Dekker, Inc., 2000.

[4] W. Zhang, M. S. Branicky, and S. M. Phillips, "Stability of networked control systems," *IEEE Contr. Syst. Mag*., vol. 21, no. 1, pp. 84-99, Feb. 2001.

[5] R. Luck and A. Ray, "An observer-based compensator for distributed delays," *Automatica*, vol. 26, no. 5, pp. 903-908, 1991.

[6] J. Nilsson, B. Bernhardsson, and B. Wittenmark, "Stochastic analysis and control of real-time systems with random time delays," *Automatica*, vol. 34, no. 1, pp. 57-64, 1998.

[7] Z.-Q. Wang, P. Lundström, and S. Skogestad, "Representation of uncertain time delays in the H_∞ framework," *Int. J. Control*, vol. 59, no. 3, pp. 627-638, 1994.

[8] K. J. Åstrom and B. Wittenmark, *Computer-Controlled Systems*, Prentice-Hall, 1990.

[9] C.-T. Chen, *Linear System Theory and Design*, Holt-Saunders, 1984.

[10] X. Liu and A. Goldsmith, "Wireless communication tradeoffs in distributed control," to appear in the *Proc. of CDC 2003*; available at http://systems.stanford.edu/Publications.html.

[11] J. Nilsson, *Real-Time Control Systems with Delays*, PhD Thesis, Lund Institute of Technology, Sweden, 1998.

[12] K. Tindell, A. Burns, and A. J. Wellings, "Calculating controller area network (CAN) message response time," *Control Eng. Practice*, vol. 3., no. 8, pp. 1163-1169, Aug. 1995.