Vibration Control of an Axially Moving String: Inclusion of the Dynamics of Electro Hydraulic Servo System

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Abstract: In this paper, an active vibration control of a translating tensioned string with the use of an electro-hydraulic servo mechanism at the right boundary is investigated. The dynamics of the moving strip is modeled as a string with tension by using Hamilton's principle for the systems with changing mass. The control objective is to suppress the transverse vibrations of the strip via boundary control. A right boundary control law in the form of current input to the servo valve based upon the Lyapunov's second method is derived. It is revealed that a time-varying boundary force and a suitable passive damping at the right boundary can successfully suppress the transverse vibrations. The exponential stability of the closed loop system is proved. The effectiveness of the control laws proposed is demonstrated via simulations.

Keywords: Axially moving system, exponential stability, boundary control, hyperbolic partial differential equation, Lyapunov method.

1. INTRODUCTION

Examples of axially moving systems are found in various engineering areas: the steel strips in thin steel sheet production lines, cables, belts, and chains in power transmission lines, the magnetic tapes of recorders, the band saws, etc. The dynamics of these systems can be differently modeled depending on the length, flexibility, and control objectives of the system considered.

In axially moving systems, the transverse vibration of the moving thing often causes a serious problem in achieving good quality. It is also known that these vibrations are often caused by the eccentricity of a pulley, and/or an irregular speed of the driving motor, and/or a non-uniform material property, and/or environmental disturbances. Since quality standard as well as productivity in the production line is getting tighter, the vibration suppression with an active or semi-active method is nowadays seriously considered.

How to model an axially moving system, i.e., as a string equation or a belt equation or a beam equation, depends on the structure of the plant and the control scheme considered. Practically, it could be modeled as a string, or a belt, or a beam depending on where the actuator is actually inserted and whether the axial deformation is considered or not.

Diverse results on the dynamics, stability, and/or active/passive controls for axially moving systems have appeared in the literature (Carrier, 1945; Bapat & Srinivasan, 1967; Wickert & Mote, 1990; Wickert, 1992; Oshima *et al*., 1997; Pellican & Zirilli, 1998; Shahruz, 1998; Shahruz, 2000; Osstveen & Curtain, 2000). Particularly, Mote (1965) modeled the dynamics of a band saw, as an axially moving string, and investigated its instability with respect to the moving speed and excitation frequency of the saw. Wickert & Mote (1988) reported a passive control strategy, by changing its damping and stiffness, for axially moving continua. Morgul (1992) investigated a boundary control law that suppresses the lateral vibration of an Euler-Bernoulli beam, but in his work the beam was not axially moving. Laousy *et al.* (1996) investigated a boundary feedback stabilization method for a rotating body-beam system. Lee & Mote (1996) demonstrated an optimal boundary force control law that dissipates the vibration energy of an axially moving string. Fung *et al*. (1999a, 1999b) reported boundary control laws for linear and nonlinear strings, in which the dynamics of the actuator has been incorporated in the control law design. An adaptive control (Fung *et al.*, 2002b) and an optimal control (Fung *et al.*, 2002a) for an axially moving string were also investigated. For a translating linear beam, Lee & Mote (1999) investigated the wave characteristics and derived boundary control laws in terms of linear velocity, linear slope, and linear force. Li & Rahn (2000) investigated an adaptive vibration control for an axially moving linear beam by splitting the moving part into two subsystems, i.e., a controlled part and an uncontrolled part. Li *et al*. (2002) applied the control strategy in (Li & Rahn, 2000) to the linear string including experimental results. Fard & Sagatun (2001) focused on the exponential stabilization of a nonlinear beam, not axially moving, by a boundary control.

Contributions of this paper are the following: First, the actuator dynamics has been incorporated in the control law design and the final control law derived gives the specific current input to the hydraulic actuator. Second, the derived boundary control law requires two information: the strip slope at the right boundary and the damping coefficient of the actuator. Hence, once the damping coefficient is properly estimated in an actuator design stage using the parameter values of the system, the final control law depends only on the slope measurement. Therefore, the use of a slope sensor enables the implementation of the control law. Finally, the exponential stability of the closed loop system has been established.

2. EQUATIONS OF MOTION

Fig. 1 shows a schematic of the plant for analyzing the dynamics and deriving a boundary control law. The strip is assumed to travel at a constant speed. The left boundary is fixed, i.e., the left boundary itself doesn't have any vertical or longitudinal movements, but allows the axial movement of the strip. The right boundary allows the transverse displacement under a control force.

Let t be the time, x be the spatial coordinate along the longitude of motion, v be the axial speed of the strip, $w(x,t)$ be the transversal displacement of the strip at spatial coordinate x and time t, and L be the length of the strip.

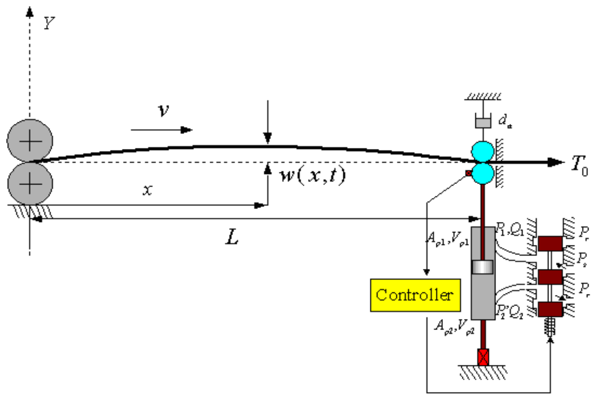


Fig. 1 An axially moving strip under the right boundary control force

Then, the absolute velocity at spatial coordinate x becomes

$$\vec{v} = vi + \frac{dw(x,t)}{dt} j = vi + \{w_t(x,t) + vw_x(x,t)\} j \quad (1)$$

where $(\cdot)_t = \partial(\cdot)/\partial t$ and $(\cdot)_x = \partial(\cdot)/\partial x$ denote the partial derivatives and $v = \partial x/\partial t$ has been used. Now, to derive the equations of motion, the Hamilton's principle for systems of changing mass (McIver, 1973) is utilized as follows:

$$\delta \int_{t_1}^{t_2} (T - U + W_{n.c.} + W_{r.b.}) dt = 0 \quad (2)$$

where T is the kinetic energy, U is the strain energy, $W_{n.c.}$ is the non-conservative work, $W_{r.b.}$ is the virtual momentum transport at the right boundary (no variation at the left boundary). The kinetic energy is

$$T = \frac{\rho A}{2} \int_0^L \{v^2 + (w_t + vw_x)^2\} dx + \frac{1}{2} m w_t^2(L,t) \quad (3)$$

where ρ is the mass per unit length, A is the cross sectional area, m is the mass of the actuator. The potential energy is

$$U = \int_0^L T_0 \varepsilon_x dx \quad (4)$$

where T_0 is a constant axial tension of the strip, ε_x is the strain. The term in (4) is due to the strip tension. If the infinitesimal distance dx is replaced by the infinitesimal length ds, the strain ε_x can be approximated as $\varepsilon_x \cong w_x^2/2$ (Benaroya, 1998). Then (4) is rewritten as follows:

$$U = \frac{T_0}{2} \int_0^L w_x^2 dx \quad (5)$$

The variations of (3) and (5) are

$$\delta T = \rho A \int_0^L (w_t + vw_x)(\delta w_t + v \delta w_x) dx + m w_t \delta w_t(L,t), \quad (6)$$

$$\delta U = T_0 \int_0^L w_x \delta w_x dx. \quad (7)$$

Also, the variations of the non-conservative work and the virtual momentum transport at the right boundary are

$$\delta W_{n.c.} = F_c \delta w(L,t) - d_a w_t(L,t) \delta w(L,t), \quad (8)$$

$$\delta W_{r.b.} = -\rho A v \{w_t(L,t) + vw_x(L,t)\} \delta w(L,t), \quad (9)$$

where d_a is the damping coefficient of the actuator and $F_c(t)$ is the control force.

The substitution of (6)-(9) into (2) yields the nonlinear equation of motion as follows:

$$\rho A w_{tt} + 2\rho A v w_{xt} - (T_0 - \rho A v^2) w_{xx} = 0. \quad (10)$$

The boundary conditions are

$$w(0,t) = 0, \quad w_x(0,t) = 0 \quad \text{and} \quad (11)$$

$$m w_{tt}(L,t) + d_a w_t(L,t) + T_0 w_x(L,t) = F_c(t). \quad (12)$$

Note that (10) is a partial differential equation representing the transverse motion, while (12) is an ordinary differential equation relating the strip motion at the right boundary and the control force.

Mote (1965) revealed that the strip moving speed v, to avoid a divergence of the solution, should be smaller than some critical speed given by

$$0 < v < v_{cr} = \sqrt{\frac{T_0}{\rho}}. \quad (13)$$

Hence, the satisfaction of (13) is also assumed in this paper. If using the parameter values in Table 1, $v_{cr} = \sqrt{T_0/\rho} = 35.33$ m/s. And the critical speed means the limit speed of the string with no vibration.

To do active control of the moving string, a hydraulic touch roll with a damper is attached at the end of the string. The rollers in the touch roll can rotate with smooth bearings, which allow the string to move axially without friction. But, the contact of the string with the roll is so tight, the displacement of the roll can be considered as the displacement of the string. As seen in Fig. 1, the control input to the system is the current to the electro-hydraulic servo valve. Hence, the dynamics of the touch roll together with the dynamics of the hydraulic servo valve have to be included in the control system design.

Regarding the touch roll as a second order mechanical system and including only the second order dynamics from the hydraulic system, the dynamics of the electro-hydraulic servo system is given by

$$\dot{x}_1 = x_2, \quad (14)$$

$$\dot{x}_2 = \frac{1}{m}\left(A_p x_3 - d_a x_2 - T_0 w_x\right), \quad (15)$$

$$\dot{x}_3 = -\alpha x_2 - \beta x_3 + (\gamma \sqrt{P_S - \mathrm{sgn}(x_4) x_3}) x_4, \quad (16)$$

$$\dot{x}_4 = -\frac{1}{\tau} x_4 + \frac{K}{\tau} u. \quad (17)$$

$$\alpha = 4 A_a \beta_e / V_t, \quad \beta = 4 C_{tm} \beta_e / V_t,$$

$$\gamma = 4 C_d \beta_e w_g / (V_t \sqrt{\rho_f}). \quad (18)$$

where $x_1 = w(L,t)$ is the piston position of the actuator, $x_2 = w_t(L,t)$ is the piston velocity of the actuator, $x_3 = P_L$ is the load pressure, $x_4 = x_v$ is the valve position, u is the input current to servo valve, P_s is the supply pressure, β_e is the effective bulk modulus, V_t is the actuator total volume, C_{tm} is the coefficient of leakage, C_d is the discharge coefficient, w_g is the spool valve area gradient, ρ_f is the fluid density, A_p is the cross-section area of the actuator.

3. BOUNDARY CONTROL LAW

In this section, a right boundary control law that suppresses the transverse vibration of the strip governed by (10)-(12), (16)-(17) is derived. The following lemmas are first stated.

Lemma 1: The total mechanical energy of the string,

$$V_{String} = \frac{\rho A}{2}\int_0^L (w_t + vw_x)^2 dx + \frac{T_0}{2}\int_0^L w_x^2 dx, \quad (19)$$

and the following function are equivalent:

$$V_B = \rho A\delta \int_0^L x w_x (w_t + vw_x) dx \quad (20)$$

$$\widetilde{V} = V_{String} + V_B. \quad (21)$$

That is, there exists constant $\delta > 0$ such that

$$(1 - C_1)V_{String} \le \widetilde{V} \le (1 + C_1)V_{String}, \quad (22)$$

where $0 < C_1 < 1$.

Proof:

$$V_B = \rho A\delta \int_0^L x w_x (w_t + vw_x) dx$$

$$= \rho A\delta L\left\{\frac{1}{\rho A}\frac{\rho A}{2}\int_0^L (w_t + vw_x)^2 dx\right\} + \rho A\delta L\left(\frac{1}{T_0}\frac{T_0}{2}\int_0^L w_x^2 dx\right)$$

$$\le \frac{\rho A\delta L}{\min(T_0, \rho A)} V_{String}. \quad (23)$$

Where

$$C_1 = \frac{\rho A\delta L}{\min(T_0, \rho A)}. \quad (24)$$

The substitution of (24) into (21) yields:

$$-C_1 V_{String} \le V_B \le C_1 V_{String}. \quad (25)$$

By add V_{String} both side of (25) it becomes

$$(1 - C_1)V_{String} \le \widetilde{V} \le (1 + C_1)V_{String}. \quad (26)$$

For V_{String} and $\widetilde{V}$ Equivalent, $1 - C_1 > 0$. By (24),

$$\delta < \frac{\min(T_0, \rho A)}{\rho A L}. \quad (27)$$

Then Lemma 1 is proved. ■

Let the error of the pressure and the error of the valve displacement are defined as

$$e_3 = x_3 - x_{3desired}, \quad (28)$$

$$e_4 = x_4 - x_{4desired}. \quad (29)$$

Now, with Lemma 1, the following Lyapunov function candidate is proposed:

$$V(t) = \widetilde{V} + V_{Actuator}, \quad (30)$$

where

$$V_{Actuator} = \frac{m}{2}\{w_t(L,t) + \psi(v + \delta L)w_x(L,t)\}^2 + \frac{1}{2}e_3^2 + \frac{1}{2}e_4^2, \quad \psi > 0. \quad (31)$$

Because the system involves a mass flow entering in and out at the boundaries, the net change of the total energy is the sum of the change in the control volume. The time derivative of the Lyapunov function candidate can derive by Reynolds translation theorem.

$$\frac{d}{dt}V(t) = V_t + vV_x\Big|_0^L. \quad (32)$$

Now, the total derivative (or the material derivative) of (30) is evaluated. First, the time derivative of $\widetilde{V}$ becomes

$$\frac{d}{dt}\widetilde{V}(t) = \frac{d}{dt}V_{String} + \frac{d}{dt}V_B. \quad (33)$$

By using (10) $\frac{d}{dt}V_{String}(t)$ can be derived

$$\frac{d}{dt}V_{String} = V_{t(Siring)}(t) + vV_{x(Srting)}(t)\Big|_0^L$$

$$= (T_0 - \rho A v^2)[w_x w_t]_0^L + \frac{v(T_0 - \rho A v^2)}{2}[w_x^2]_0^L$$

$$- \frac{\rho A v}{2}[w_t^2]_0^L + \frac{\rho A v}{2}[(w_t + vw_x)^2]_0^L + \frac{vT_0}{2}[w_x^2]_0^L$$

$$= T_0 w_x(L,t)w_t(L,t) + vT_0[w_x^2]_0^L. \quad (34)$$

And $\frac{d}{dt}V_B$ is

$$\frac{d}{dt}V_B = V_{t(B)}(t) + vV_{x(B)}(t)\Big|_0^L$$

$$= v\rho A\delta \int_0^L (x w_{xt} + x w_t w_{xx} + w_x w_t) dx$$

$$+ v^2 \rho A\delta \int_0^L x w_x w_{xx} dx + \rho A\delta \int_0^L x w_{xt} w_t dx$$

$$+ \delta \int_0^L x w_x (\rho A w_{tt} + 2\rho A v w_{xt} + \rho A v^2 w_{xx}) dx$$

$$+ v^2 \rho A\delta \int_0^L w_x^2 dx. \quad (35)$$

Lemma 2: Because $w(x,t)$ satisfies (11), the following equalities hold,

$$\int_0^L x w_{xt} w_t dx = \frac{L}{2}w_t^2(L,t) - \frac{1}{2}\int_0^L w_t^2 dx, \quad (36a)$$

$$\int_0^L x w_x w_{xx} dx = \frac{L}{2}w_x^2(L,t) - \frac{1}{2}\int_0^L w_x^2 dx, \quad (36b)$$

$$\int_0^L (x w_{xt} w_x + x w_{xx} w_t + w_x w_t) dx = [x w_x w_t]_0^L$$

$$= L w_x(L,t) w_t(L,t). \quad (36c)$$

Proof: The integration by parts yields all above equalities. ■

Now, by using Lemma 2, (33) is modified as follows:

$$\frac{d}{dt}\widetilde{V}(t) = T_0 w_x(L,t)w_t(L,t) + vT_0[w_x^2]_0^L + \rho A v\delta L w_x(L,t)w_t(L,t)$$

$$+ \frac{1}{2}(\rho A v^2 \delta L + T_0 \delta L)w_x^2(L,t) + \frac{\rho A v^2 \delta}{2}\int_0^L w_x^2 dx$$

$$+ \frac{\rho A\delta L}{2}w_t^2(L,t) - \frac{\rho A\delta}{2}\int_0^L w_t^2 dx - \frac{\delta T_0}{2}\int_0^L w_x^2 dx. \quad (37)$$

On the other hand, the time derivative of (31) becomes

$$\frac{d}{dt}V_{Actuator} = \{w_t(L,t) + \psi(v + \delta L)w_x(L,t)\}$$
$$\times\{m w_{tt}(L,t) + \psi m(v + \delta L)w_{xt}(L,t)\} + e_3\dot{e}_3 + e_4\dot{e}_4. \quad (38)$$

Let the right boundary control force $F_c(t)$ is defined as

$$F_c(t) = -\psi m(v + \delta L)w_{xt}(L,t). \quad (39)$$

Using (12), (39) can be written as

$$\frac{d}{dt}V_{Actuator} = \{w_t(L,t) + \psi(v + \delta L)w_x(L,t)\}$$
$$\times\{-d_a w_t(L,t) - T_0 w_x(L,t)\} + e_3\dot{e}_3 + e_4\dot{e}_4$$

$$= \{w_t(L,t) + \psi(v + \delta L)w_x(L,t)\}\{-d_a w_t(L,t) - T_0 w_x(L,t)\}$$
$$+ e_3(-\alpha x_2 - \beta x_3 + \gamma\sqrt{P_S - \mathrm{sgn}(x_4)x_3}\, x_{4desired})$$
$$+ e_3(\gamma\sqrt{P_S - \mathrm{sgn}(x_4)x_3}\, e_4 - \dot{x}_{3desired})$$

$$+e_4(-\frac{x_4}{\tau}+\frac{K}{\tau}u-\dot{x}_{4desired}). \tag{40}$$

The position of valve $x_{4desired}$ is defined as

$$x_{4desired}=\frac{1}{\gamma\sqrt{P_S-\mathrm{sgn}(x_4)x_3}}\{\alpha x_2+\beta x_3\}$$
$$+\frac{1}{\gamma\sqrt{P_S-\mathrm{sgn}(x_4)x_3}}\{\dot{x}_{3desired}-s_3e_3\}, \tag{41}$$

where $s_3>0$. The substitution of (41) into (40) yields:

$$\frac{d}{dt}V_{Actuator}=\{w_t(L,t)+\psi(v+\delta L)w_x(L,t)\}$$
$$\times\{-d_a w_t(L,t)-T_0 w_x(L,t)\}-s_3e_3^2+\gamma\sqrt{P_S-\mathrm{sgn}(x_4)x_3}\,e_3e_4$$
$$+e_4\left(-\frac{x_4}{\tau}+\frac{K}{\tau}u-\dot{x}_{4desired}\right) \tag{42}$$

Then, from (37) and (42), the total derivative of (30) becomes

$$\frac{d}{dt}V(t)=\frac{d}{dt}\left(\tilde{V}+V_{Actuator}\right)$$
$$\le T_0w_x(L,t)w_t(L,t)+vT_0\left[w_x^2\right]_0^L+\rho Av\delta Lw_x(L,t)w_t(L,t)$$
$$+\frac{1}{2}\left(\rho Av^2\delta L+T_0\delta L\right)w_x^2(L,t)+\frac{\rho Av^2\delta}{2}\int_0^L w_x^2dx$$
$$-\frac{\rho A\delta}{2}\int_0^L w_t^2dx-\frac{\delta T_0}{2}\int_0^L w_x^2dx-d_aw_t^2(L,t)$$
$$-d_a\psi(v+\delta L)w_x(L,t)w_t(L,t)-T_0w_x(L,t)w_t(L,t)$$
$$-T_0\psi(v+\delta L)w_x^2(L,t)\ -s_3e_3^2+\gamma\sqrt{P_S-\mathrm{sgn}(x_4)x_3}\,e_3e_4$$
$$+\frac{\rho A\delta L}{2}w_t^2(L,t)\ +e_4\left(-\frac{x_4}{\tau}+\frac{K}{\tau}u-\dot{x}_{4desired}\right). \tag{43}$$

Finally, the main theorem of this paper is stated as follows:

Theorem: Consider the system (10)-(12) and (16)-(17). Let the right boundary control force $F_c(t)$ and the damping coefficient of the actuator d_a in (12) be given, respectively, by

$$d_a\ge\frac{\rho A\delta L}{2}, \tag{44}$$

$$d_a\ge\frac{v\rho A\delta L}{\psi(v+\delta L)}=\frac{\rho A\delta L}{\psi(1+\delta L/v)}. \tag{45}$$

Then the range of d_a is

$$d_a\ge\max\left\{\frac{\rho A\delta L}{2},\frac{\rho A\delta L}{\psi(1+\delta L/v)}\right\}. \tag{46}$$

And

$$1-\psi<0, \tag{47}$$

$$\rho Av^2-T_0<0, \tag{48}$$

$$\frac{1}{2}\rho Av^2+T_0-T_0\psi<0. \tag{49}$$

Then we can choose the value of constant ψ and string area A as table 1. Also by the table 1, we can provide the control force $F_c(t)$. And the control input u which makes $\frac{d}{dt}V(t)$ as negative definite can archived.

$$u=\frac{\tau}{K}\left[\frac{x_4}{\tau}+\dot{x}_{4desired}-s_4e_4-\gamma\sqrt{P_S-\mathrm{sgn}(x_4)x_3}\,e_3\right]. \tag{50}$$

The explicit value of control input is

$$x_{4desired}=\frac{1}{\gamma\sqrt{P_S-\mathrm{sgn}(x_4)x_3}}\{\alpha\, w_t(L,t)+\beta x_3\}$$
$$+\frac{1}{\gamma\sqrt{P_S-\mathrm{sgn}(x_4)x_3}}(\dot{x}_{3desired}-s_3e_3), \tag{51}$$

where $s_4>0$.

4. STABILITY ANALYSIS

In this section, the exponential stability of string system which applied the control input (39) and the damping coefficient (46) is proved. The time derivative of Lyapunov function is expressed as,

$$\frac{d}{dt}V(t)\le X+Y. \tag{52}$$

Here

$$X=-\left(\frac{\rho Av\delta L}{\psi(v+\delta L)}-\frac{\rho A\delta L}{2}\right)w_t^2(L,t)$$
$$-\frac{\delta L}{2}\left(T_0-\rho Av^2\right)w_x^2(L,t)\ -s_3e_3^2-\frac{1}{\tau}e_4^2, \tag{53}$$

$$Y=-\frac{\rho A\delta}{2}\int_0^L w_t^2dx-\frac{\delta T_0}{2}\int_0^L w_x^2dx. \tag{54}$$

At first, X is

$$X=-\left(\frac{\rho Av\delta L}{\psi(v+\delta L)}-\frac{\rho A\delta L}{2}\right)w_t^2(L,t)$$
$$-\frac{\delta L}{2}\left(T_0-\rho Av^2\right)w_x^2(L,t)\ -s_3e_3^2-\frac{1}{\tau}e_4^2$$
$$\le-\min(C_2,C_3,C_4,C_5)\left[\frac{m}{2}\{w_t(L,t)+\psi(v+\delta L)w_x(L,t)\}^2\right]$$
$$-\min(C_2,C_3,C_4,C_5)\left(\frac{1}{2}e_3^2+\frac{1}{2}e_4^2\right)$$
$$=-\min(C_2,C_3,C_4,C_5)V_{Actuator}, \tag{55}$$

where,

$$C_2=\frac{1}{m}\left(\frac{\rho Av\delta L}{\psi(v+\delta L)}-\frac{\rho A\delta L}{2}\right), \tag{56a}$$

$$C_3=\frac{\delta L}{2\psi\, m(v+\delta L)^2}\left(T_0-\rho Av^2\right), \tag{56b}$$

$$C_4=2s_3,\quad C_5=\frac{\tau}{2}. \tag{56c, d}$$

Y is

$$Y=-\frac{\rho A\delta}{2}\int_0^L w_t^2dx-\frac{\delta T_0}{2}\int_0^L w_x^2dx$$
$$\le-\min(C_6,C_7,C_8)\left(\frac{T_0}{2}\int_0^L w_x^2dx+\frac{1}{2}\int_0^L(w_t+vw_x)^2dx\right)$$
$$=-\min(C_6,C_7,C_8)V_{String}, \tag{57}$$

where,

$$C_6=\frac{\rho A\delta}{2},\quad C_7=\frac{\delta T_0}{4v^2},\quad C_8=\frac{\delta}{2}. \tag{58 a, b, c}$$

By using (18), (50) expressed as

$$Y\le-\frac{\min(C_6,C_7,C_8)}{1+C_1}\left(V_{String}+V_B\right). \tag{59}$$

Then from (55) and (59), time derivative of Lyapunov function (52) can express as

$$\frac{d}{dt}V(t)\le X+Y$$

$$\le -\min(C_2, C_3, C_4, C_5)V_{Actuator}$$
$$-\frac{\min(C_6, C_7, C_8)}{1+C_1}\left(V_{String} + V_B\right)$$
$$= -\lambda V(t) \, . \qquad (60)$$

Here, $\lambda = \min\left(C_2,\ C_3, C_4, C_5, \frac{C_6}{1+C_1}, \frac{C_7}{1+C_1}, \frac{C_8}{1+C_1}\right)$. Then, the dynamics of the closed loop system is exponentially stable. Hence, it is seen that the total mechanical energy (19) of the strip decays exponentially, which again implies that all state variables decay exponentially in time.

5. IMPLEMENTATION OF THE CONTROL LAW

The implementation of (39), (46) requires two things: the design of control force $F_c(t)$ and the satisfaction of a damping coefficient d_a. Since the satisfaction of a damping coefficient is related to the design of an actuator, it must be answered ahead. Note that δ should satisfy (27). Because all terms in the right hand side and left hand side of (27) are already known, the range of the damping coefficient can be achieved. Once d_a is determined, δ is chosen as explained above. The implementation of $w_{xt}(L,t)$ in (39) can be achieved by backwards differencing of $w_x(L,t)$ measured at each step.

6. SIMULATIONS

To demonstrate the performance of the closed loop system, computer simulations using the finite difference scheme have been performed. The values used in simulations are tabulated in Table 1.

By (27) δ is calculated as follows:

$$\delta < \min\left\{\frac{9800000}{7850 \times 1.4 \times 0.0045}, \frac{1}{20}\right\} = 0.05 \, , \qquad (61)$$

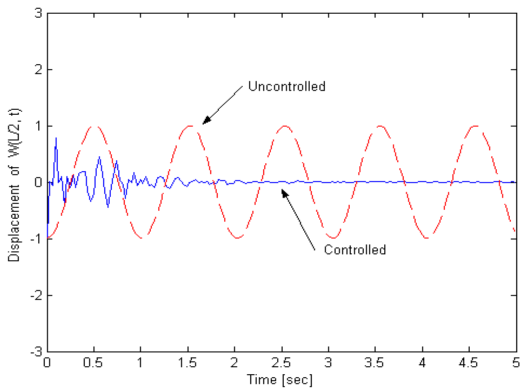
Let $\delta = 0.04$, see (Rao, 1990). Then, from (46), d_a is calculated. So let

$$\psi = 100 \, , \quad d_a = 3000 \, . \qquad (62)$$

Let the initial conditions be

$$w(x,0) = \sin(3\pi) \ \text{ m and } \ w_t(x,0) = 0 \ \text{ m/s.} \qquad (63)$$

Now, simulations using (61)-(63) have been performed for 5 seconds. Fig. 2 shows the transverse displacement at $x = L/2$, L . And Fig. 3 shows the control force and desired force at $x = L$, respectively. As shown in Fig. 3, the lateral vibration has been suppressed within 3 seconds. Fig. 4 shows the decay of the total mechanical energy of the strip in time. It shows that the total energy with control decays exponentially, while the energy without control sustains in time.



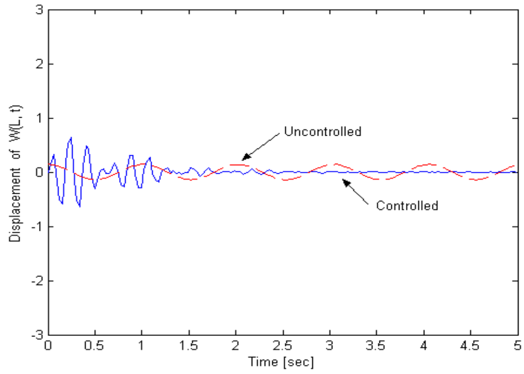


Fig. 2. The transverse displacement, $w(L/2,t)$, $w(L,t)$ with damping coefficient $d_a = 3000$.

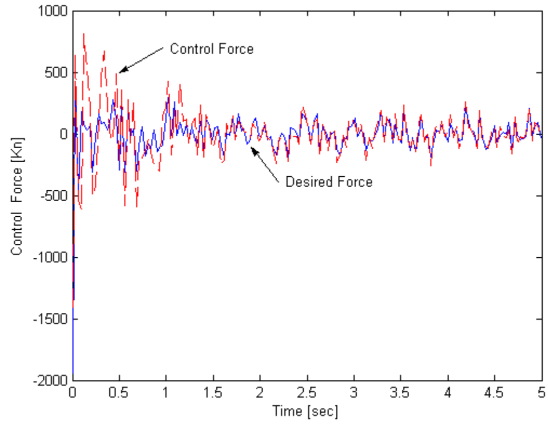


Fig. 3. The force of the system.

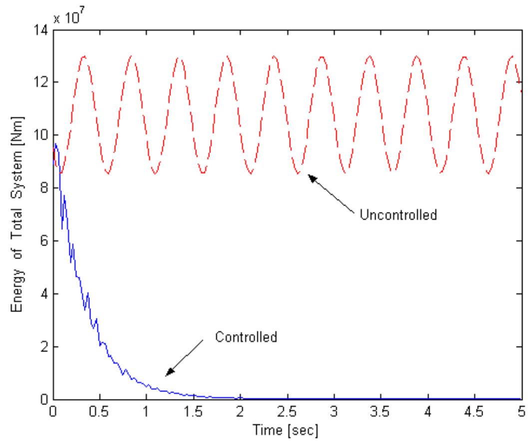


Fig. 4. The exponential decay of the total energy.

7. CONCLUSIONS

This paper investigates a boundary control law for suppressing the transverse vibration of an axially moving steel strip in the zinc galvanizing line. Because the strip was modeled as a string equation with a linear tension, the method developed is general in the sense that it can be applied to any system in a similar form. Once the range of damping coefficient is established, an appropriate value for δ can be selected for given system parameters. Achieving the exponential stability by using one sensor and one actuator is the main contribution of the algorithm proposed. And it provides the input, which can drive the actuator.

Table 1 The plant parameters used for simulations.

symbol	definition	values
A	cross section area	$1.4\times 0.0045\ m^2$
L	length of the controlled part	20 m
T_0	tension of the strip	9,800 kN
M	mass of the actuator	25 kg
v	strip moving speed	1.8 m/s
ρ	mass per unit area	7,850 kg/m^2
d_a	damping coefficient	3000 Ns/m

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