Discrete Representation Method of Nonlinear Time-Delay System in Control

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Abstract: A new discretization method for nonlinear system with time-delay is proposed. It is based on the well-known Taylor series expansion and the zero-order hold (ZOH) assumption. We know that a discretization of linear system can be obtained with the ZOH assumption and within the sampling interval. A similar line of thinking is available in nonlinear case. The mathematical structure of the new discretization method is explored and under the structure, the sampled-data representation of nonlinear system including time-delay is computed. Provided that the discrete form of the single input nonlinear system with time-delay is derived, this result is easily extended to nonlinear system with multi-input time-delay. For simplicity two inputs are considered in this study. It is enough to generalize that of multiple inputs. Finally, the time-discretization of non-affine nonlinear system with time-delay is investigated for apply all nonlinear system

Keywords: Nonlinear System, Time-Delay, Taylor-Series, Multi-Input, Non-affine System

1. Introduction

Currently, as the interest of the Networked control system or embedded control systems which require the communication between the systems and the complex computation function in controllers increase, time-delay becomes more important. In the design of the model-based controller, time-discretization of either the controller or system model is required. Thus, the development of the time-discretization method for continuous-time nonlinear system with time-delay is needed.

This paper expands the well-known time discretization of the linear time-delay system [1],[7] to nonlinear control systems with multiple time-delay in control. The suggested discretization method applies the Taylor series expansion according to the mathematical structure developed for the delay-free nonlinear system [4],[5]. The conventional numerical techniques such as the Euler, Runge-Kutta method have been used for obtaining the sampled-data representation for the original continuous-time system [1], which does not have delay. All these approaches need small time steps to meet the required accuracies and this may not be the case in control applications where slow sampling and large sampling periods are inevitably introduced due to physical and technical limitations [4],[5],[7]. Another interesting result for the discretization of the delay-free nonlinear system can be found in the Carleman linearization method [6]. However, this method is useful only for the low-dimensional system since its dimensionality increases rapidly with the continuous model's dimension and the degree of the desired accuracy.

There is no existing time-discretization method for continuous-time nonlinear system with time-delay, so this paper proposes a discretization of nonlinear system with input time delay.

In particular, this paper makes the following contribution:

- a new method is proposed for the discretization of affine/non-affine nonlinear systems with single-input/multi-input time-delay in the control. The resulting discrete system is finite-dimensional, thus allowing the direct application of existing nonlinear control system design techniques.

2. Nonlinear System with Single-Input Time-Delay in Control

Single-input nonlinear continuous-time control system can be expressed with the following state-space representation of the form:

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t)u(t-D) \tag{1}$$

where $x \in X \subset R^n$ is the vector of the states and X an open and connected set, $u \in R$ is the input variable and D is the system's constant time-delay. It is assumed that $f(x)$, $g(x)$ are real analytic vector fields on X.

An equidistant grid on the time axis with mesh $T = t_{k+1} - t_k > 0$ is considered, where $[t_k, t_{k+1}) = [kT, (k+1)T)$ is the sampling interval and T is the sampling period. It is also assumed that system (1) is driven by an input that is piecewise constant over the sampling interval, i.e. the zero-order hold (ZOH) assumption holds true:

$$u(t) = u(kT) \equiv u(k) = constant \tag{2}$$

for $kT \leq t < kT + T$. Furthermore, let:

$$D = qT + \gamma \tag{3}$$

where $q \in \{0, 1, 2, ...\}$ and $0 < \gamma \leq T$. Equivalently, the time-delay D is customarily represented as an integer multiple of the sampling period plus a fractional part of T [1], [7]. Under the ZOH assumption and the above notation, it is rather straightforward to verify that the "delayed" input variable attains the following two distinct values within the sampling interval [7]:

$$\begin{aligned} &if \quad kT \leq t < kT + \gamma, \\ &\quad u(t-D) = u(kT - qT - T) \equiv u(k-q-1) \\ &if \quad kT + \gamma \leq t < kT + T, \\ &\quad u(t-D) = u(kT - qT) \equiv u(k-q) \end{aligned} \tag{4}$$

Under above preliminaries, we discuss briefly the discretization of nonlinear system with delay-free and consider the time-discretization of single-input nonlinear system with time-delay. Initially, delay-free ($D = 0$) nonlinear control systems are considered with a state-space representation of the form:

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t))u(t) \tag{5}$$

Under the ZOH assumption and within the sampling interval, the solution of (5) is expanded in a uniformly convergent Taylor series [8] and the resulting coefficients can be easily computed by taking successive partial derivatives of the right hand-side of (5):

$$x(k+1) = x(k) + \sum_{\ell=1}^{\infty} \frac{T^\ell}{\ell!} \frac{d^\ell x}{dt^\ell}\bigg|_{t_k} = x(k) + \sum_{\ell=1}^{\infty} A^{[\ell]}(x(k), u(k)) \frac{T^\ell}{\ell!} \tag{6}$$

where $x(k)$ is the value of the state vector x at time $t = t_k = kT$ and $A^{[\ell]}(x, u)$ are determined recursively by:

$$\begin{aligned} A^{[1]}(x,u) &= f(x) + ug(x) \\ A^{[\ell+1]}(x,u) &= \frac{\partial A^{[\ell]}(x,u)}{\partial x}(f(x) + ug(x)) \end{aligned} \tag{7}$$

with $\ell = 1, 2, 3, ...$
When the delayed input is applied to the equation (6), the sampled data representation of the single input nonlinear system with delayed input can be derived. Results are following Equations[3].

- $kT \le t < kT + \gamma$

$$x(kT+\gamma) = x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), u(k-q-1)) \frac{\gamma^l}{l!} \tag{8}$$

- $kT + \gamma \le t < kT + T$

$$x(kT+T) = x(kT+\gamma) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma), u(k-q)) \frac{(T-\gamma)^l}{l!}. \tag{9}$$

where $x(k)$ and $A^{[l]}(x, u)$ are the same as above delay-free case. Thus, time-discretization of nonlinear control system with delayed input is computed by

$$x(k+1) = x(k) + \sum_{l=1}^{\infty} A^l(x(k), u(k-q-1)) \frac{\gamma^l}{l!} + \sum_{l=1}^{\infty} A^l$$
$$((x(k) + \sum_{i=1}^{\infty} A^i(x(k), u(k-q-1)) \frac{\gamma^i}{i!}), u(k-q)) \frac{(T-\gamma)^l}{l!} \tag{10}$$

3. Nonlinear System with Multi-Input Time-Delay inControl

As shown in [3], a discrete-time representation of nonlinear system with input time-delay can be obtained using Taylor series, and it has been extended to n dimensional system. Similarly the single input case can be expanded to multi-input case. The discretization method of general nonlinear system with multi-input delay is developed using Taylor series expansion. In this section the difference of input delays is within one sampling period or larger than one sampling period. A two-input nonlinear continuous-time control system can be expressed with the following state-space form.

$$\frac{dx(t)}{dt} = f(x(t)) + g_1(x(t))u_1(t - D_1) + g_2(x(t))u_2(t - D_2) \tag{11}$$

The delays of the inputs are derived from Eq.(3),

$$\begin{aligned} &u_1(t - D_1) - - > (D_1 = q_1T + \gamma_1) \\ &u_2(t - D_2) - - > (D_2 = q_2T + \gamma_2) \end{aligned} \tag{12}$$

where $q_2 = q_1 + n$, n is an integer. If the difference of input delays is less than one sampling period, the inputs are as follows;

- $u_1(t - D_1)$

$$\begin{cases} u(kT - q_1T - T) \equiv u(k - q_1 - 1) & \text{if } kT \le t < kT + \gamma_1 \\ u(kT - q_1T) \equiv u(k - q_1) & \text{if } kT + \gamma_1 \le t < kT + T \end{cases}$$

- $u_2(t - D_2)$

$$\begin{cases} \text{if } kT \le t < kT + \gamma_2 \\ \quad u(kT - (q_1 + n)T - T) \equiv u(k - (q_1 + n) - 1) \\ \text{if } kT + \gamma_2 \le t < kT + T \\ \quad u(kT - (q_1 + n)T) \equiv u(k - (q_1 + n)) \end{cases} \tag{13}$$

It is necessary to specify how the sampled data representation is affected if the multi-input with delay are applied in the system. We consider differences of delays about two cases. First, the delays are within one sampling period and second, the delays are larger than one sampling period.

3.1. The difference of input delays are less than one sampling period

In this section, the input delays are less than one sampling period. Since the difference of the input delays are less than one sampling period, all the inputs are located in the same sampling interval. There are two cases in the input delays such as $\gamma_1 \le \gamma_2$, $\gamma_2 < \gamma_1$, the values of q_1 and q_2 are identical in Eq.(12). There are three intervals in one sampling period such as $kT \le t < kT + \gamma_1$, $kT + \gamma_1 \le t < kT + \gamma_2$ and $kT + \gamma_2 \le t < kT + T$. The input values depend on the time interval. The input values and the corresponding states are obtained in the followings.

- $\gamma_1 \le \gamma_2$

- if t is in $kT \le t < kT + \gamma_1$
$u_1(t - D_1) = u_1(k - q_1 - 1)$, $u_2(t - D_2) = u_2(k - q_2 - 1)$

$$x(kT+\gamma_1) = x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), u_1(k - q_1 - 1), u_2(k - q_2 - 1)) \frac{\gamma_1^l}{l!} \tag{14}$$

The input values in the interval of $kT \le t < kT + \gamma_1$ are assigned by the input values of the one sampling period ahead.

For the second interval,
- if t is in $kT + \gamma_1 \le t < kT + \gamma_2$

$u_1(t - D_1) = u_1(k - q_1)$, $u_2(t - D_2) = u_2(k - q_2 - 1)$

$$x(kT+\gamma_2) = x(kT+\gamma_1) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma_1), u_1(k - q_1), u_2(k - q_2 - 1)) \frac{(\gamma_2 - \gamma_1)^l}{l!} \tag{15}$$

In the third interval,
- if t is $kT+\gamma_2 \le t < kT+T$,

$$u_1(t-D_1) = u_1(k-q_1), \quad u_2(t-D_2) = u_2(k-q_2)$$

$$x(kT+T) = x(kT+\gamma_2) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma_2), u_1(k-q_1), u_2(k-q_2))\frac{(T-\gamma_2)^l}{l!} \quad (16)$$

In the case that the delay of u_2 is larger than the delay of u1, the location of the γ_1, γ_2 are just switched and computational procedures are identical.

3.2. Difference of input delay is more than one sampling period and less than two sampling period

In the last section, it has been investigated that difference of the input delays is less than one sampling period. In this section, it will be derived that the difference of the input delays is larger than one sampling period and less than two sampling period with similar way as in the previous section. Since there is one sampling period difference in the input delays, two sampling period should be considered obtaining the input values and the state values. The number of intervals should be considered are six since there are two inputs and two sampling period must be considered. In this derivation, we assumed that a value of the k is fixed.
All the inputs and the state values are as follows;

- if $kT \le t < kT+\gamma_1$,
then $u_1 = u_1(k-q_1-1)$, $u_2 = u_2(k-q_1-2)$

$$x(kT+\gamma_1) = x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), u_1(k-q_1-1), u_2(k-q_1-2))\frac{\gamma_1^l}{l!} \quad (17)$$

- if $kT+\gamma_1 \le t < kT+\gamma_2$,
then $u_1 = u_1(k-q_1)$, $u_2 = u_2(k-q_1-2)$

$$x(kT+\gamma_2) = x(kT+\gamma_1) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma_1), u_1(k-q_1), u_2(k-q_1-2))\frac{(\gamma_2-\gamma_1)^l}{l!} \quad (18)$$

- if $kT+\gamma_2 \le t < kT+T$,
then $u_1 = u_1(k-q_1)$, $u_2 = u_2(k-q_1-1)$

$$x(kT+T) = x(kT+\gamma_2) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma_2), u_1(k-q_1), u_2(k-q_1-1))\frac{(T-\gamma_2)^l}{l!} \quad (19)$$

- if $kT+T \le t < kT+T+\gamma_1$,
then $u_1 = u_1(k-q_1)$, $u_2 = u_2(k-q_1-1)$

$$x(kT+T+\gamma_1) = x(kT+T) + \sum_{l=1}^{\infty} A^l(x(kT+T), u_1(k-q_1), u_2(k-q_1-1))\frac{\gamma_1^l}{l!} \quad (20)$$

- if $kT+T+\gamma_1 \le t < kT+T+\gamma_2$,
then $u_1 = u_1(k-q_1+1)$, $u_2 = u_2(k-q_1-1)$

$$x(kT+T+\gamma_2) = x(kT+T+\gamma_1) + \sum_{l=1}^{\infty} A^l (x(kT+T+\gamma_1), u_1(k-q_1+1), u_2(k-q_1-1))\frac{(\gamma_2-\gamma_1)^l}{l!} \quad (21)$$

- if $kT+T+\gamma_2 \le t < kT+2T$,
then $u_1 = u_1(k-q_1+1)$, $u_2 = u_2(k-q_1)$

$$x(kT+2T) = x(kT+T+\gamma_2) + \sum_{l=1}^{\infty} A^l(x(kT+T+\gamma_2), u_1(k-q_1+1), u_2(k-q_1))\frac{(T-\gamma_2)^l}{l!} \quad (22)$$

From the sections 3.1 and 3.2 , the nonlinear discretization equation can be derived for the delayed two-input nonlinear system. The Eq. (14), (17) are same and the Eq. (20) are derived from Eq. (17) simply by replacing k by $k+1$ since they are located in second sampling period. Similarly, Eq. (15), (18) are in the first sampling period and Eq. (21) is located in second sampling period, the input values and the states values are obtained similar way. Also the Eq. (16), (19), (22) are obtained in similar way. The input and state values are expressed in same method no matter where the input is located. Therefore, equations for nonlinear system with two-input time-delay can be obtained as follows depends on the interval of time t.

- if $kT \le t < kT+\gamma_1$,
then $u_1 = u_1(k-q_1-1)$, $u_2 = u_2(k-(q_1+n)-1)$

$$x(kT+\gamma_1) = x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), u_1(k-q_1-1), u_2(k-(q_1+n)-1))\frac{\gamma_1^l}{l!} \quad (23)$$

- if $kT+\gamma_1 \le t < kT+\gamma_2$,
then $u_1 = u_1(k-q_1)$, $u_2 = u_2(k-(q_1+n)-1)$

$$x(kT+\gamma_2) = x(kT+\gamma_1) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma_1), u_1(k-q_1), u_2(k-(q_1+n)-1))\frac{(\gamma_2-\gamma_1)^l}{l!} \quad (24)$$

- if $kT+\gamma_2 \le t < kT+T$,
then $u_1 = u_1(k-q_1)$, $u_2 = u_2(k-(q_1+n))$

$$x(kT+T) = x(kT+\gamma_2) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma_2), u_1(k-q_1), u_2(k-(q_1+n)))\frac{(T-\gamma_2)^l}{l!} \quad (25)$$

where $k = 0, 1, 2, ...$ and $q_2 = q_1 + n$.

3.3. The general equations

From the section 3.1 and section 3.2 the general discretization method of nonlinear system with multi-input time-delay can be derived. That is, the general discretized nonlinear equation for the delayed multi-input system can be obtained as follows :

- if $kT \le t < kT+\gamma_1$

$$x(kT+\gamma_1) = x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), u_1(k-q_1-1), \cdots, u_n(k-q_n-1))\frac{\gamma_1^l}{l!} \quad (26)$$

$$\vdots$$

- if $kT+\gamma_i \le t < kT+\gamma_{i+1}$ where $1 \le i \le$ n-1

$$x(kT+\gamma_{i+1}) = x(kT+\gamma_i) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma_i), u_1(k-q_1), \cdots, u_i(k-q_i), u_{i+1}(k-q_{i+1}-1), \cdots, u_n(k-q_n-1)) \frac{(\gamma_{i+1}-\gamma_i)^l}{l!} \quad (27)$$

$\vdots$

- if $kT+\gamma_n \le t < kT+T$

$$x(kT+T) = x(kT+\gamma_n) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma_n), u_1(k-q_1), \cdots u_{n-1}(k-q_{n-1}), u_n(k-q_n)) \frac{(T-\gamma_n)^l}{l!} \quad (28)$$

4. Non-Affine Nonlinear System with Time-Delay in Control

The equations of the single input non-affine nonlinear control system are as follows [10];

$$\dot{x} = f_0(x) + g_1(x)u + g_2(x)u^2 + \cdots + g_l(x)u^l \text{ and } \dot{x} = f(x,u) \quad (29)$$

where $x \in R^n$ is the state, $u \in R$ is the control input,

$$f_0 : R^n \to R^n, \quad g_i : R^n \to R^n, \quad i = 1, 2, \cdots, l$$
and $f : R^n \times R \to R^n$ are smooth mappings.

The non-affine system has nonlinear control inputs, whereas the affine system has linear control input. Furthermore, the non-affine nonlinear system also can be discretized using Taylor series expansion. The mathematical structure of the discretized non-affine nonlinear system is the same as the affine case since the input u is assumed to be constant in the sampling interval in this research. All the partial differentials used in the affine case also can be used in this non-affine case. Thus the related equations are as follows;

$$\begin{aligned} x(k+1) &= x(k) + \sum_{\ell=1}^{\infty} \frac{T^\ell}{\ell!} \frac{d^\ell x}{dt^\ell}\bigg|_{t_k} \\ &= x(k) + \sum_{\ell=1}^{\infty} A^{[\ell]}(x(k), u(k)) \frac{T^\ell}{\ell!} \end{aligned} \quad (30)$$

$$\begin{aligned} \dot{x} &= f_0(x) + g_1(x)u + g_2(x)u^2 + \cdots + g_l(x)u^l \\ &= A^{[l]}(x,u) = f(x,u) \\ \ddot{x} &= \dot{f_0}(x)\dot{x} + \dot{g_1}(x)u\dot{x} + \dot{g_2}(x)u^2\dot{x} + \cdots + \dot{g_l}(x)u^l\dot{x} \\ &= (\dot{f_0}(x) + \dot{g_1}(x)u + \dot{g_2}(x)u^2 + \cdots + \dot{g_l}(x)u^l)\dot{x} \\ &= A^{[2]}(x,u) = \frac{\partial A^{[1]}(x,u)}{\partial x} f(x,u) \\ &\vdots \end{aligned} \quad (31)$$

The generalized coefficients are represented as follows;

$$\begin{aligned} A^{[1]}(x,u) &= f(x,u) \\ A^{[\ell+1]}(x,u) &= \frac{\partial A^{[\ell]}(x,u)}{\partial x} f(x,u) \end{aligned} \quad (32)$$

Now we will consider time-discretization of nonaffine nonlinear system with delayed input.
Single-input non-affine nonlinear systems with input delay can be expressed in the following state-space representation of the form:

$$\frac{dx(t)}{dt} = f(x(t), u(t-D)) \quad (33)$$

From the ZOH assumption and equation (30) through (32), we obtain as follows;

- $kT \le t < kT+\gamma$

$$x(kT+\gamma) = x(kT) + \sum_{l=1}^{\infty} A^l(x(kT), u(k-q-1)) \frac{\gamma^l}{l!} \quad (34)$$

- $kT+\gamma \le t < kT+T$

$$x(kT+T) = x(kT+\gamma) + \sum_{l=1}^{\infty} A^l(x(kT+\gamma), u(k-q)) \frac{(T-\gamma)^l}{l!} \quad (35)$$

Also, when the Taylor series expansion is applied to each subinterval, the sampled-data representations for non-affine nonlinear system are identical to the equations (8) and (9). Finally, the discretization method using Taylor series expansion can be used for the non-affine nonlinear system.

5. Simulations

The performance of the suggested method is evaluated in this section through computer simulation of the nonlinear system. These numerical experiments are performed for a fixed truncation order: various input time-delays and fixed sampling periods. Throughout this example the truncation order is chosen as $N = 3$ for all simulations since the simulation results show that truncation orders greater than 3 do not significantly improve the accuracy.

5.1. Affine system

Consider a simplified model of maneuvering an automobile shown in Fig.1. The middle of the axis linking the front wheels has position $(x_1, x_2) \in R^2$, while the rotation of this axis is given by the angle x_3. If x_1, x_2 related with rolling are directly controlled with input u_1 and the state x_3 related with rotation is directly controlled by u_2, the governing nonlinear differential equation can be obtained as followings[2];

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \sin x_3 \\ \cos x_3 \\ 0 \end{bmatrix} u_1(t-D_1) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_2(t-D_2) \quad (36)$$

Three different sets of inputs will be applied onto the system. Delayed inputs u_1 and u_2 are in the same sampling period that is the simplest case and two delayed inputs are not in the same sampling interval. The difference of the input delays is one sampling period and two sampling period will be considered in the latter case. All the inputs are step functions with magnitudes $u_1 = 1$ and $u_2 = 2.5$. The simulation result of the first case is shown in Table 1.
All the simulations, the initial conditions are assumed to be $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 30°$ and the sampling

period(T) is assumed to be 0.001s. The delays of the first case are 0.0005 for u_1 and 0.0008 for u_2, respectively the simulation results are shown in Table 1 as mentioned above. In this case, the inputs u_1 and u_2 lie in the same sampling period. The numerical differences for state x_1 range from -0.8×10^{-5} to 0.7×10^{-5} and those for state x_2 range from -0.7×10^{-5} to 0.6×10^{-5}. In the second case, delay of inputs are $D_1 = 0.0005$ and $D_2 = 0.0018$. The difference between delay of u_1 and delay of u_2 is about one sampling period. The numerical differences for state x_1 range from -2×10^{-4} to 0.6×10^{-4} and the differences for state x_2 range from -2×10^{-4} to 0. In third case, delay of inputs are $D_1 = 0.0005$ and $D_2 = 0.0028$. The difference of each delay is about two sampling period. The numerical differences for state x_1 lie in the range from -0.6×10^{-4} to 1.6×10^{-4} and -0.07×10^{-4} to 2×10^{-4} for state x_2.

5.2. Non-Affine System

Now, we consider the non-affine nonlinear system with time-delay. The sampling period is fixed with 0.01sec and also, three different delay cases are examined. The system is considered as a single-input nonlinear system [9]

$$\begin{aligned} \dot{x_1} &= -x_1^3 + x_1 \exp^{x_2} u(t-D)^2 \\ \dot{x_2} &= x_2^2 u(t-D) \end{aligned} \quad (37)$$

The above equation is globally asymptotically stabilized by the smooth state feedback control law

$$u = -\frac{x_2^3}{1 + x_1^2 \exp^{x_2}} \quad (38)$$

The initial conditions are $x_1(0) = 1.0$ and $x_2(0) = -1.0$. The system runs for 100 seconds and the responses are compared with the Matlab results. In this simulations, the delays are 0.005, 0.015, 0.025 seconds.

Fig.2 shows the state error responses computed using the Matlab solver and the proposed method when the sampling period is $T = 0.01$ and the input time-delay is $D = 0.005$ ($q = 0$); input time-delay is smaller than the sampling period. The numerical differences between the Matlab and the Taylor method for state x_1 lies in the range -0.0006 to 0.0003, and -0.0022 to 0.0001 for state x_2. In second case, sampling period is $T = 0.01$ and delay is $D = 0.015$ ($q = 1$). The numerical differences for state x_1 are from -0.0012 to 0.0003 and the differences for state x_2 are from -0.0035 to 0.0001. Similarly the system is simulated for $T = 0.01$ and delay $D = 0.025$ ($q = 2$). In this third case, the numerical differences for state x_1 range form -0.002 to 0.0001 and those for state x_2 range from -0.005 and 0.0001.

The above numerical experiments for various combinations of the time-delay and the sampling-period demonstrate that the proposed Taylor series expansion method discretizes a nonlinear system with input time-delay quite accurately.

6. Conclusions

A new approach is proposed for the derivation of a discrete-time representation of a nonlinear control system with time-delay. It is based on the Taylor series expansion method.

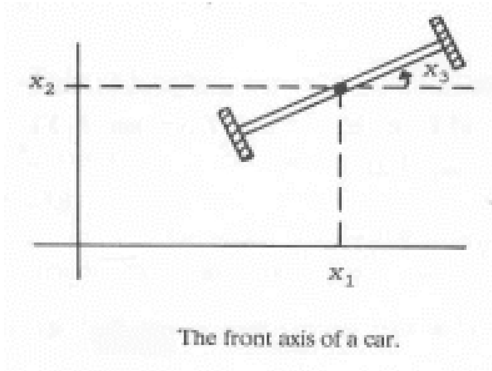

Fig. 1. The front axis of automobile

In particular, the proposed discretization method relies on a Taylor series expansion for the solution of the continuous-time system and the ZOH assumption. The mathematical structure of the new discretization scheme is explored. The resulting time-discretization provides a finite-dimensional representation for nonlinear systems with time-delay enabling the possible application of existing nonlinear controller design techniques. As applications of time-delay nonlinear control systems proliferate, such a transformation becomes an indispensable tool of a control designer's repertoire. The performance of the proposed method is evaluated by computer simulations. These simulations demonstrate the accuracy of the proposed discretization method. Extensions of the proposed approach to systems with state and/or output delays is feasible, and it will be the subject of future publication.

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Table 1. The responses of system in the simplest case($T = 0.001s$, $D_1 = 0.0005$, $D_2 = 0.0008$)

Time step	Matlab (x_1)	Taylor (x_1)	Matlab (x_2)	Taylor (x_2)
200	0.1377	0.1377	0.1414	0.1414
400	0.3268	0.3268	0.1997	0.1997
600	0.5208	0.5208	0.1603	0.1602
800	0.6721	0.6721	0.0326	0.0.0326
1000	0.7436	0.7436	-0.1518	-0.1518
1200	0.7180	0.7180	-0.3481	-0.3481
1400	0.6014	0.6014	-0.5080	-0.5080
1600	0.4224	0.4224	-0.5924	-0.5924
1800	0.2248	0.2248	-0.5807	-0.5807
2000	0.0570	0.0570	-0.4757	-0.4757

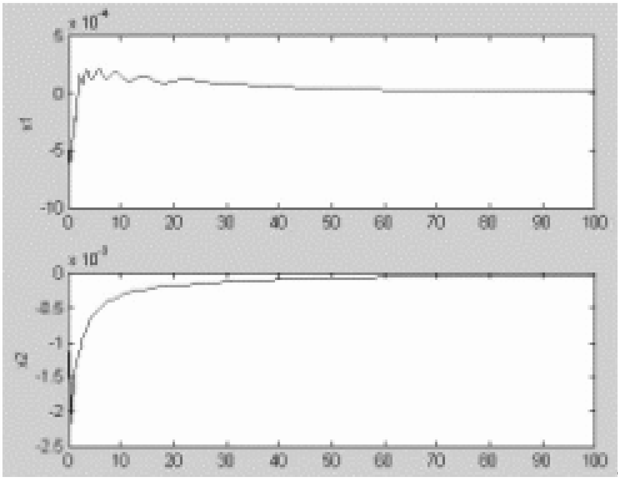

Fig. 2. State error response for the first case(x:time(s), y:error)

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