Stable Intelligent Control of Chaotic Systems via Wavelet Neural Network

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Abstract: This paper presents a design method of the wavelet neural network based controller using direct adaptive control method to deal with a stable intelligent control of chaotic systems. The various uncertainties, such as mechanical parametric variation, external disturbance, and unstructured uncertainty influence the control performance. However, the conventional control methods such as optimal control, adaptive control and robust control may not be feasible when an explicit, faithful mathematical model cannot be constructed. Therefore, an intelligent control system that is an on-line trained WNN controller based on direct adaptive control method with adaptive learning rates is proposed to control chaotic nonlinear systems whose mathematical models are not available. The adaptive learning rates are derived in the sense of discrete-type Lyapunov stability theorem, so that the convergence of the tracking error can be guaranteed in the closed-loop system. In the whole design process, the strict constrained conditions and prior knowledge of the controlled plant are not necessary due to the powerful learning ability of the proposed intelligent control system. The gradient-descent method is used for training a wavelet neural network controller of chaotic systems. Finally, the effectiveness and feasibility of the proposed control method is demonstrated with application to the chaotic systems.

Keywords: Direct adaptive control, Wavelet neural network, Chaotic systems, Gradient-descent method, Discrete-type Lyapunov Stability theorem

1. INTRODUCTION

In the last few decades, chaos has received increasing attention in various areas such as mathematics, engineering, physics, biology, and economics. One attractive topic concerning chaos is chaos control, which is needed to prevent a chaotic system from becoming unstable or being trapped in performance-degraded situations due to the unpredictability and irregularity of chaos.

In 1990, Ott et al. proposed the so-called OGY method, by which the chaotic phenomenon of a dynamical system can be stabilized by a small perturbation of an accessible system parameter when the chaotic orbit approaches a periodic orbit near a saddle point. Since then, a number of successful control methods and techniques for controlling chaotic systems have been developed (see, for example, [1-4]). Among these chaos control techniques, the conventional control techniques such as feedback control, optimal control, and robust control were introduced to control the chaotic systems, and these kinds of techniques confirmed the effectiveness of chaos control [5,6]. But most of these techniques can be applied to control chaotic systems when the exact or at least the approximate mathematical model for chaotic systems is available. To overcome this shortage of them, the direct/indirect adaptive control methods can be used for controlling chaotic systems. For example, Park et al. presented a generalized predictive control method based on an ARMAX model for the control for discrete-time chaotic systems [7].

On the other hand, the intelligent control techniques based on neural networks and fuzzy logic are developed to control chaotic nonlinear systems [8]. Even though these intelligent control strategies have shown the effectiveness especially for unknown chaotic system, they have some drawbacks, which come from their own inherent characteristics. Therefore, the intelligent control techniques using the wavelet transform (WT), which has an excellent time-frequency analysis ability, are introduced [9]. Wavelets have been applied successfully to multi-scale analysis and synthesis, time-frequency signal analysis in signal processing, and function approximation. And, wavelets are well suited to approximate functions with local nonlinearities and fast variations because of their intrinsic properties of finite support and self-similarity. As a result, wavelet theory can have useful applications in nonlinear control system design..

In this paper, we propose the design method of the wavelet neural network (WNN) based controller using direct adaptive control method to deal with a stable intelligent control of chaotic systems. An intelligent control system that is an on-line trained WNN controller based on direct adaptive control method with adaptive learning rates is proposed to control chaotic systems whose mathematical models are not available. The adaptive learning rates are derived in the sense of discrete-type Lyapunov stability theorem, so that the convergence of the tracking error can be guaranteed in the closed-loop system. Finally, in order to evaluate the performance of our controller, we apply the proposed method to the continuous-time chaotic system.

2. DIRECT ADAPTIVE CONTROL SYSTEM

The direct adaptive control system is shown in Fig. 1, in which the stable intelligent control system comprises a WNN controller and an on-line training mechanism with adaptive learning rates.

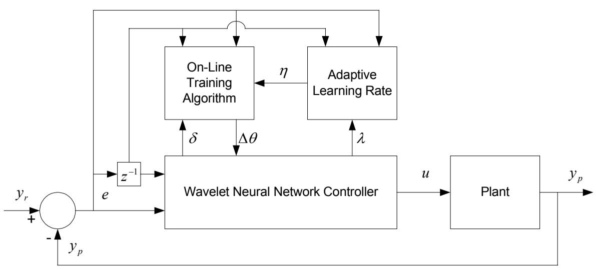

Fig. 1 Block diagram of the direct adaptive control system

2.1 Wavelet neural network controller

The theory of wavelets was first proposed by Mallat in the field of multi-resolution analysis (MRA) [10]. A three-layer WNN shown in Fig. 2, which is comprised of an input node (the i layer), a wavelet node (the j layer), and an output node (the k layer), is adopted to implement the proposed WNN controller. The control problem is to design the WNN controller so that the plant output can track any desired output. To achieve this control objective, define the tracking error $e = y_r - y_p$, in which y_r represents a desired output. The inputs of the WNN controller are e and $e(z^{-1})$, where z^{-1} is a time delay; the output of the WNN controller is the control input u.

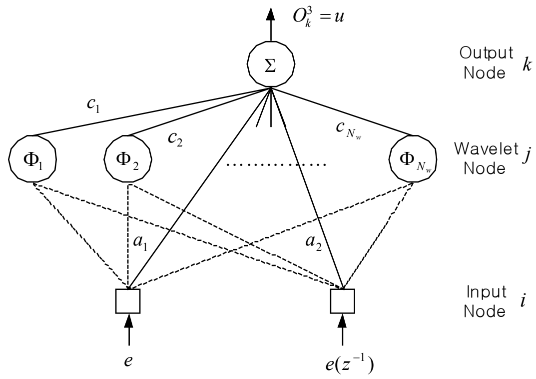

Fig. 2 Configuration of the WNN

For every node i in the input layer, the input and output value are represented as:

$$
\begin{aligned}
I_i^1 &= x_i \\
O_i^1 &= I_i^1 = x_i
\end{aligned}
\tag{1}
$$

where, $i = 1,2$, $x_1 = e$, and $x_2 = e(z^{-1})$.

A family of wavelets is constructed by translations and dilations performed on a single fixed function called the mother wavelet. For the wavelet layer, the input and output value of each node are represented as:

$$
\begin{aligned}
I_j^2 &= O_i^1 = x_i \\
O_j^2 &= \Phi_j = \prod_{i=1}^{2} \phi(z_{ij}), \quad with\ z_{ij} = \frac{x_i - m_{ij}}{d_{ij}}
\end{aligned}
\tag{2}
$$

where, $j = 1,2,\cdots,N_w$, N_w is the total number of wavelets. The translation factor m_{ij} and the dilation factor d_{ij} are real numbers ($d_{ij} > 0$). And, we choose the first derivative of a Gaussian function as a mother wavelet:

$$
\phi(x) = -x\exp\left(-\frac{1}{2}x^2\right) \tag{3}
$$

Furthermore, the single node k in the output layer is labeled as $\sum$, which computes the overall output as the summation of all input signals.

$$
\begin{aligned}
I_k^3 &= c_j O_j^2 \text{ and } a_i O_i^1 \\
O_k^3 &= u = \sum_{j=1}^{N_w} c_j O_j^2 + \sum_{i=1}^{2} a_i O_i^1 = \sum_{j=1}^{N_w} c_j \Phi_j + \sum_{i=1}^{2} a_i x_i
\end{aligned}
\tag{4}
$$

where, a_i is connection weight between input nodes and output nodes, and c_j is connection weight between wavelet nodes and output nodes.

2.2 On-line training algorithm

The central part of the training algorithm for a WNN controller concerns how to recursively obtain a gradient vector in which each element in the training algorithm is defined as the derivative of a cost function with respect to a parameter of the network. This is done by means of the chain rule, and the method is generally referred to as the back-propagation learning rule, because the gradient vector is calculated in the direction opposite to the flow of the output of each node. To describe the on-line training algorithm of the WNN controller using the supervised gradient descent method, first the cost function is defined as follows:

$$
E = \frac{1}{2}e^2 = \frac{1}{2}(y_r - y_p)^2 \tag{5}
$$

where, y_p is the output value of the plant and y_r is desired output value.

The connection weights between input nodes and output nodes are updated according to the following equation:

$$
\begin{aligned}
a_i(N+1) &= a_i(N) + \Delta a_i(N) \\
&= a_i(N) - \eta_a \frac{\partial E}{\partial a_i(N)} \\
&= a_i(N) - \eta_a \frac{\partial E}{\partial u}\frac{\partial u}{\partial a_i(N)} \\
&= a_i(N) + \eta_a \cdot \delta \cdot x_i
\end{aligned}
\tag{6}
$$

where, $\delta = -\frac{\partial E}{\partial u} = -\frac{\partial E}{\partial e}\frac{\partial e}{\partial y_p}\frac{\partial y_p}{\partial u}$, η_a is the learning rate of the parameter a_i.

The training of the connection weights between wavelet nodes and outputs nodes is calculated by:

$$
\begin{aligned}
c_j(N+1) &= c_j(N) + \Delta c_j(N) \\
&= c_j(N) - \eta_c \frac{\partial E}{\partial c_j(N)} \\
&= c_j(N) - \eta_c \frac{\partial E}{\partial u}\frac{\partial u}{\partial c_j(N)} \\
&= c_j(N) + \eta_c \cdot \delta \cdot \Phi_j
\end{aligned}
\tag{7}
$$

where, η_c is the learning rate of the parameter c_j.

And, the training of the translation parameters is calculated by:

$$
\begin{aligned}
m_{ij}(N+1) &= m_{ij}(N) + \Delta m_{ij}(N) \\
&= m_{ij}(N) - \eta_m \frac{\partial E}{\partial m_{ij}(N)} \\
&= m_{ij}(N) - \eta_m \frac{\partial E}{\partial u}\frac{\partial u}{\partial m_{ij}(N)} \\
&= m_{ij}(N) - \eta_m \cdot \delta \cdot \frac{c_j}{d_{ij}} \cdot \frac{1 - z_{ij}^2}{z_{ij}} \cdot \Phi_j
\end{aligned}
\tag{8}
$$

where, η_m is the learning rate of the translation parameters of the mother wavelet.

Finally, the dilation parameters of the mother wavelet are updated as follows:

$$\begin{aligned} d_{ij}(N+1) &= d_{ij}(N) + \Delta d_{ij}(N) \\ &= d_{ij}(N) - \eta_d \frac{\partial E}{\partial d_{ij}(N)} \\ &= d_{ij}(N) - \eta_d \frac{\partial E}{\partial u}\frac{\partial u}{\partial d_{ij}(N)} \\ &= d_{ij}(N) - \eta_d \cdot \delta \cdot \frac{c_j}{d_{ij}} \cdot (1-z_{ij}^2) \cdot \Phi_j \end{aligned} \tag{9}$$

where, η_d is the learning rate of the dilation parameters of the mother wavelet.

The exact calculation of the Jacobian of the system, $\partial y_p / \partial u$, cannot be determined due to the nonlinear dynamics of the plant. Though the identifier can be implemented to calculate the Jacobian of the system, heavy computation effort is required. To overcome this problem and to increase the on-line learning rate of the weights, an approximation law is adopted as follows:

$$\delta \cong e + e(z^{-1}) \tag{10}$$

2.3 Convergence analysis

Selection of the values for the learning rates has a significant effect on the network performance. In order to train the WNN effectively, adaptive learning rates, which guarantee the convergence of tracking error based on the analysis of a discrete-type Lyapunov function, are derived in this section. The convergence analyses are to derive specific learning rates for specific types of network parameters to assure convergence of the tracking error.

Theorem 1. *Let* η_c *be the learning rate of the WNN weights between wavelet node and output node, and let* $P_{c\max}$ *be defined as* $P_{c\max} \equiv \max_N \|P_c(N)\|$ *, where* $P_c(N) = \partial O_k^3 / \partial c_j$ *and* $\|\cdot\|$ *is the Euclidean norm in* $\Re^n$*. The convergence is guaranteed if* η_c *is chosen as* $\eta_c = \lambda / P_{c\max}^2 = \lambda / |\Phi_j|_{\max}^2$ *, in which* λ *is a positive constant gain.*

Proof 1. Since

$$P_c(N) = \frac{\partial O_k^3}{\partial c_j} = \Phi_j \tag{11}$$

Thus

$$\|P_c(N)\| \le |\Phi_j|_{\max} \tag{12}$$

Then, a discrete-type Lyapunov function is selected as:

$$V(N) = \frac{1}{2} e^2(N) \tag{13}$$

The change in the Lyapunov function is obtained by

$$\begin{aligned} \Delta V(N) &= V(N+1) - V(N) \\ &= \frac{1}{2}\left[e^2(N+1) - e^2(N)\right] \end{aligned} \tag{14}$$

The error difference can be represented by [11]

$$e(N+1) = e(N) + \Delta e(N) = e(N) + \left[\frac{\partial e(N)}{\partial c_j}\right]^T \Delta c_j \tag{15}$$

where, $\dfrac{\partial e(N)}{\partial c_j} = \dfrac{\partial e(N)}{\partial O_k^3}\dfrac{\partial O_k^3}{\partial c_j} = -\dfrac{1}{e(N)}\delta P_c(N)$

Therefore, the error difference can be rewritten by

$$e(N+1) = e(N) - \left[\frac{1}{e(N)}\delta P_c(N)\right]^T \eta_c \delta P_c(N) \tag{16}$$

Then

$$\begin{aligned} \| e(N+1) \| &= \left\| e(N)\left[1 - \eta_c \left[\frac{\delta}{e(N)}\right]^2 P_c^T(N) P_c(N)\right]\right\| \\ &\le \| e(N) \| \left\| 1 - \eta_c \left[\frac{\delta}{e(N)}\right]^2 P_c^T(N) P_c(N) \right\| \end{aligned} \tag{17}$$

If η_c is chosen as $\eta_c = \lambda / P_{c\max}^2 = \lambda / |\Phi_j|_{\max}^2$, the term $\left\| 1 - \eta_c [\delta / e(N)]^2 P_c^T(N) P_c(N) \right\|$ in the above equation is less than 1. Therefore, the Lyapunov stability of $V > 0$ and $\Delta V < 0$ is guaranteed. The tracking error will converge to zero as $t \rightarrow \infty$. This completes the proof of the theorem. □

Theorem 2. *Let* η_m *be the learning rate of the translation of the mother wavelet for the WNN, and let* $P_{m\max}$ *be defined as* $P_{m\max} = \max_N \| P_m(N) \|$ *, where* $P_m(N) = \partial O_k^3 / \partial m_{ij}$ *and* $\|\cdot\|$ *is the Euclidean norm in* $\Re^n$*. The convergence is guaranteed if* η_m *is chosen as* $\eta_m = \eta_c \left[|c_j|_{\max} |1 - z_{ij}^2|_{\max} / (|d_{ij}|_{\min} |z_{ij}|_{\min}) \right]^{-2}$*, in which* λ *is a positive constant gain and* $|\cdot|$ *is the absolute value.*

Proof 2. Since

$$\begin{aligned} P_m(N) &= \frac{\partial O_k^3}{\partial m_{ij}} = \frac{\partial O_k^3}{\partial \Phi_j}\frac{\partial \Phi_j}{\partial m_{ij}} \le \left|\frac{\partial O_k^3}{\partial \Phi_j}\right| \cdot \left|\frac{\partial \Phi_j}{\partial m_{ij}}\right| \\ &\le |c_j| \cdot \left|\frac{\partial \Phi_j}{\partial z_{ij}}\right| \cdot \left|\frac{\partial z_{ij}}{\partial m_{ij}}\right| = |c_j| \cdot \left|\frac{1}{d_{ij}}\right| \cdot \left|\frac{\partial \Phi_j}{\partial z_{ij}}\right| \\ &= \left|\frac{c_j}{d_{ij}}\right| \cdot \left|\frac{1-z_{ij}^2}{z_{ij}} \cdot \Phi_j\right| \le \left|\frac{c_j}{d_{ij}}\right| \cdot \left|\frac{1-z_{ij}^2}{z_{ij}}\right| \cdot |\Phi_j| \end{aligned} \tag{18}$$

Thus

$$\left\|P_m(N)\right\| < \left|\Phi_j\right|_{\max}\left[\frac{|c_j|_{\max}|1-z_{ij}^2|_{\max}}{|d_{ij}|_{\min}|z_{ij}|_{\min}}\right] \tag{19}$$

The error difference can also be represented by

$$e(N+1) = e(N) + \Delta e(N) = e(N) + \left[\frac{\partial e(N)}{\partial m_{ij}}\right]^T \Delta m_{ij} \tag{20}$$

where Δm_{ij} represents a translation change of the mother wavelet, and

$$\frac{\partial e(N)}{\partial m_{ij}} = \frac{\partial e(N)}{\partial O_k^3}\frac{\partial O_k^3}{\partial m_{ij}} = -\frac{1}{e(N)}\delta P_m(N) \tag{21}$$

Therefore, the error difference can be rewritten by

$$\begin{aligned} e(N+1) &= e(N) + \Delta e(N) \\ &= e(N) - \left[\frac{\delta}{e(N)}P_m(N)\right]^T \eta_m \delta P_m(N) \end{aligned} \tag{22}$$

Then

$$\begin{aligned} \|e(N+1)\| &= \left\| e(N)\left[1-\eta_m\left(\frac{\delta}{e(N)}\right)^2 P_m^T(N)P_m(N)\right]\right\| \\ &\le \|e(N)\|\left\|1-\eta_m\left(\frac{\delta}{e(N)}\right)^2 P_m^T(N)P_m(N)\right\| \end{aligned} \tag{23}$$

If η_m is chosen as

$$\begin{aligned} \eta_m &= \lambda \Big/ P_{m\max}^2 = \lambda \Big/ \left[\left|\Phi_j\right|_{\max}\left(\frac{|c_j|_{\max}|1-z_{ij}^2|_{\max}}{|d_{ij}|_{\min}|z_{ij}|_{\min}}\right)\right]^2 \\ &= \eta_c\left[|c_j|_{\max}|1-z_{ij}^2|_{\max} \Big/ (|d_{ij}|_{\min}|z_{ij}|_{\min})\right]^{-2} \end{aligned} \tag{24}$$

the term $\left\|1-\eta_m(\delta/e(N))^2 P_m^T(N)P_m(N)\right\|$ in the above equation is less then 1. Therefore, the Lyapunov stability of $V > 0$ and $\Delta V < 0$ is guaranteed. The tracking error will converge to zero as $t \to \infty$. □

Theorem 3. *Let η_d be the learning rate of the dilation of the mother wavelet for the WNN, and let $P_{d\max}$ be defined as $P_{d\max} = \max_N \| P_d(N) \|$, where $P_d(N) = \partial O_k^3 / \partial d_{ij}$ and $\|\cdot\|$ is the Euclidean norm in $\Re^n$. The convergence is guaranteed if η_d is chosen as $\eta_d = \eta_m |z_{ij}|_{\min}^{-2}$, in which λ is a positive constant gain and $|.|$ is the absolute value.*

Proof 3. Since

$$\begin{aligned} P_d(N) &= \frac{\partial O_k^3}{\partial d_{ij}} = \frac{\partial O_k^3}{\partial \Phi_j}\frac{\partial \Phi_j}{\partial d_{ij}} \le \left|\frac{\partial O_k^3}{\partial \Phi_j}\right| \cdot \left|\frac{\partial \Phi_j}{\partial d_{ij}}\right| \\ &\le |c_j| \cdot \left|\frac{\partial \Phi_j}{\partial z_{ij}}\right| \cdot \left|\frac{\partial z_{ij}}{\partial d_{ij}}\right| = |c_j| \cdot \left|\frac{z_{ij}}{d_{ij}}\right| \cdot \left|\frac{\partial \Phi_j}{\partial z_{ij}}\right| \\ &= \left|\frac{c_j z_{ij}}{d_{ij}}\right| \cdot \left|\frac{1-z_{ij}^2}{z_{ij}} \cdot \Phi_j\right| \le \left|\frac{c_j}{d_{ij}}\right| \cdot \left|1-z_{ij}^2\right| \cdot \left|\Phi_j\right| \end{aligned} \tag{25}$$

Thus

$$\left\|P_d(N)\right\| < \left|\Phi_j\right|_{\max}\left[\frac{|c_j|_{\max}|1-z_{ij}^2|_{\max}}{|d_{ij}|_{\min}}\right] \tag{26}$$

The error difference can also be represented by

$$e(N+1) = e(N) + \Delta e(N) = e(N) + \left[\frac{\partial e(N)}{\partial d_{ij}}\right]^T \Delta d_{ij} \tag{27}$$

where Δd_{ij} represents a translation change of the mother wavelet, and

$$\frac{\partial e(N)}{\partial d_{ij}} = \frac{\partial e(N)}{\partial O_k^3}\frac{\partial O_k^3}{\partial d_{ij}} = -\frac{1}{e(N)}\delta P_d(N) \tag{28}$$

Therefore, the error difference can be rewritten by

$$\begin{aligned} e(N+1) &= e(N) + \Delta e(N) \\ &= e(N) - \left[\frac{\delta}{e(N)}P_d(N)\right]^T \eta_d \delta P_d(N) \end{aligned} \tag{29}$$

Then

$$\begin{aligned} \|e(N+1)\| &= \left\| e(N)\left[1-\eta_d\left(\frac{\delta}{e(N)}\right)^2 P_d^T(N)P_d(N)\right]\right\| \\ &\le \|e(N)\|\left\|1-\eta_d\left(\frac{\delta}{e(N)}\right)^2 P_d^T(N)P_d(N)\right\| \end{aligned} \tag{30}$$

If η_d is chosen as

$$\begin{aligned} \eta_d &= \lambda \Big/ P_{d\max}^2 = \lambda \Big/ \left[\left|\Phi_j\right|_{\max}\frac{|c_j|_{\max}|1-z_{ij}^2|_{\max}}{|d_{ij}|_{\min}}\right]^2 \\ &= \eta_m |z_{ij}|_{\min}^{-2} \end{aligned} \tag{31}$$

the term $\left\|1-\eta_d(\delta/e(N))^2 P_d^T(N)P_d(N)\right\|$ in the above equation is less then 1. Therefore, the Lyapunov stability of $V > 0$ and $\Delta V < 0$ is guaranteed. The tracking error will converge to zero as $t \to \infty$. □

Theorem 4. *Let η_a be the learning rate of the dilation of the mother wavelet for the WNN, and let $P_{a\max}$ be defined as $P_{a\max} = \max_N \| P_a(N) \|$, where $P_a(N) = \partial O_k^3 / \partial a_i$ and $\| \cdot \|$ is the Euclidean norm in $\Re^n$. The convergence is guaranteed if η_a is chosen as $\eta_a = \lambda / |x_i|_{\max}^2$, in which λ is a positive constant gain and $| . |$ is the absolute value.*

Proof 4. Since

$$P_a(N) = \frac{\partial O_k^3}{\partial a_i} = x_i \tag{32}$$

Thus

$$\|P_a(N)\| \le |x_i|_{\max} \tag{33}$$

The error difference can also be represented by

$$e(N+1) = e(N) + \Delta e(N) = e(N) + \left[\frac{\partial e(N)}{\partial a_i}\right]^T \Delta a_i \tag{34}$$

where Δa_i represents a translation change of the mother wavelet, and

$$\frac{\partial e(N)}{\partial a_i} = \frac{\partial e(N)}{\partial O_k^3}\frac{\partial O_k^3}{\partial a_i} = -\frac{1}{e(N)}\delta P_a(N) \tag{35}$$

Therefore, the error difference can be rewritten by

$$\begin{aligned} e(N+1) &= e(N) + \Delta e(N) \\ &= e(N) - \left[\frac{\delta}{e(N)} P_a(N)\right]^T \eta_a \delta P_a(N) \end{aligned} \tag{36}$$

Then

$$\begin{aligned} \|e(N+1)\| &= \left\| e(N)\left[1 - \eta_a\left(\frac{\delta}{e(N)}\right)^2 P_a^T(N) P_a(N)\right]\right\| \\ &\le \|e(N)\| \left\| 1 - \eta_a\left(\frac{\delta}{e(N)}\right)^2 P_a^T(N) P_a(N)\right\| \end{aligned} \tag{37}$$

If η_a is chosen as

$$\eta_a = \lambda / P_{a\max}^2 = \lambda / |x_i|_{\max}^2 \tag{38}$$

the term $\left\|1 - \eta_a (\delta/e(N))^2 P_a^T(N) P_a(N)\right\|$ in the above equation is less then 1. Therefore, the Lyapunov stability of $V > 0$ and $\Delta V < 0$ is guaranteed. The tracking error will converge to zero as $t \to \infty$. □

3. SIMULATION RESULTS

In this section, we present some simulation results to validate the control performance of proposed controller for the continuous-time chaotic system. We consider the Duffing system as the controlled chaotic system.

The state equation of the Duffing system is as follows:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} y(t) \\ -p_1 x(t) - x^3(t) - p y(t) + q\cos(wt) \end{bmatrix} \tag{39}$$

where, the parameter set is as follows:

$$\{p, p_1, q, w\} = \{0.4, -1.1, 1.8, 1.8\}$$

The control objective for Duffing system is to follow the periodic solution of the Duffing system. In tracking the Duffing system, we define the initial system state as (1,0) and the reference signal as the periodic solution in case of q=2.3 of Eqn. (39).

The parameters used in this simulation are shown in Table 1.

Table 1 Parameters used in this simulation

Number of the input	2
Number of the wavelet function	14
Number of the output	1
Sampling time	0.01
A positive constant gain (λ)	0.1

In the simulation, in order to evaluate the performance of the proposed controller, we compare the results of the proposed stable intelligent control system with those of the WNN controller with fixed learning rates (0.0001).

Figures 3 and 4 show the tracking control results of state x and y for Duffing system using the direct adaptive control technique based on the WNN controller with fixed learning rate [12]. Also, the tracking control results of Duffing system using the proposed stable intelligent control system are shown in Figs. 5 and 6. And, Table 2 shows the tracking control results of the proposed controller.

From the results obtained the above, we can see that the proposed stable intelligent control system has the better control performance, and that it is faster, more effective and stable, as compared with the intelligent control system based on the WNN controller with fixed learning rate.

Consequently, selection of values for the learning rates has a significant effect on the network performance. If small values are given for the learning rates, convergence will be assured at a low speed. On the other hand, if large values are given for the learning rates, the system may become unstable.

Table 2 Tracking control results for Duffing system

WNN controller with adaptive learning rates [ours]	State x	0.0332
	State y	0.1386
WNN controller with fixed learning rates [12]	State x	0.1518
	State y	0.5483

4. CONCLUSIONS

In this paper, we have proposed the design method of the wavelet neural network (WNN) based controller using direct adaptive control method to deal with a stable intelligent control of chaotic systems whose mathematical models are not available. In our method, the intelligent control system is an on-line trained WNN controller with adaptive learning rates. The adaptive learning rates are derived in the sense of discrete-type Lyapunov stability theorem, so that the convergence of the tracking error can be guaranteed in the closed-loop system. The major merits of our control system are that the strict constrained conditions and prior knowledge of the controlled plant are not necessary in the whole design process, and convergence of the tracking error in the control system can be guaranteed. From the simulation results, we can see that the proposed stable intelligent control system has the better control performance, and that it is faster, more effective and stable, as compared with the intelligent control system based on the WNN controller with fixed learning rate.

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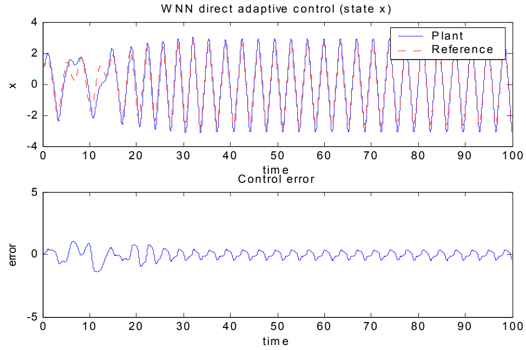

Fig. 3 Track control result of Duffing system (state x) using the WNN controller with fixed learning rate (0.0001)

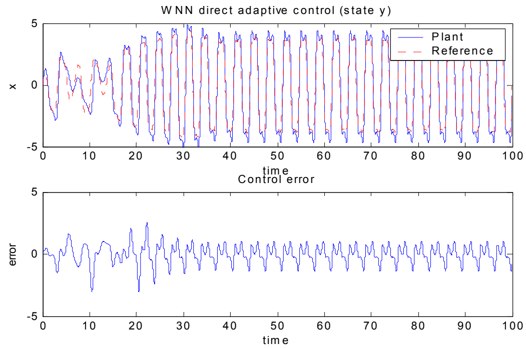

Fig. 4 Track control result of Duffing system (state y) using the WNN controller with fixed learning rate (0.0001)

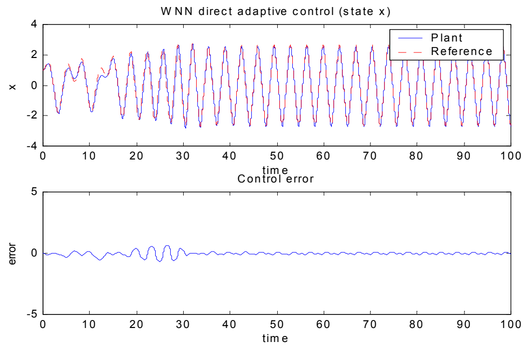

Fig. 5 Track control result of Duffing system (state x) using the WNN controller with adaptive learning rate

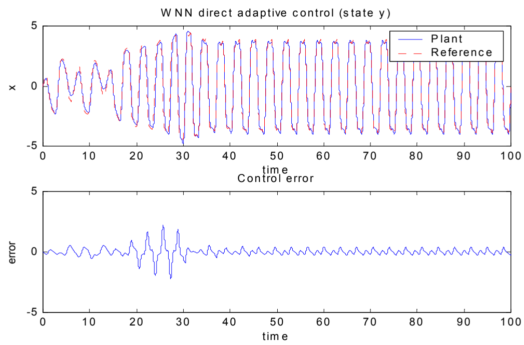

Fig. 6 Track control result of Duffing system (state y) using the WNN controller with adaptive learning rate