Design of Enhanced Min-Max Control using Feedforward Control

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Abstract: This paper deals with robust control problems of linear systems with matched nonlinear uncertainties. In order to handle the uncertainties, a Lyapunov min-max control approach can usually be adopted. By the way, the min-max control input is required to be switched and provokes chattering phenomena which limit the practical implementation. The magnitude of switching control input which cause chattering is dependent on the size of uncertainties. In this paper, it is shown that the magnitude of the min-max control input can be made small using a well-known disturbance observer technique and only considers the disturbance observing errors. The chattering phenomena can be reduced as small as possible by selecting a high diturbance observer gain. The simulations show that the min-max control with a disturbance observer can reduce chattering phenomena much smaller and guarantee much better robust performance rather than the one without a disturbance observer.

Keywords: Min-Max control, Disturbance observer, Nonlinear Uncertainty, Chattering

1. Introduction

Most control systems have their own disturbance sources and model uncertainties. In order to overcome these sources of uncertainties, there have been developed a lot of robust control methods last three decades, e.g., LQG/LTR, H_∞ control, sliding mode control, and adaptive control.

In this paper, robust control of linear systems with matched uncertainties by a Lyapunov min-max approach is considered. Basically, a Lyapunov min-max approach requires the switching of the control input, which cause chattering phenomena. See [1], [2] and [3]. In this paper, we suggest how to improve the chattering phenomena of the min-max approach by using a disturbance observer technique. In the fields of motion control, the disturbance observer techniques have been widely used and shown good performances. See [4]-[10]. In the conventional disturbance observer design, the inverse of the model is used for the design of the disturbance observer. If the plant has nonminimum phase zeros, then the disturbance observer must be internally unstable. Even for strictly proper systems, the disturbance observer includes derivative operation. So, the high-frequency noise might be amplified. On the otherhand, using Internal model(IM), these kinds of problems would be removed. Recently,Zhu et al. suggested the enhanced internal model control , which enhanced the robustness of the disturbance observer [8].

In this paper, a disturbance observer is first designed using the structure suggested by Zhu et al. [8] in order to estimate the system uncertainties. But, the disturbance observing error is unavoidable. In order to compensate for this error, the min-max control input is designed and applied to the plant with the disturbance observer output. It will be shown that the magnitude of the designed min-max control can be made small by selecting high observer gain. The asymptotic stability of the closed-loop system is also analyzed using Lyapunov method. Through computer simulations, the advantages of the proposed method will be shown over the well-known min-max control method without a disturbance observer.

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2. Conventional Lyapunov Min-Max Control Approach

Consider the following linear systems with nonlinear uncertainties.

$$\dot{x}(t) = Ax(t) + Bu(t) + Bf(x,t) \quad (1)$$

where $x(t) \in R^n$ is a state vector and $u(t) \in R$ is an input. $f : R^n \to R$ is a nonlinear uncertainty function which satisfies the following assumptions.

(A1) The function f is bounded by $\epsilon_1 > 0$ for all x and $t \geq 0$ as follows.

$$\|f(x,t)\| \leq \epsilon_1 \quad (2)$$

(A2) The time-derivative of the function f is bounded by $\epsilon_2 > 0$ for all x and $t \geq 0$ as follows.

$$\|\frac{f(x,t)}{dt}\| \leq \epsilon_2 \quad (3)$$

(A3) The system (1) is controllable.

From Assumption *(A3)*, if the system matrix A is unstable, then it can be made stable by state feedback control. For convenience, we assume the matrix A is stable throughout the paper.

(A4) The system matrix A is stable.

The problem considered in this paper is to find a robust controller which stabilizes the uncertain system described by (1). Gutman (1979) suggested the Lyapunov min-max approach in which the control input was given as follows.

$$u = \begin{cases} -\epsilon_1 \frac{B^T PX}{\|B^T PX\|} & \text{if } B^T PX \neq 0 \\ 0 & \text{if } B^T PX = 0 \end{cases} \quad (4)$$

As seen in (4), the min-max control input is dependent on the uncertainty bounds and required to be switched in order to make the derivative of Lyapunov function negative [1]. Thus, the min-max control input may cause the chattering phenomena which bring about difficulties in the realization of

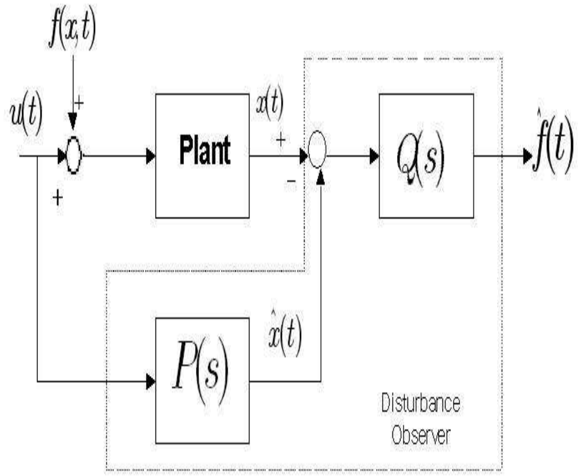

Fig. 1. Block Diagram of Disturbance Observer

the min-max control and performance degradation in the actual performance. In the next section, new design approach will be proposed.

3. Design of enhanced conventional min-max control approach

In order to enhance the performance of the original min-max approach, firstly the unknown function $f(x,t)$ is estimated using a simple disturbance observer technique and this estimated value will be subtracted for the compensation of the unknown function $f(x,t)$. If the min-max control approach is applied only to the compensation for the estimation error, the required switching magnitude of the control can be reduced. In fact, the estimation error is shown to be made arbitrarily small if the estimator gain is chosen large enough and the chattering phenomena caused by the min-max control can be made small enough.

3.1. Design of a disturbance observer

Now, we design the estimator of the function f using the well-known disturbance observer technique. The structure of the estimator is described in Fig. 1. The estimator is designed to perform as if it is a low-pass filter so that the transfer function from the unknown function $f(x,t)$ to the estimator output $\hat{f}(t)$ satisfies the following equation.

$$G_D(s) \triangleq \frac{\pounds[\hat{f}(t)]}{\pounds[f(x,t)]} = \frac{K}{s+K}, \quad K > 0 \qquad (5)$$

where $\pounds[(\cdot)]$ means the Laplace transform of $(\cdot)$. In Figure 1, let us design the subsystems whose transfer functions $P(s)$ and $Q(s)$ satisfy the following equations.

$$\begin{aligned} P(s) &\triangleq \frac{\hat{X}(s)}{U(s)} = (sI-A)^{-1}B \\ Q(s) &\triangleq \frac{\pounds[\hat{f}(t)]}{X(s)-\hat{X}(s)} = \frac{K}{s+K}C(sI-A) \end{aligned} \qquad (6)$$

where $X(s), U(s), \hat{X}(s)$ are Laplace transforms of $x(t), u(t), \hat{x}(t)$ and C is chosen such that $CB = 1$. Then, the transfer function, $G_D(s)$ satisfies (5) since $X(s) = (sI-A)^{-1}B$.

Lemma 1: : The estimation error $e_d(t) \triangleq f(x,t) - \hat{f}(t)$ is bounded and satisfies the following inequality.

$$e_d(t) \leq (\epsilon_1 - \hat{f}(0))e^{-Kt} + \frac{\epsilon_2}{K}(1-e^{-Kt}) \qquad (7)$$

Note that the steady-state estimation error can be made arbitrary small if K is chosen large enough.

3.2. Design of a new min-max control

In this section, a control input is designed so that the disturbance estimation error can be rejected.

First, the estimation output for the unknown function is feedforward to the plant input which subtracts the effect of the unknown function and then the additional control input is applied to compensate the effect of the estimation error. Define a control input by

$$\begin{aligned} u(t) &= -K_c x(t) - \hat{f}(t) \\ &\quad - \frac{B^T P x(t)}{\|B^T P x(t)\|}\{(\epsilon_1 - \hat{f}(0))e^{-Kt} + \frac{\epsilon_2}{K}(1-e^{-Kt})\} \end{aligned} \qquad (8)$$

where K_c is chosen so that there exist positive definite matrices P and Q satisfying

$$(A - BK_c)^T P + P(A - BK_c) = -Q. \qquad (9)$$

Theorem 1 shows the stability of the proposed control (8).

Theorem 1: : The system (1) is asymptotically stabilized by the input given by (8).

Proof) If the input (8) is applied to the system (1), then, the closed-loop equation can be written by

$$\begin{aligned} \dot{x}(t) &= (A - BK_c)x(t) + B[f(x,t) - \hat{f}(t) \\ &\quad - \{(\epsilon_1 - \hat{f}(0))e^{-Kt} + \frac{\epsilon_2}{K}(1-e^{-Kt})\}\frac{B^T P x(t)}{\|B^T P x(t)\|}] \end{aligned} \qquad (10)$$

Define a Lyapunov function candidate V as follows.

$$V \triangleq \frac{1}{2}x^T P x \qquad (11)$$

Then, the time-derivative of the Lyapunov function V can be written by the following equation.

$$\begin{aligned} \dot{V} &= \frac{1}{2}x^T\{(A - BK_c)^T P + P(A - BK_c)\}x \\ &\quad + x^T PB[f(x,t) - \hat{f}(t) \\ &\quad - \{(\epsilon_1 - \hat{f}(0))e^{-Kt} + \frac{\epsilon_2}{K}(1-e^{-Kt})\}\frac{B^T P x(t)}{\|B^T P x(t)\|}] \end{aligned} \qquad (12)$$

From Lemma 1, the time-derivative of the Lyapunov function V satisfies the following inequality.

$$\begin{aligned} \dot{V} &\leq \frac{1}{2}x^T\{(A - BK_c)^T P + P(A - BK_c)\}x \\ &\quad + \|x^T PB\|\|f(x,t) - \hat{f}(t)\| \end{aligned}$$

$$
\begin{aligned}
& -\{(\epsilon_1 - \hat{f}(0))e^{-Kt} + \frac{\epsilon_2}{K}(1-e^{-Kt})\}\frac{x^T PBB^T Px(t)}{\|B^T Px\|} \\
\leq \; & \frac{1}{2}x^T\{(A-BK_c)^T P + P(A-BK_c)\}x \\
& + \|x^T PB\|\|e_d(t)\| \\
& -\{(\epsilon_1 - \hat{f}(0))e^{-Kt} + \frac{\epsilon_2}{K}(1-e^{-Kt})\}\|B^T Px(t)\| \\
= \; & -\frac{1}{2}x^T Qx \qquad (13)
\end{aligned}
$$

for all $t > 0$ and $x \neq 0$. Therefore, the closed loop system is globally asymptotically stable. □

Even if we get rid of the term related to ϵ_1 in the control input (8), the asymptotic stability of the closed-loop system is guaranteed.

Theorem 2: *:* The system (1) is asymptotically stabilized by the input defined by

$$
\begin{aligned}
u(t) &= -K_c x(t) - \hat{f}(t) \\
&\quad - \frac{B^T Px(t)}{\|B^T Px(t)\|}\frac{\epsilon_2}{K}(1-e^{-Kt}).
\end{aligned} \qquad (14)
$$

Note that if the estimator gain K is chosen large enough, the magnitude of the switching input part $\frac{B^T Px(t)}{\|B^T Px(t)\|}\frac{\epsilon_2}{K}(1-e^{-Kt})$ can be arbitrarily small. So, the control input defined in (14) does not result in a severe chattering phenomenon. Through simulations, the advantage of the proposed control method will be shown compared to the well-known min-max control method[1], [2], [3].

4. Example

In order to show the effectiveness of the proposed method, its performance will be compared to that of the well-known min-max control approach[1]. In (1), the matrices and unknown function are chosen as follows.

$$
\begin{aligned}
A &= \begin{bmatrix} 0 & 1 \\ -100 & -20 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
f(x,t) &= sin(2\pi t)
\end{aligned} \qquad (15)
$$

Then, the Lyapunov equation is satisfied for the following positive definite matrices, P and Q.

$$
P = \begin{bmatrix} 6.2625 & -25 \\ -25 & 126.25 \end{bmatrix}, Q = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix} \qquad (16)
$$

Since $f(x,t) = sin(2\pi t)$, the assumptions *(A1)* and *(A2)* are satisfied with $\epsilon_1 = 1$ and $\epsilon_2 = 2\pi$. Since the system matrix A is stable as in *(A4)*, the well-known min-max control input[1] can be defined by

$$
u = \begin{cases} -\epsilon_1 \frac{B^T PX}{\|B^T PX\|} & \text{if } B^T PX \neq 0 \\ 0 & \text{if } B^T PX = 0 \end{cases} \qquad (17)
$$

In this section, the performance of the control (8) is compared with that of (17). Throughout the simulations, the disturbance observer gain K is 150.

Fig. 2 and 4 represent the graphs of control inputs, and Fig. 3 and 5 show the graphs of the state trajectories. In each figure, dotted line represents the results when the min-max control input is applied without disturbance observer and solid one means those when the proposed control input is applied. In Fig. 2 and 4, figure (a) is the result of the proposed control and figure (b) is that of the min-max control. As seen in Fig. 2 and 3 , both control methods guarantee the asymptotic stability, but the min-max control approach requires tremendous switching ability compared with the proposed method. The control input of the proposed method is much smoother, which results in much less chattering phenomena rather than the min-max approach.

Fig. 4 and 5 are the graphs of the results when the control inputs are filtered with the following low pass filter.

$$
\frac{\pounds[u(t)]}{\pounds[u_c(t)]} = \frac{k_f}{s+k_f} \qquad (18)
$$

where $u_c(t)$ is a control input command, $u(t)$ is an actual control input, and k_f is a filter gain which is set to 70 in this simulation. Fig. 4 shows the typical advantages of the proposed method. The performance of min-max approach is deteriorated by the filter action, but the proposed method is not. Fig. 4 shows the graphs of the control inputs and the min-max control input seems to approach to the one of the proposed method which is much smoother. Compared Fig. 5 with Fig. 3, the trajectory of the state x_2 in the filtered case of the min-max control approach is shown to be more regulated than that of the original case of the min-max control approach.

5. Conclusion

In this paper, a robust control problem is considered for linear systems with matched nonlinear uncertainties. To compensate for the uncertainties, a well-known Lyapunov min-max approach has been adopted. It has been shown that the chattering phenomena of the min-max approach can be reduced by being incorporated with a disturbance observer technique.

In this paper, the uncertainties were considered as disturbances and a simple disturbance observer has been designed first. Since the suggested disturbance observer is a kind of low-pass filter, it has inevitably estimation errors and a min-max control input has been designed to consider these errors. Because the disturbance observing error can be made arbitrarily small by increasing the observer gain, the chattering phenomena of min-max control which is composed of the disturbance observing error bounds can be reduced small enough. From the viewpoint of implemetation, the proposed approach is practically much more realizable rather than the conventional min-max control. Through computer simulations, the advantages of the proposed approach have been shown.

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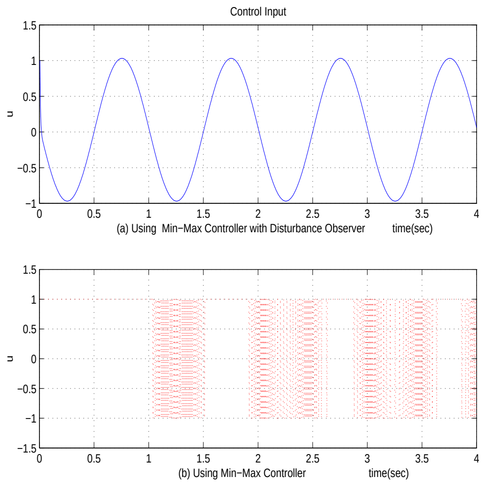

Fig. 2. Control Input

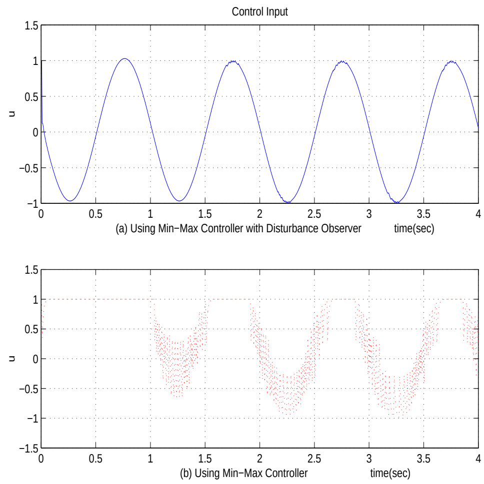

Fig. 4. Control Input using an Actuator Filter

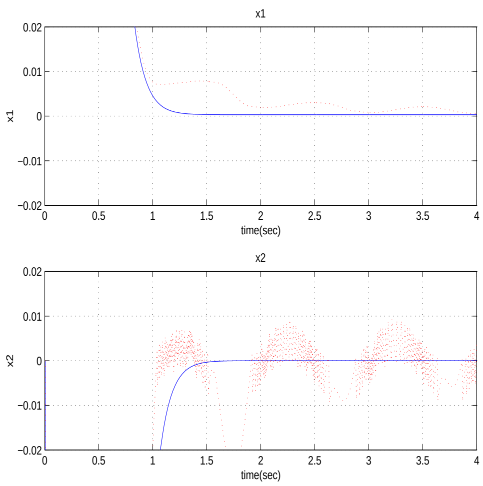

Fig. 3. state Trajectories

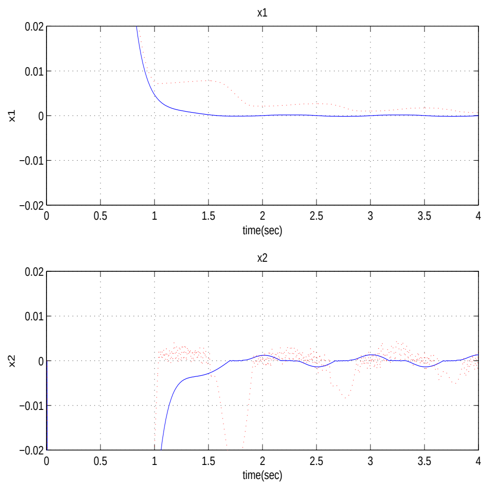

Fig. 5. State Trajectories using an Actuator Filter

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