Stereo cameras calibration bases on Epipolar Rectification and its Application

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Abstract : The constraints necessary guarantee using the comparison of these extrinsic parameters, which each Rotation matrix and Translation Vector must be equal to the either, except the X-axis Translation Vector. Thus, we can not yet calculate the 3D-range measurement in the end of camera calibration. To minimize this disadvantage, the Epipolar Rectification has been proposed in the literature. This paper aims to present the development of Epipolar Rectification to calibrate Stereo cameras. The required computation of the transformation mapping between points in 3D-space is based on calculating the image point that appears on new image plane by using calibrated parameters. This computation is assumed from the rotating the old ones around their optical center until focal planes becomes coplanar, thereby containing the baseline, and the Z-axis of both camera coordinate to be parallel together. The optical center positions of the new extrinsic parameters are the same as the old camera, whereas the new orientation differs from the old ones by the suitable rotations. The intrinsic parameters are the same for both cameras. So that, after completed calibration process, immediately can calculate the 3D-range measurement. And the rectification determines a transformation of each image plane such that pairs of conjugate Epipolar lines become collinear and parallel to one of the image axis. From the experimental results verify the proposed technique are agreed with the expected specifications.

Keywords: Passive stereo, Epipolar geometry, Rectification Transformation

1. INTRODUCTION

Three dimensional range measurements determined by using passive stereo method is most widely used for computer and robot vision. From the specification of stereo image, to obtain the accurate calculation of 3D-range measurement, the X-axis of focal plane must be parallel to the line called base line, which represents the distance between two cameras. Moreover, the Z-axis of both camera coordinates must also be parallel. These requirements can be checked from the extrinsic parameters after camera calibration

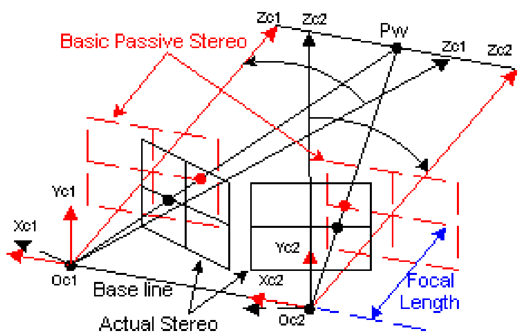


Fig. 1 camera calibration bases on Epipolar rectification. Transformation of the image planes two images the rectification process.

The idea camera calibration base on Epipolar rectification is to define two new extrinsic parameters obtained by rotating the old ones around their optical centers until focal plane become coplanar, thereby containing the baseline. This ensures that epipole are at infinity, hence epipolar lines are parallel. To have horizontal epipolar lines, the base line must be parallel to the new x axis of both cameras. In addition, to have a proper rectification, conjugate points must have the same vertical coordinate. This is obtained by requiring that new cameras have the same intrinsic parameters.

Note that, being the focal length the same, image planes are coplanar too, as in Fig. 1

The use of binocular stereo vision leads, however, to two difficulties of practical order. In first place, the evaluation of all the necessary characteristics of the camera, constituted of its intrinsic and extrinsic parameters (position and orientation in relation to the external coordinate system) is denominated calibration, and constitutes one of the central problems in stereo vision. The second critical aspect is with respect to the determination of the homologue points in the different images, that is, of those that correspond to projections of a same point in the surface of the represented object.

2. FUNDAMENTALS

2.1 Epipolar geometry

The epipolar geometry between two views is essentially the geometry of the intersection of the image plane with the pencil of planes having the baseline as axis (the baseline is the line joining the camera centers). Considering the search for corresponding point in stereo matching usually motivates this geometry, and we will start from that objective here.

Let us consider a stereo rig composed by two-pinhole cameras Fig.1 Let Oc1 and Oc2 be the optical centers of the left and right cameras respectively. A 3-D point Pw in projected onto both image plane, to point x1 and x2, which constitute a conjugate pair. Give a point x1 in the left image plane, its conjugate point in the right image is constrained to lie on a line called the epipolar line. Since x1 may be the projection of an arbitrary point on its optical ray, the epipolar line is the projection through Oc2 of the optical ray of x1 . All the epipolar lines in one image plane pass through a common point e1 and e2 respectively called the epipole, which is the projection of the optical center of the other camera.

When Oc1 is in the focal plane of the right cameras, the right epipole is at infinity, and the epipolar lines form a bundle of parallel lines in the right image. A very special case is when both epipole are at infinity, that happens when the baseline Oc1 and Oc2 is constrained in both focal plane, i.e., the retinal planes are parallel to the baseline. Epipolar line, then. Form a bundle of parallel line in both images. Any pair of images can be transformed so that epipolar line is parallel and horizontal in each image. This procedure is called rectification.

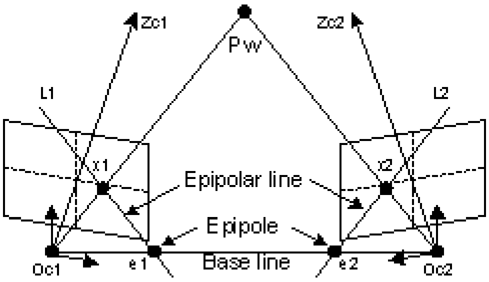


Fig 2. Epipolar geometry

2.2 Constraining of passive stereo

We describe the constraints necessary to camera calibration. We define our extrinsic parameters as.

$$Me_1 = \begin{bmatrix} a_{1i} & & a_{14} \\ a_{2i} & | & a_{24} \\ a_{3i} & & a_{34} \end{bmatrix} \quad (1)$$

$$Me_2 = \begin{bmatrix} b_{1i} & & b_{14} \\ b_{2i} & | & b_{24} \\ b_{3i} & & b_{34} \end{bmatrix} \quad (2)$$

The specification of stereo image is to correct calculation of 3D-range measurement. The first X-axis of focal plane might be parallel with line that is distance between cameras (Base line)and epipolar line are parallel if.

$$\begin{aligned} a_{3i} &= b_{3i} \\ a_{34} &= b_{34} \end{aligned} \quad (3)$$

Second, the Z-axis of both camera coordinates must also be parallel.

$$\begin{aligned} a_{2i} &= b_{2i} \\ a_{24} &= b_{24} \\ a_{1i} &= b_{1i} \end{aligned} \quad (4)$$

Thus, from the specification of stereo we can not yet calculate the 3D-range measurement in the end of camera calibration. To minimize this disadvantage, the Epipolar Rectification has been proposed in the literature.

3. PROCEDURE OF CAMERA CALIBRATION

3.1 Camera Calibration

Consider calibration pattern it consists of two orthogonal grids equally spaced black square drawn on white , perpendicular planes. Assume that the world coordinates is centered at the lower left corner of the left grid, with axes parallel to the three directions identified by the calibration pattern.

Give the size of the planes, their angle the number of squares all know by construction the coordinates of each vertex can be computed in the world coordinate using trigonometry. The projection of the vertices on the image can be found by intersection the edge line of the corresponding square sides or through corner detection.

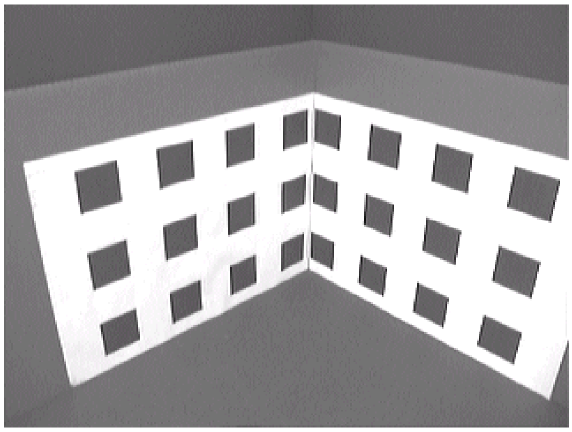


Fig. 3 Pattern with reference point in orthogonal planes used during calibration of a CCD camera in positions left and right.

The relationship between 3D coordinates and image coordinate can be written as

$$\begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix} \quad (5)$$

From Eq.(5) it can be rewritten perspective projection equation as

$$x_i = \frac{m_{11}x_w + m_{12}y_w + m_{13}z_w + m_{14}}{m_{31}x_w + m_{32}y_w + m_{33}z_w + m_{34}} \quad (6)$$

$$y_i = \frac{m_{21}x_w + m_{22}y_w + m_{23}z_w + m_{24}}{m_{31}x_w + m_{32}y_w + m_{33}z_w + m_{34}} \quad (7)$$

A system of 2N equations is obtained whose unknowns are the 12 elements. Although 6 points are sufficient, 20 black squares distributed on 2 perpendicular white planes were used to give more robustness to the calculation.

These equation will lead to a homogeneous system of equation Am=0. If $A = UDV^T$ the system has a nontrivial solution m_{ij} which is proportional to the column of V corresponding to the smallest singular value of A

(SVD(A)). For easy of reference later on we will subdivide the matrix M, let's define the following vector.

$$v_1 = [m_{11} \quad m_{12} \quad m_{13}]^T \tag{8}$$

$$v_2 = [m_{21} \quad m_{22} \quad m_{23}]^T \tag{9}$$

$$v_3 = [m_{31} \quad m_{32} \quad m_{33}]^T \tag{10}$$

$$v_4 = [m_{14} \quad m_{24} \quad m_{34}]^T \tag{11}$$

Compute intrinsic parameters and extrinsic parameters with following formula. Define s=sign (m34).

$$Mi = \begin{bmatrix} -\sqrt{v_1^T v_1 - (v_1^T v_3)^2} & 0 & v_1^T v_3 \\ 0 & -\sqrt{v_2^T v_2 - (v_2^T v_3)^2} & v_2^T v_3 \\ 0 & 0 & 1 \end{bmatrix} \tag{12}$$

$$R_{old} = \begin{bmatrix} s\left(v_1^T v_3 \left[v_3^T\right] - \left[v_1^T\right]\right) \Big/ -\sqrt{v_1^T v_1 - v_1^T v_3} \\ s\left(v_2^T v_3 \left[v_3^T\right] - \left[v_2^T\right]\right) \Big/ -\sqrt{v_2^T v_2 - v_2^T v_3} \\ s v_3^T \end{bmatrix} \tag{13}$$

$$T_{old} = \begin{bmatrix} s\left(v_1^T v_3 m_{34} - m_{14}\right) \Big/ -\sqrt{v_1^T v_1 - v_1^T v_3} \\ s\left(v_2^T v_3 m_{34} - m_{24}\right) \Big/ -\sqrt{v_2^T v_2 - v_2^T v_3} \\ s m_{34} \end{bmatrix} \tag{14}$$

The same result applies to the right camera. From camera parameters, i.e., the extrinsic parameters Me_{old1} and Me_{old2} are know. After rectification is to define two new Me_{new1} and Me_{new2} obtained by rotating the old one around their optical until focal planes become coplanar. Thereby the new x axis parallel to the baseline.

$$V1 = \left(-R_{old1}^{-1} T_{old1} - (-R_{old2}^{-1} T_{old2})\right) \tag{15}$$

The new y axis orthogonal to x axis

$$V2 = s v_3 \times V1 \tag{16}$$

The new z axis orthogonal to xy

$$V3 = V1 \times V2 \tag{17}$$

Then the new extrinsic parameters can be written as.

$$R_{new} = \begin{bmatrix} norm\left(V1\right)^{-1} V1^T \\ norm\left(V2\right)^{-1} V2^T \\ norm\left(V3\right)^{-1} V3^T \end{bmatrix} \tag{18}$$

$$T_{new1} = R_{new}(R_{old1}^{-1} T_{old1}) \tag{19}$$

$$T_{new2} = R_{new}(R_{old2}^{-1} T_{old2}) \tag{20}$$

$$Me_{new1} = \left[R_{new} \mid \ T_{new1}\right] \tag{21}$$

$$Me_{new2} = \left[R_{new} \mid \ T_{new2}\right] \tag{22}$$

3.2 Rectification Transformation

In order to rectify the left image, we need to compute the transformation mapping the old image plane onto new image plane. It is useful to think of an image as the intersection of the image plane with the cone of ray between points in 3D space and the optical center. We are moving the image plane with the core of rays. We will see that the sought transformation is the collinearity (linear transformation of the projective plane) give by the matrix. The same result applies to the right image. The relationship between 3D coordinates and image coordinate (pixel) can be written as.

$$P_p = MP_w \tag{23}$$

$$Me_{old} = \left[Q_{old} \mid q_{old}\right] \tag{24}$$

$$Me_{new} = \left[Q_{new} \mid q_{new}\right] \tag{25}$$

$$Oc = -Q^{-1} q \tag{26}$$

$$P_w = Oc + Q_{old}{}^{-1} P_{pold} \tag{27}$$

$$P_w = Oc + Q_{new}{}^{-1} P_{pnew} \tag{28}$$

The equations of the optical rays are the following since rectification does not the move optical center hence

$$P_{pnew} = KQ_{new} Q_{old}^{-1} P_{pold} \tag{29}$$

The matrix K depends on the intrinsic parameters only, and has the following form

$$K = \begin{bmatrix} \alpha_x & 0 & c_x \\ 0 & \alpha_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \tag{30}$$

Where $\alpha_x = -f_x s_x$ and $\alpha_y = -f_y s_y$ are the focal lengths in horizontal and vertical pixel, respectively f is millimeters, sx and sy are the effective number of pixel per millimeter along the x and y axis , $c_x = s_x o_x + x_0$ and $c_y = s_y o_y + y_0$ are the coordinate of the principal point, give by the intersection of the optical axis with the image plane.

The transformation is then applied to the original left image to produce the rectified image, as in Fig. 4. Note that the pixel integer coordinate position of the rectified image correspond, in general, to non-integer position on the original image plane. Therefore, the gray levels of the rectified image are computed by bilinear interpolation.

4. EXPERIMENTAL RESULTS

In this research, we chose to apply this technique base on epipolar rectification. From the experimental of calibration results verify the proposed technique are agree with the expected specifications in Table 1. It was then possible to evaluate the result of the calibration and to determine when experimental errors were due to wrong matching or to calibration procedure or image acquisition. Then we show the results errors obtained with a determined three dimensional range measurement using passive stereo in 3D-world coordinate from points of the coplanar plane Fig. 5 (Gray level and edge detection). When performed from the rectified images directly. Fig. 6

Table 1 Camera parameters obtained with calibration rectified images directly.

Original Left image Me_{old1}	$\begin{bmatrix} 0.9984 & -0.0416 & -0.0380 & -229.9401 \\ -0.0348 & -0.9858 & 0.1645 & 209.1187 \\ -0.0443 & -0.1629 & -0.9867 & 329.7693 \end{bmatrix}$
Original Right image Me_{old2}	$\begin{bmatrix} 0.9957 & 0.0060 & 0.0921 & -208.8629 \\ -0.0091 & -0.9865 & 0.1633 & 203.3184 \\ 0.0918 & -0.1635 & -0.9823 & 307.1385 \end{bmatrix}$
Rectification Left image Me_{new1}	$\begin{bmatrix} 0.9963 & -0.0251 & -0.0827 & -220.9192 \\ -0.0113 & -0.9865 & 0.1635 & 203.6505 \\ -0.0857 & -0.1620 & -0.9831 & 339.2227 \end{bmatrix}$
Rectification Right image Me_{new2}	$\begin{bmatrix} 0.9963 & -0.0251 & -0.0827 & -150.8330 \\ -0.0113 & -0.9865 & 0.1635 & 203.6505 \\ -0.0857 & -0.1620 & -0.9831 & 339.2227 \end{bmatrix}$


Left image


Right image

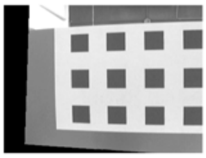

Rectification left image

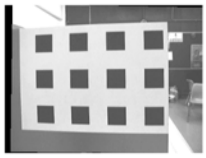

Rectification Right image

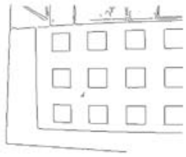
Fig. 4 the original stereo pair (top) and rectified pair (bottom). The left picture plot the epipolar lines corresponding to the point masked in the right picture.

5. CONCLUSION

In this paper aims to present the development of Epipolar Rectification to calibrate Stereo cameras. The required computation of the transformation mapping between points in 3D-space is based on calculating the image point that appears on new image plane by using calibrated parameters. So that, after completed calibration process, immediately can calculate the 3D-range measurement. And the rectification determines a transformation of each image plane such that pairs of conjugate Epipolar lines become collinear and parallel to one of the image axis. From the experimental results verify the proposed technique are agreed with the expected specifications.


left image


right image


edge detection on left image

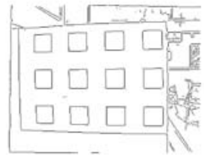

edge detection on right image

Fig. 5 we ran test to verify that the algorithm performed rectification correctly and also to check that the accuracy 3D by passive stereo when performed from the rectified images directly.

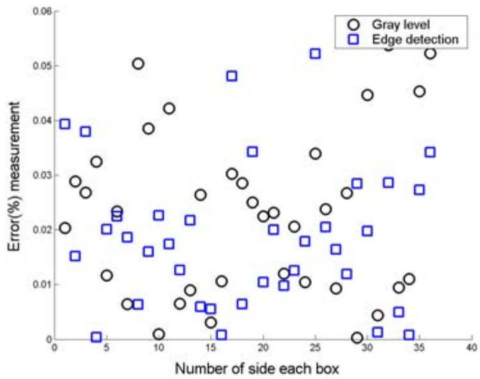


Fig. 6 Percent Error Vs size each box on coplanar plane

Table 2. Result of size each box between gray level and edge detection (Statistics on Fig. 6)

Unit(%)	Gray level	Edge detection
Min	0.00027	0.00034
Max	0.05377	0.00521
Mean	0.02292	0.01858
Std	0.01519	0.01307

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