Dynamic Trajectory Control of a Biped Robot with Curved Soles

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Abstract: This paper proposes a desired trajectory and a control algorithm for a biped robot with curved soles. Firstly, we derived the desired trajectory from a model called the Moving Inverted Pendulum Mode (MIPM) of which a contact point of the foot is moving in the horizontal direction. A biped robot with curved soles is under-actuated system, because it has one contact point with the ground during the single supporting phase. Therefore, to solve the under-actuated problem, we changed control variables, used modified dynamic equations and used the computed torque control. The simulation results show that a biped robot with curved soles walks stably. Also, fast walking and natural motion of a biped robot can be implemented.

Keywords: Biped robot, Curved soles, Under-actuated system, MIPM, Computed Torque Control

1. INTRODUCTION

One of the most important functions out of a biped robot is to walk naturally like human, so toes have very important role for a biped robot to make its walking motion pattern like human [1]. Many researchers are studying on toed biped robots with this reason. Effectiveness of toe joints is discussed in three aspects. One is utilizing it to speed up the walking, another is using it to enable a biped robot to go up higher steps, and the other is using it to whole-body action in which knees are contacting to the ground [2].

Several researches of toe joint utilization in bipedal locomotion have been proposed such as passive toe joints and active toe joints. In case of a biped robot with passive toe joints, the edge of foot is hinged to the ground [3]. Its motion is ballistic walking during single supporting. This kind of model becomes a non-holonomic constraints system. The biped robot with active toe joints has more active motion, however it requires more actuators [2] and leads to many difficult problems, such as the increase of a robot's weight, controlling, and choosing actuators.

A biped robot with curved soles has good properties without additional actuator such as the biped robot with active toe joints. The research on curved soles has started from passive walking. McGeer's results ([4], [5]) on passive dynamic walking suggest that the mechanical parameters have a greater effect on the existence. Another paper [6] proposed a self-excited biped walking mechanism. In these papers, the curved sole can roll smoothly and steadily along a level surface, maintaining any speed without loss of energy for supporting phase. We need the structure of a passive walking robot to obtain smooth motion, cutting down energy consumption and the effectiveness of toe joints.

However, there are some problems. In single supporting phase, a biped robot with curved soles has a point contact. Hence, the robot is under-actuated system as a toed robot without actuator. To control a biped robot with curved soles, we have to solve under-actuated problems. There are various studies about the control of an under-actuated biped. One method [7] is based on the definition of the reference trajectory for outputs, not as a function of time, but as a function of a configuration variable independent of the outputs. With such a control, the configuration of the robot at impact time is the desired configuration, but its velocities can differ from the desired velocities. Another approach involves parameterized reference trajectories. In this case, one derivative of the parameter is used as a supplementary input. The parameter is used to satisfy some constraints on the reaction between the feet and the ground.

In this paper, we modeled a biped robot with curved soles in Section 2. To get the desired trajectory of a biped robot with curved soles, we made a model called the Moving Inverted Pendulum Mode (MIPM) to generate this biped locomotion pattern in Section 3. It is similar to the Linear Inverted Pendulum Mode (LIPM) ([8], [9]), but robot's contact point is moving in the horizontal direction. To solve under-actuated problems, we changed control variables, solved modified dynamic equations, and used the computed torque control in Section 4. In Section 5, the computer simulations of a biped robot with curved soles to compare the performance of the proposed model with that of a biped robot with flat soles. We finally summarize conclusions in Section 6.

2. ROBOT MODELING

2.1 The structure of the biped robot with curved soles

Fig.1 shows the structure of a biped robot with curved soles. This robot is similar to the 6DOF biped robot with flat soles, but these soles are rounded. This biped robot walks in the sagittal plane. It is composed of a trunk and two identical legs.

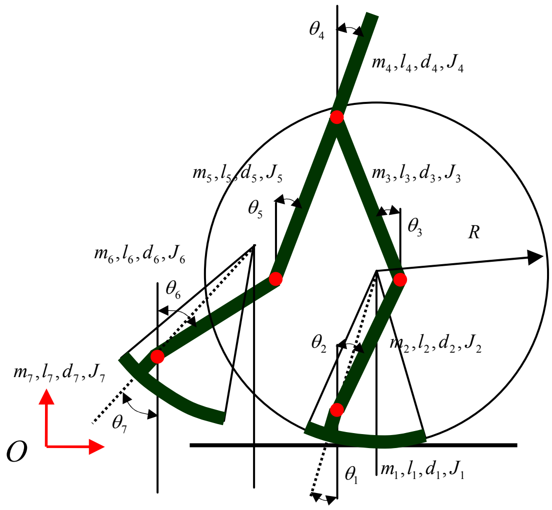

Fig.1 A biped robot with curved soles and its coordinates

Each leg is composed of three links that include curved sole. Each ankle, knee and hip is one-degree-of-freedom rotational joints. The walking cycle is composed of single support phases. During the single support phase, the vector $q=[\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7]^T$ describes the configuration of the biped. Variable θ_1 and θ_7 describes the angle of links fixed on curved soles. The others describe each joint angle in absolute coordinates. All links are assumed massive and rigid. The parameters are:

$$m_1=m_2=m_3=m_5=m_6=m_7=1[kg],\ m_4=6[kg]$$
$$l_1=l_7=0.05[m],\ l_2=l_3=l_5=l_6=0.35[m],\ l_4=0.3[m]$$
$$d_i=\frac{l_i}{2}[m],\ I_i=\frac{m_i l_i^2}{12}[kgm^2]\ \ (i=1\sim 7)$$
$$R=0.3[m]$$

where m_i is mass of the link, l_i is length of the link, d_i is length from low edge of the link to center of link mass, I_i is mass moment of inertia, and R is radius of sole.

2.2 Dynamic modeling

This model is composed of single support phases separated by instantaneous double support phases, and it has some assumptions: There is no slipping on the contact point, a landing leg is contacted on the ground, and impact force does not exist.

Dynamic equations for the model are obtained by solving the Lagrange's equation. The Lagrange's equation relates to the angular acceleration of each joint and the applied torque of each actuator; then it is convenient to obtain a nonlinear state space model, taking the angular positions and velocities as state variables.

Lagrange's equations are expressed as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\bar{q}}_i}\right)-\frac{\partial L}{\partial \bar{q}_i}=\Xi_i\quad (i=1\sim 7) \tag{3}$$

where

$$L=T-U$$
$$\Xi_i=\frac{\partial(\delta W)}{\partial\theta_i},\quad \delta W=\sum_{j=2}^{7}\tau_{j-1}\delta(\theta_j-\theta_{j-1})$$

In single support phase, the dynamic model can be written as

$$M(\bar{q})\ddot{\bar{q}}+H(\bar{q},\dot{\bar{q}})\dot{\bar{q}}+G(\bar{q})=PT \tag{4}$$

with

$$P=\begin{bmatrix}-1&0&0&0&0&0\\1&-1&0&0&0&0\\0&1&-1&0&0&0\\0&0&1&-1&0&0\\0&0&0&1&-1&0\\0&0&0&0&1&-1\\0&0&0&0&0&1\end{bmatrix},\quad T=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\\\tau_4\\\tau_5\\\tau_6\end{bmatrix}$$

where $M(7\times7)$ is the inertia matrix, $H(7\times7)$ is the vector of Coriolis and centrifugal, and $G(7\times1)$ is the vector of gravity effects. T is a (6×1) torque matrix, and P is a (7×6) matrix. The number of actuator is six but there are seven independent configuration variables.

3. REFERENCE TRAJECTORY

3.1 Moving Inverted Pendulum Mode (MIPM)

The biped locomotion is generated by the Linear Inverted Pendulum Mode (LIPM) in which the contact point is moving in the horizontal direction. In this case, we called it the MIPM (Moving Inverted Pendulum Mode). The MIPM is based on a simple biped model which consists of one particle. It is from the assumption that most of the weight of the biped robot is concentrated on its base link. Fig.2 shows the one-particle model for the MIPM. Where M denotes the mass of the base link. p_1 is a vector to a contact point. p_2 is a vector from a contact point to M. ϕ is angle of the base link.

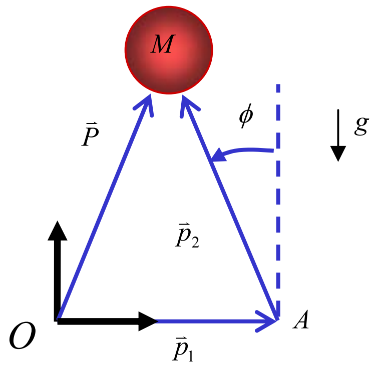

Fig.2 MIPM Model

$$\bar{p}_1=[x_1,0],\quad \bar{p}_2=[x_2,z_2],\quad \bar{g}=[0,-g]$$

From this model, we can easily derive the angular momentum equation. Angular momentum with respect to the fixed coordinate is

$$\begin{aligned}\bar{H}_o&=\bar{P}\times M\dot{\bar{P}}\\&=(\bar{p}_1+\bar{p}_2)\times M(\dot{\bar{p}}_1+\dot{\bar{p}}_2)\end{aligned} \tag{5}$$

If differentiating Eq.5

$$\begin{aligned}\dot{\bar{H}}_o&=(\dot{\bar{p}}_1+\dot{\bar{p}}_2)\times M(\dot{\bar{p}}_1+\dot{\bar{p}}_2)\\&\quad+(\bar{p}_1+\bar{p}_2)\times M(\ddot{\bar{p}}_1+\ddot{\bar{p}}_2)\\&=M[x_1+x_2,z_2]^T\times[\ddot{x}_1+\ddot{x}_2,\ddot{z}_2]^T\end{aligned} \tag{6}$$

It is assumed that the robot moves only in the sagittal plane and the height of the base link remains constant, z=Hz

Table 1: Assumption parameter

$\bar{P}=[X,Z]$	$Z=H_z$
$\bar{p}_1=[x_1,z_1]$	$z_1=0$
$\bar{p}_2=[x_2,z_2]$	$z_2=H_z$

where $\dot{z}_2 = \ddot{z}_2 = 0$ because z_2 is constant.

So, the differential angular momentum is

$$\dot{\bar{H}}_0 = MH_z(\ddot{x}_1 + \ddot{x}_2)\,. \tag{7}$$

In this mode, the moment is

$$\begin{aligned}\bar{M}_o &= \bar{P}\times M\bar{g} - \bar{p}_1 M\bar{g} \\ &= M(\bar{p}_1 + \bar{p}_2)\times \bar{g} - \bar{p}_1 M\bar{g} \\ &= M[x_1 + x_2, z_2]^T \times [0, -g]^T - Mgx_1 \\ &= Mgx_2\end{aligned} \tag{8}$$

We get a simple equation that describes the dynamics along x-axis.

$$\begin{aligned}\dot{\bar{H}}_o &= \bar{M}_o \\ MH_z(\ddot{x}_1 + \ddot{x}_2) &= Mgx_2 \\ \ddot{x}_1 + \ddot{x}_2 &= \frac{g}{H_z}x_2\end{aligned} \tag{9}$$

where x_1 and x_2 are functions of link angle ϕ. Thus,

$$\begin{aligned}x_1 &= x_1(0) - R(\phi - \phi(0)) \\ x_2 &= H_z\phi\end{aligned} \tag{10}$$

Therefore, Eq.9 is derived as

$$\ddot{\phi} = \omega\phi, \qquad \omega = \frac{g}{H_z - R} \tag{11}$$

As the result, reference locomotion trajectory is

$$\begin{aligned}&\phi(t) = C_1 e^{\omega t} + C_2 e^{-\omega t} \\ &C_1 = \frac{1}{2}\left\{\phi(0) + \frac{\dot{\phi}(0)}{\omega}\right\}, C_2 = \frac{1}{2}\left\{\phi(0) - \frac{\dot{\phi}(0)}{\omega}\right\}\end{aligned} \tag{12}$$

$\phi(0)$ is initial position of the base link and initial velocity is $\dot{\phi}(0) = \dfrac{(1+e^{\omega t})}{(1-e^{\omega t})}\omega\phi(0)\,.$

In the result, the particle in the MIPM has reference locomotion such as

$$\begin{aligned}X &= x_1 + x_2 \\ &= x_1(0) - R(\phi - \phi(0)) - H_z\phi \\ &= (H_z - R)\phi + x_1(0) + R\phi(0)\end{aligned} \tag{13}$$

This trajectory is the hip position in biped robot.

3.2 Free leg trajectory

As far as the free leg is concerned, any appropriate trajectory can be selected. The free leg trajectory is based on the contact point. In this paper, the following trajectory is selected for the free leg

$$\begin{aligned}x_f(t) &= x_1 - S\cos\{\omega_f t\} \\ z_f(t) &= \frac{h_f}{2}\left[1 - \cos\{2\omega_f t\}\right]\end{aligned} \tag{14}$$

where $S + (x_1(T) - x_1(0))$ is the stride, h_f is the maximum foot height, T is the one step period, and the stride frequency is $\omega_f = \dfrac{\pi}{T}$.

And we have to make angle trajectory of each link fixed on the curved soles. The foot angle θ_1 of the supporting leg is

$$\begin{aligned}\Delta\theta_1 &= \Delta\phi \\ \theta_1 &= \phi - \phi(0) + \theta_1(0)\end{aligned} \tag{15}$$

where $\theta_1(0)$ and $\phi(0)$ is each initial value.

The foot angle of the swing leg, θ_7, is very important. θ_7 has to connect the supporting leg's angle θ_1 smoothly, therefore we make the third order polynomial satisfied with following conditions.

$$\begin{aligned}&\theta_7 = At^3 + Bt^2 + Ct + D \\ &\theta_7(0) = \theta_1(T),\ \theta_7(T) = \theta_1(0) \\ &\dot{\theta}_7(0) = \dot{\theta}_1(T),\ \dot{\theta}_7(T) = \dot{\theta}_1(0)\end{aligned} \tag{16}$$

4. CONTROL

This paper presents a control law for the tracking of a cyclic reference trajectory. It is based upon the computed torque control. This controller uses error dynamics of the link positions and joint angles.

4.1 Modified dynamic equation

The sole of a supporting leg contacts to the floor as a point. This non-actuated point is a free joint that cannot easily balance a robot's body and stabilize robot's motions. This instability condition is caused by the under-actuated system which is difficult to control the angular momentum and body posture.

Eq.4 is the dynamic equation of a curved sole biped robot, where T is a (6×1) torque matrix, and P is a (7×6) matrix. The number of torques is six but there are seven independent configuration variables. The degree of under-actuation is the one of them during the single support phase.

The matrix P is a (7×6) full rank matrix. Thus, there exist (1×7) matrices denoted $P^\perp$ such that $P^\perp P = 0$. The matrix M is invertible, but the matrix P is not invertible. P is a (7×6) full rank matrix, thus its pseudo-inverse P^+ is such that $P^+P = I$. By definition of P^+ and $P^\perp$, the matrix $[P^\perp\ P^+]^T$ is a (7×7) invertible matrix. Thus Eq.4 is equivalent to the following set of equations:

$$P^{\perp}\left(M(\bar{q})\ddot{\bar{q}}+H(\bar{q},\dot{\bar{q}})\dot{\bar{q}}+G(\bar{q})\right)=0 \tag{17}$$

$$P^{+}\left(M(\bar{q})\ddot{\bar{q}}+H(\bar{q},\dot{\bar{q}})\dot{\bar{q}}+G(\bar{q})\right)=T \tag{18}$$

The matrix P is constant. Therefore P^{+} and $P^{\perp}$ are constant, and can be calculated offline. This system is not singular.

4.2 Control algorithm

The control is based on computed torque control. To solve the under-actuated problem in this paper, we change the control variables: hip-position (x_h, z_h), swing leg ankle-position (x_f, z_f), trunk-angle (θ_h), and foot-angle of the swing leg (θ_f). The changed control variable vector is

$$\begin{aligned} & q=[\theta_1,\theta_2,\theta_3,\theta_4,\theta_5,\theta_6,\theta_7]^T \\ & \Rightarrow r=[x_h,z_h,x_f,z_f,\theta_h,\theta_f]^T \end{aligned},$$

therefore we have six actuators, and we have six control variables.

Error dynamics is

$$\begin{aligned} & \ddot{e}+K_1\dot{e}+K_2e=0 \\ & \ddot{e}=\ddot{r}_d-\ddot{r} \\ & \ddot{r}=\ddot{r}_d+K_1\dot{e}+K_2e \end{aligned}. \tag{19}$$

Vector r is composed of dynamic equation parameters, q.

Then, we have to eliminate one parameter to solve the under-actuated problems. We chooses θ_1. $\theta_4,\theta_5,\theta_6,\theta_7$ is depended on six control variables, and θ_2,θ_3 is decided by θ_1. Therefore, we arrange Eq.17 to $\ddot{\theta}_1$, and cancel $\ddot{\theta}_1$ in Eq.19, we have a equation such as

$$\begin{bmatrix}\ddot{x}_h\\ \ddot{z}_h\\ \ddot{x}_f\\ \ddot{z}_f\\ \ddot{\theta}_4\\ \ddot{\theta}_f\end{bmatrix}=A\begin{bmatrix}\ddot{\theta}_2\\ \ddot{\theta}_3\\ \ddot{\theta}_4\\ \ddot{\theta}_5\\ \ddot{\theta}_6\\ \ddot{\theta}_7\end{bmatrix}+B\begin{bmatrix}\dot{\theta}_2^{\,2}\\ \dot{\theta}_3^{\,2}\\ \dot{\theta}_4^{\,2}\\ \dot{\theta}_5^{\,2}\\ \dot{\theta}_6^{\,2}\\ \dot{\theta}_7^{\,2}\end{bmatrix}+C \tag{20}$$

Using the error dynamics, we have $\ddot{\theta}_2,\ddot{\theta}_3,\ddot{\theta}_4,\ddot{\theta}_5,\ddot{\theta}_6,\ddot{\theta}_7$. This values are inserted in Eq.17, we have $\ddot{\theta}_1$. In the result, we can solve the torque equation, Eq.18.

This control low is used alternately during every step.

5. SIMULATION

Simulations are executed in order to show the stable walking and compare the performance of the proposed model with that of a biped robot with flat soles. The parameters for generating the walking pattern are also shown in Table 2.

This model is composed of single support phases, and it has some assumptions: There is no slipping on the contact point, a landing leg is contacted on the ground, and impact force does not exist.

Table 2: walking pattern parameters

Parameters	Symbol	Dimension
Step Time	T	0.6 (sec)
CG Height	H_z	0.7 (m)
Stride	$S+\left(x_1(T)-x_1(0)\right)$	$0.25+\alpha$ (m)
Maximum Foot Height	h_f	0.05 (m)
Radius of sole	R	0.3 (m)

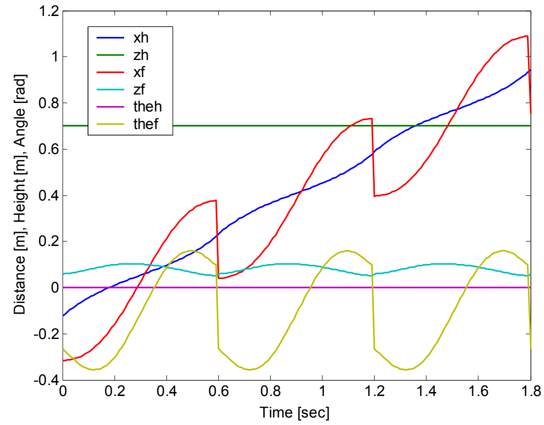

Fig. 3 Desired Trajectory

Fig.3 shows the desired trajectory in where 'xh' and 'zh' are hip-position (x_h, z_h), 'xf' and 'zf' are swing leg ankle-position (x_f, z_f), 'theh' is trunk-angle (θ_h), and 'thef' is foot-angle of the swing leg (θ_f). On single supporting phase simulation, the landing of a swing leg happens at the same time when supporting leg takes off. From this reason, the three values out of the six control values are not continuous and these cause errors.

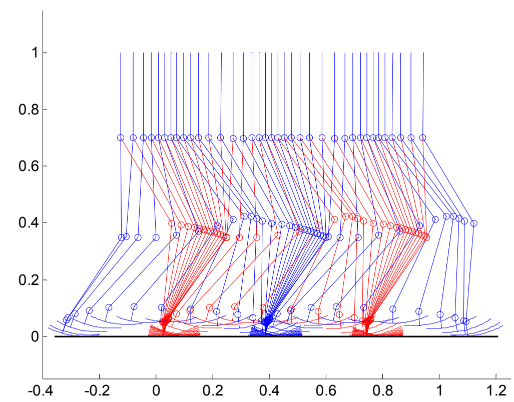

Fig. 4 Stick diagram for the locomotion

Fig.4 shows a stick graph of a simulation result. Each stick shows each link. The dots are each joint. There isn't reference position about the contact point, however a biped robot with curved soles can walk stably. In case of higher walking speed, the simulation shows stable walking. The curved soles cause the same effect as dynamic link whose length changes.

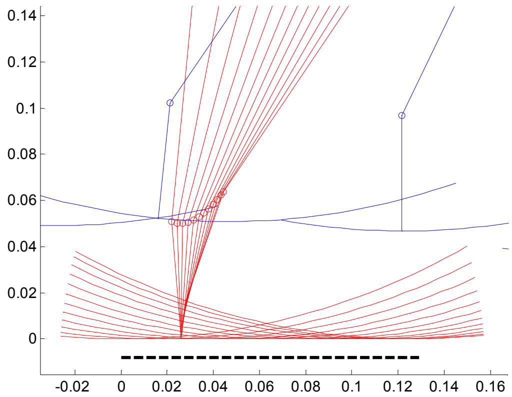

Fig. 5 Supporting Leg

Fig.5 shows supporting leg, and its contact point is moving horizontally. This figure is that the curved sole rolls on the ground. Initial contact point is 0 (m), and final contact point is 0.13 (m).

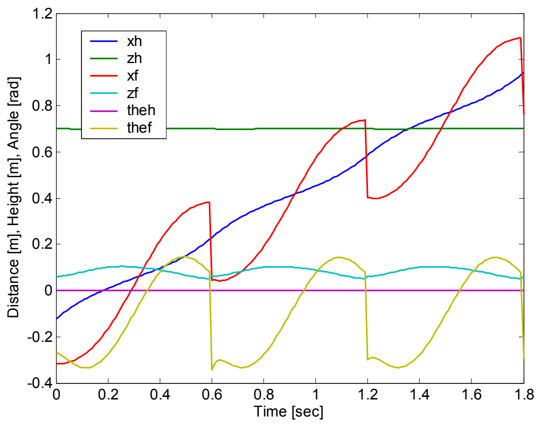

Fig. 6 Simulation Trajectory Result

As the result of simulations, Fig.6 shows the control variable trajectories. Especially, θ_f makes errors at the contact moment. It is caused by changing the swing legs.

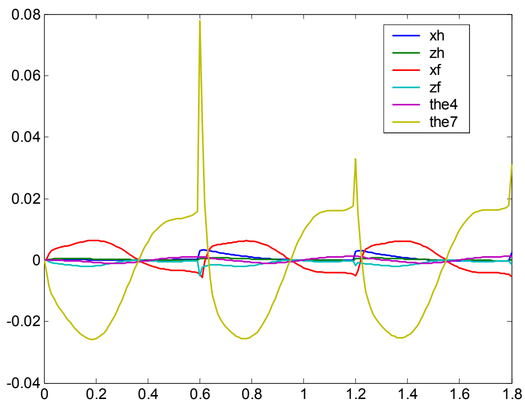

Fig. 7 Error for 3 steps

At the first contact moment, the largest error is coming. The reason is that $\theta_1(T)$ is not equal to $\theta_7(0)$, however $\theta_7(T)$ is equal to $\theta_1(0)$, thus the error is periodic after the second step.

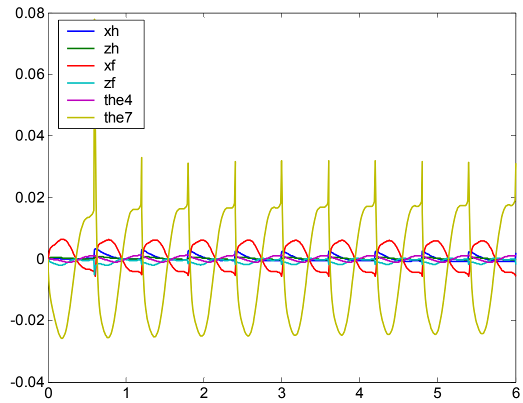

Fig. 8 Error for 10 steps

Fig.8 show the errors of a simulation for 10 steps. After the second step, the magnitude of the hip position errors are within 0.003(m) and 0.001(m), it of the trunk angle error is within 0.001(rad), and it of the swing foot positions are within 0.006(m), 0.002 (m).

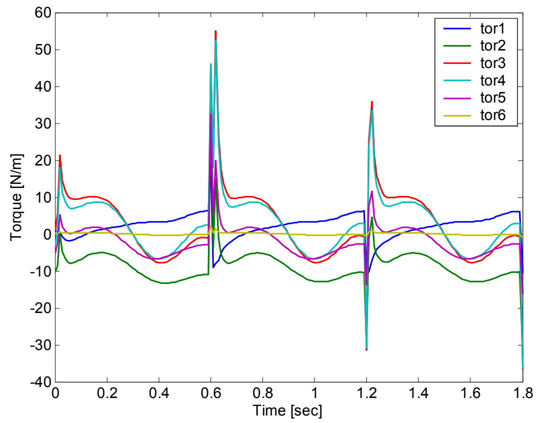

Fig. 9 Torque for 3 steps

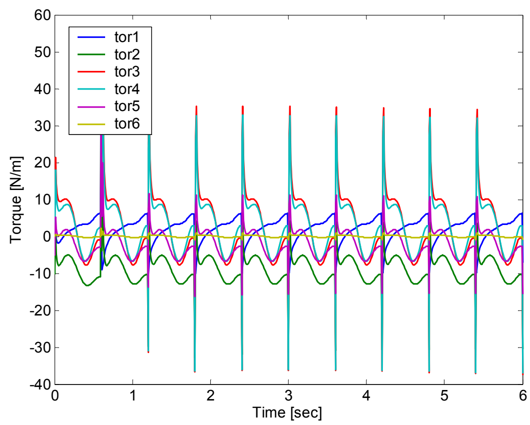

Fig. 10 Torque for 10 steps

Fig. 9 shows an applied torque in this simulation. It shows that an applied torque is realistic and reasonable. For the single supporting phase, the torque is smaller than that of a biped robot with flat soles except for contact moments. After the second step, the torque has the same figure.

6. CONCLUSION

In this paper, we proposed the desired trajectory for a biped robot with curved soles and the control algorithm to solve the under-actuated problems, and then we simulated it. To get the desired trajectory, we proposed the MIPM in which the contact point is moving in the horizontal direction. To solve the under-actuated problems, we changed control variables, used modified dynamic equations and used the computed torque control.

Simulation results show that a biped robot with curved soles can walk stably, though the periodical error occurs. And fast walking and natural motion of biped robot can be implemented. A biped robot with curved soles can take place of the toed biped robot without additional actuator.

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