Energy Optimization of a Biped Robot for Walking a Staircase Using Genetic Algorithms

Kweon Soo Jeon* and Jong Hyeon Park**

* Department of Precision Mechanical Engineering, Hanyang University, Seoul, Korea
(Tel : +82-2-2297-3786; E-mail: jks0422@ihanyang.ac.kr)
** School of Mechanical Engineering, Hanyang University, Seoul, Korea
(Tel : +82-2-2297-3786; E-mail: jongpark@hanyang.ac.kr)

Abstract: In this paper, we generate a trajectory minimized the energy gait of a biped robot for walking a staircase using genetic algorithms and apply to the computed torque controller for the stable dynamic biped locomotion. In the saggital plane, a 6 degree of freedom biped robot that model consists of seven links is used. In order to minimize the total energy efficiency, the Real-Coded Genetic Algorithm (RCGA) is used. Operators of genetic algorithms are composed of a reproduction, crossover and mutation. In order to approximate the walking gait, the each joint angle is defined as a 4-th order polynomial of which coefficients are chromosomes. Constraints are divided into equality and inequality. Firstly, equality constraints consist of position conditions at the end of stride period and each joint angle and angular velocity condition for periodic walking. On the other hand, inequality constraints include the knee joint conditions, the zero moment point conditions for the x-direction and the tip conditions of swing leg during the period of a stride for walking a staircase.

Keywords: Biped Robot, Genetic Algorithm, Staircase, Trajectory, Optimization

1. INTRODUCTION

Human beings walk stably under the various locomotion conditions of the environment and generate a walking pattern for minimizing energy. Therefore, many researches for a biped robot have been studied for the structure similar to human, the adaptability in various terrains and the generation of energy-efficient trajectories.

Generation of a gait trajectory for a biped robot has been proposed by many methods. Park and Kim [1] proposed the gravity-compensated inverted pendulum mode (GCIPM).

In the study to generate low energy gaits, various optimization algorithms are used and the variables of basis functions to approximate the walking gait are used for design variables generally. Choi et al. [2] proposed that optimal via-points data are found using genetic algorithm which minimizes the sum of deviation of velocities and accelerations as well as jerk. Cheng and Lin [3] proposed that the design of the controller and the gait is formulated as a parameter search problem and a genetic algorithm is applied to help the design. Park and Choi [4] proposed a method that minimizes the energy consumption by finding the optimal locations of the mass centers of the links, and the optimal trajectory of the legs. Chevallereau et al. [5] searched for the optimal stride and period to generate optimal trajectories.

Walking a staircase is one of the irregular ground conditions. Shih [6] proposed to synthesize an efficient walking pattern for a practical biped robot when ascending and descending stairs for a biped robot with 7 DOF.

In this paper, we generate a trajectory minimized energy gait of a biped robot for walking a staircase using genetic algorithms and apply to the computed torque controller for the stable dynamic biped locomotion. In the saggital plane, a 6 degree of freedom biped robot model that consists of seven links is used. In order to minimize the total energy, the Real-Coded Genetic Algorithm (RCGA) is used.

The dynamics of a biped robot is described in Section 2. The constraints and formulas for walking a staircase are presented in Section 3. Section 4 describes computer simulation and comparisons of the modified GCIPM method, followed by conclusions in Section 5.

2. BIPED ROBOT MODEL

In the saggital plane, a 6 degree of freedom biped robot model that consists of seven links, as shown in Fig. 1, is used in the study [7, 8].

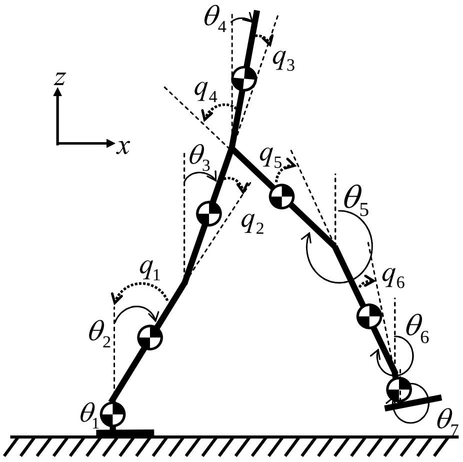


Fig.1 6 DOF biped robot model

The dynamic equation applied to a biped robot is derived from Lagrange's equation substituted from the kinetic and potential energies of each link. The dynamic equation can be written as:

$$M(\theta)\ddot{\theta} + C(\theta,\ \dot{\theta}) + G(\theta) = E\tau \qquad (1)$$

where $M(\theta)$ is the (6 × 6) inertia matrix, $C(\theta,\ \dot{\theta})$ is the (6 × 1) vector of coriolis and centrifugal force, $G(\theta)$ is the (6 × 1) vector of gravity force, and E is the (6 × 6) matrix.

$$E = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (2)$$

The parameters of the biped robot model are listed in Table 1.

Table 1 The parameters of the biped robot model

Link No.	Length (m)	Mass (kg)
1	0.1	1
2	0.4	5
3	0.4	4
4	0.5	6
5	0.4	4
6	0.4	5
7	0.1	1

We assume that the mass of link is concentrated on the center of each link. We also assume that walking cycle of a biped robot is divided into two phases that repeat alternately, i.e., the single support phase and double support phase.

3. ENERGY OPTIMAL TRAJECTORY BY GENETIC ALGORITHMS

3.1 Genetic algorithms

Genetic algorithm has three operators. Reproduction increases average fitness values, crossover exchanges chromosomes' information and mutation prevents convergence to a local minimum/maximum [9]. Parameters of genetic algorithm are listed in Table 2.

Table 2 Parameters used in the genetic algorithm

Parameters	Values
Maximum Generations	5,000
Population	30
Chromosome Length	14
Crossover Ratio	0.9
Mutation Ratio	0.02

In this study, instead of a Binary-Coded Genetic Algorithms (BCGA), we used a Real-Coded Genetic Algorithms (RCGA) because BCGA has many problems in the practical applications [10].

3.2 Equality constraints

In order to ascend stairs for a biped robot, the tip position of swing leg must be satisfied with following conditions at the start and end of the stride period.

At $t = 0 : x_{tip}(0) = -S$ (3)

$t = 0 : z_{tip}(0) = -H$ (4)

At $t = t_f : x_{tip}(t_f) = S$ (5)

$t = t_f : z_{tip}(t_f) = H$ (6)

where t_f is a stride period, S is a stride and H is a stair height.

For the biped robot to have steady and repeatable walking pattern, the following repeatability conditions should be satisfied.

$$q_i(0) = q_{7-i}(t_f) \quad (i = 1, \cdots, 6) \qquad (7)$$

$$\dot{q}_i(0) = \dot{q}_{7-i}(t_f) \quad (i = 1, \cdots, 6) \qquad (8)$$

3.3 Inequality constraints

In order to ascend stairs for a biped robot during a stride period, the tip and toe of swing leg must be satisfied with following stair conditions, Fig. 2 :

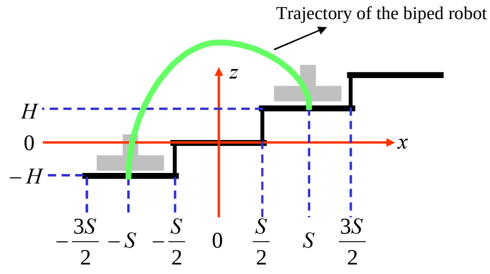


Fig. 2 Stair conditions

$$\begin{aligned} &\text{if } -\frac{3S}{2} < x_{tip}(t) \le -\frac{S}{2} \text{ then } z_{tip}(t) > -H + \delta h \\ &\text{if } -\frac{S}{2} < x_{tip}(t) \le \frac{S}{2} \text{ then } z_{tip}(t) > \delta h \\ &\text{if } \frac{S}{2} < x_{tip}(t) \le \frac{3S}{2} \text{ then } z_{tip}(t) > H + \delta h \end{aligned} \qquad (9)$$

$$\begin{aligned} &\text{if } -H < z_{toe}(t) \le 0 \text{ then } x_{toe}(t) > -\frac{S}{2} - \delta s \\ &\text{if } 0 < z_{toe}(t) \le H \text{ then } x_{toe}(t) > \frac{S}{2} - \delta s \\ &\text{if } H < z_{toe}(t) \le 2H \text{ then } x_{toe}(t) > \frac{3S}{2} - \delta s \end{aligned} \qquad (10)$$

where δh and δs are a safe boundary in order that it may be avoided that the tip and toe of swing leg collide with the stair. And time t didn't include $t = 0$ and $t = t_f$.

The following constraints to have a human-like locomotion and to avoid any singularity are knee joint conditions :

$$\theta_2 - \theta_3 > \delta\theta \qquad (11)$$

$$\theta_6 - \theta_5 > \delta\theta \qquad (12)$$

where $\delta\theta$ is a safe boundary.

The final constraint is related to the stability of a biped robot. We can know the stability of a biped robot by the zero-moment point (ZMP). When the ZMP exists within the foot print of the supporting leg, we can say that the biped robot is stable.

$$\|x_{ZMP}\| < \frac{\Delta}{2} \tag{13}$$

where Δ is a safe boundary, and

$$x_{ZMP} = \frac{\sum_{i=1}^{6} m_i(\ddot{z}_i + g)x_i - \sum_{i=1}^{6} m_i \ddot{x}_i z_i - \sum_{i=1}^{6} I_i \ddot{\theta}_i}{\sum_{i=1}^{6} m_i(\ddot{z}_i + g)} \tag{14}$$

where $(x_i,\ z_i)$ is the position of the mass center of link i and I_i is the inertia moment of link i.

3.4 Optimization methods

In order to approximate the walking gait, the each joint angle is defined as a 4-th order polynomial of time t.

$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \\ \theta_7 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{bmatrix} 1 \\ t \\ t^2 \\ t^3 \\ t^4 \end{bmatrix} \tag{15}$$

where coefficients, $a_{i,j}$ ($i = 1,\cdots,6$, $j = 1,\cdots,5$) are design variables. Therefore, the total number of design variables is 14 except for 16 for equality constraints, Eqs. (3) to (8).

The performance index to be minimized is :

$$J = \frac{1}{2}\int_0^{t_f} p^T Q p dt \tag{16}$$

where $p = \tau\dot{q}$ describes the powers applied at each joint and $Q = diag\left[\omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4 \quad \omega_5 \quad \omega_6\right]^T$ is positive definite matrix and the elements of Q, $\omega_{1\sim6}$, are the weighting factor on control torque of relative driving actuators.

The inequality constraints, Eqs. (9) to (13), are as follows :

$$g_j(\alpha) \le 0 \quad (j = 1,\cdots,n) \tag{17}$$

Transformation methods convert the constraint optimization problem defined in Eqs. (9) to (13) into an unconstraint problem for the transformation function :

$$F(\alpha,\ r) = J(\alpha) + P(g(\alpha),\ r) \tag{18}$$

where r is a vector of penalty parameters and P is a real valued function whose action of imposing the penalty is controlled by r. The form of penalty function P depends on the transformation method used. The transformation method used in this paper is exterior penalty function method :

$$P(g(\alpha),\ r) = \sum_{j=1}^{n} r_j [g_j^{\ +}(\alpha)]^2 \tag{19}$$

where $g_j^{\ +}(\alpha) = \max(0,\ g_j(x))$, and r_j is a scalar. $g_j^{\ +}(\alpha)$ is zero if inequality is inactive, $(g_j(\alpha) < 0)$ and it is positive if inequality is violated, i.e. $(g_j(\alpha) > 0)$.

4. SIMULATION

We used Matlab program in order to simulate locomotion of a biped robot for walking a staircase. Parameters used in the simulation are following Table 3:

Table 3 Parameters used in the simulation

Parameters	Values
S	0.3m
H	0.05m
δs	0.001m
δh	0.001m
$\delta\theta$	0.0001rad
Δ	0.19m

4.1 Modified GCIPM method

In this paper, we modified that trajectories of the swing leg and hip are proposed by [1] as the following :

Trajectory of the swing leg :

$$x_{tip}(t) = -S\cos(\omega_f t) \tag{20}$$

$$z_{tip}(t) = \frac{h_f}{2}[1 - \cos(\omega_f t)] + 2Ht - H \tag{21}$$

Trajectory of the hip :

$$x_{hip}(t) = C_1 e^{\omega t} + C_2 e^{-\omega t} + \eta\cos(\omega_f t) \tag{22}$$

$$z_{hip}(t) = H_z + Ht \tag{23}$$

where $\omega = \sqrt{\dfrac{g}{H_z}}$, $\omega_f = \dfrac{\pi}{T}$, T is a step time, H_z is a center of gravity height, h_f is a maximum foot height, and C_1, C_2, η are coefficients.

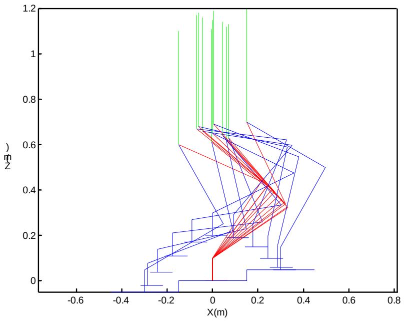


Fig. 3 The Stick diagram of a biped robot using modified GCIPM

Fig. 3 is the locomotion of a biped robot for walking a staircase using modified GCIPM. And Fig. 4 is torque and power at the joints with the locomotion generated by the modified GCIPM.

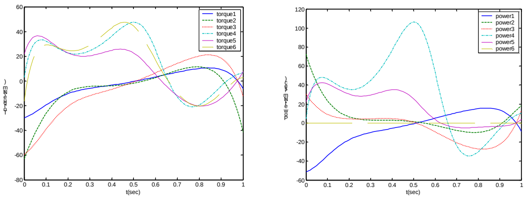


Fig. 4 Torque and power on each joint

4.2 Method using genetic algorithms

The parameters of genetic algorithm are as previously shown in Table 2. Genetic operators are used that reproduction is a gradient-like selection method, crossover is a modified simple crossover method, and mutation is boundary mutation method. If the number of generation comes to the maximum generation and the value of the cost function is constant for 50 generations, simulations come to stop.

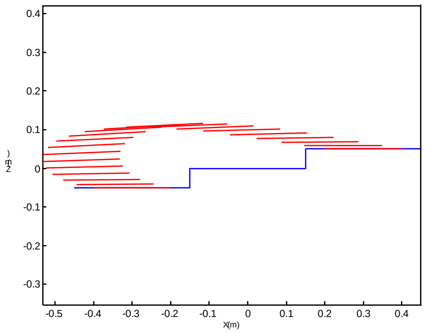


Fig. 5 The foot diagram in locomotion

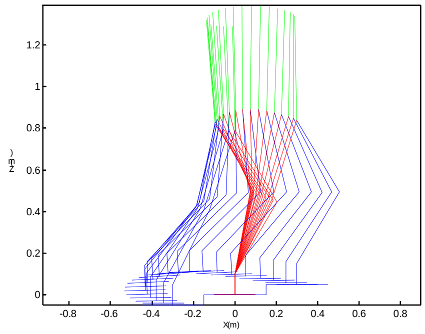


Fig. 6 The Stick diagram of a biped robot

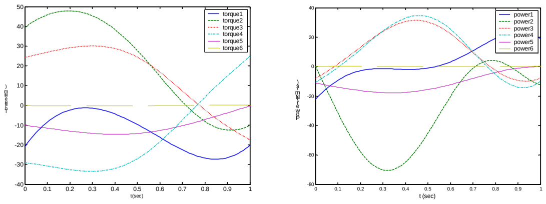


Fig. 7 Torque and power on each joint

Fig. 5 and Fig. 6 are the locomotion of a biped robot for walking a staircase using genetic algorithms. They are similar to locomotion of a human for ascending stairs. Especially, we know that the foot moves to some backward during beginning of a stride period in order to avoid that it collide with the stair. And Fig. 7 is torque and power at the joints with the locomotion generated by genetic algorithms

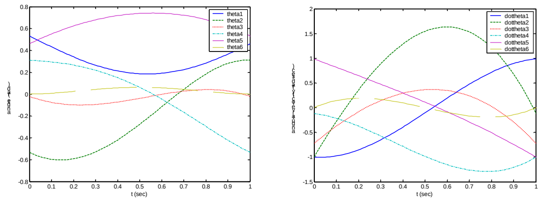
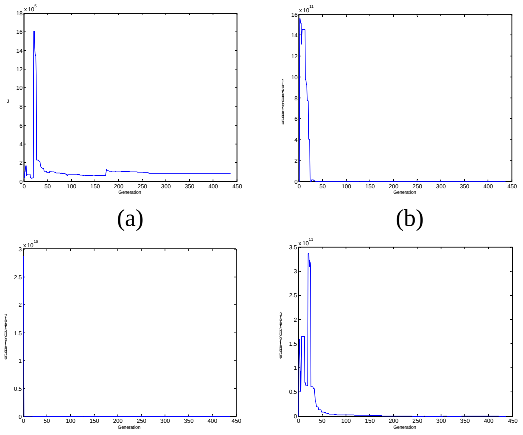


Fig. 8 Angle and angular velocity on each joint

Fig. 8 is angle and angular velocity on each joint which are satisfied with repeatability condition for the biped robot to have steady and repeatable walking pattern.



(a) (b)

(c) (d)

Fig. 9 Cost functions of the inequality constraints

Fig. 9 is compared with cost functions of the inequality constraints. Fig. 9 (a) is the performance index, (b) is the stair condition, (c) is the knee joint condition, and (d) is the ZMP condition.

4.3 The comparison of the modified GCIPM and GA

We compare the modified GCIPM with GA method using the energy efficiency.

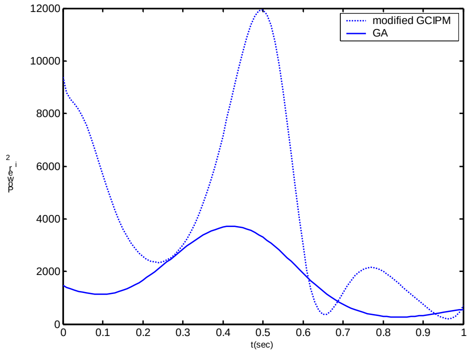


Fig. 10 Comparison of the energy efficiency

As Fig. 10, we know that the GA method is more efficient than the modified GCIPM.

4.4 Simulation of walking a staircase

The computed torque controller is used the simulation. We use desired angle, angular velocity, and angular acceleration on each joint which has been searched by genetic algorithms. Fig 11 is the locomotion of a biped robot for walking staircases.

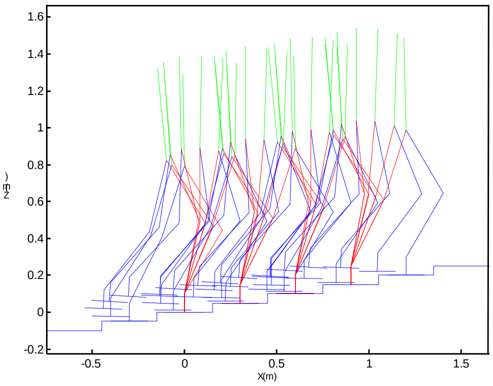


Fig. 11 The simulation of a biped robot

Fig. 12 is the errors between desired values and estimated values on each joint.

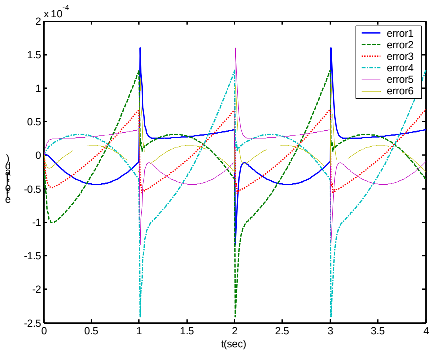


Fig. 12 The errors between desired and estimated values

5. CONCLUSION

In this paper, genetic algorithms are used for trajectory optimization for walking a staircase of a biped robot. In order to find out the optimal trajectory, the coefficients of 4-th order polynomials are used as design variables. And this research compares the energy efficiency between modified gravity-compensated inverted pendulum model for walking a staircase and the proposed method. We show that the latter is more efficient.

As a future work, we plan to generate for various height of stairs using this method.

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