Biped Gait Generation based on Linear Inverted Pendulum Mode On Flexible Terrain

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Abstract: In this paper, gait generation algorithm based on Linear Inverted Pendulum Mode is extended considering that the terrain is uncertain and flexible. Deformation of the soft terrain by the weight of the biped robot is taken into account to design the desired motion of the swing leg. Landing time disagreement caused by dynamics of the robot is also considered and a method to adjust gait is proposed. Results of numerical simulation show the effectiveness of the proposed method.

Keywords: biped robot, flexible terrain, linear inverted pendulum mode, gait generation,

1.Introduction

It is necessary for biped robots to achieve dynamic walk in order to be efficient and practical. For a couple of decades, dynamic walk has been studied by many researchers. Vukobratovic [1] considered Zero Moment Point (ZMP) firstly which showed stability margin of dynamic gait. ZMP is one of the most widely used concept to control dynamic biped gait. Another useful model of dynamic gait is Inverted Pendulum Mode (IPM) which gives a good approximation of human's walking motion. Miyazaki and Arimoto [2] studied the nature of dynamic walk and they revealed that the motion could be modeled by a single inverted pendulum. Furushou et.al [3] developed a gait synthesis algorithm based on IPM. Considering geometrical constraints on the path of the dynamic walk, Kajita [4][5][6] showed that dynamics of center of mass (COM) of a biped robot with mass-less legs were able to become a conservative linear system without any approximation. Gait (walking speed, stopping, and changing walking direction and so on) could be controlled by controlling the value of the constant of the motion which was called orbital energy. Owing to this simplicity, especially for small humanoids it is practical to implement gait generation algorithm based on LIPM.

Considering practical use of biped robots, they are required to be able to walk on rough and flexible terrain such as soft carpets, swamps and so on which can be found in human's daily environment. To this end, in this paper uncertainty of the environment caused by flexible terrain is taken into account. This uncertainty causes earlier (or later) contact between the tip of the swing leg and the terrain than the expected moment. Since the moment when the biped robot switches the support leg and the swing leg is significantly important in order to control the value of the orbital energy, this disagreement in the contact leads poor performance or it even makes the biped robot fall down. In order to overcome this difficulty, in this study an adaptive scheme to control the timing of the contact is proposed to generate robust gait.

The disagreement in contact between the swing leg and the terrain is also caused by dynamical effect of the motion of the swing leg. This effect was neglected in the original LIPM because the biped robot was modeled with mass-less legs. It is difficult to control the tip of the swing leg of the biped robot on uncertain flexible terrain. This paper shows that the adaptive gait generation algorithm is also effective to this tracking error of the swing leg from the desired trajectory.

In the following section, the dynamical model of the biped robot is shown. The adaptive gait generation algorithm is proposed in Section3. Numerical simulation in Section4 validates the proposed method and conclusions follow (Section5)

2. Biped Robot System

In this study, the biped robot is modeled by 4 links corresponding to shank and thigh as shown Fig.1. It is assumed that the biped robot moves in 2-dimensional sagittal plane. The attitude of the biped robot is denoted by angles of Joints $(\theta_1,\theta_2,\theta_3,\theta_4)$ are shown in Fig.1. The tip of the support leg of the biped robot is denoted as (x_0, y_0) in the inertial frame. And it is supposed that each link is uniform rigid body, and that the COM of each link lies middle of it. Denote length and mass of link1 and link2 as L_1, L_2 m_1, m_2 respectively. It is assumed that the biped robot does not have any *a priori* information of the terrain, e.g. irregular. Actuators are attached on each joint and tips of legs of the biped robot. Therefore, the attitude of the biped robot is controlled by control inputs(torque) generated by each motor. The motion equation of the biped robot is given by

$$\boldsymbol{M}(\boldsymbol{q}(t))\ddot{\boldsymbol{q}}(t)+\boldsymbol{C}(\boldsymbol{q}(t),\dot{\boldsymbol{q}}(t))\dot{\boldsymbol{q}}(t)+\boldsymbol{G}(\boldsymbol{q}(t))=\boldsymbol{\tau}(t)+\boldsymbol{R}(t) \tag{2.1}$$

where $\boldsymbol{q}=(x_0, y_0, \boldsymbol{\theta}^{\mathrm{T}})^{\mathrm{T}}$ and $\boldsymbol{\theta}^{\mathrm{T}}=(\theta_1,\theta_2,\theta_3,\theta_4)$. $\dot{\boldsymbol{q}}(t)$ denotes time derivative of $\boldsymbol{q}(t)$.

$\boldsymbol{M}$: matrix of intertia, $\boldsymbol{C}\dot{\boldsymbol{q}}$: centrifugal forces, $\boldsymbol{G}$: gravity forces
$\boldsymbol{\tau}$: control inputs, $\boldsymbol{R}$: reaction forces from the terrain

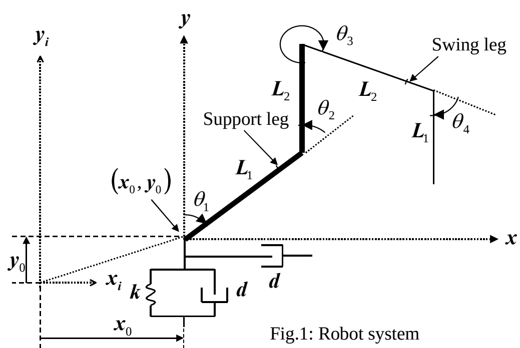

Fig.1: Robot system

The terrain is modeled by soft spring-damper system, where the spring coefficient and damper coefficient are k and d respectively. Values of k and d are set to model the flexible terrain. Let F_1 and F_2 denote reaction forces exerting on the support leg and the swing leg from the terrain respectively. Let

V_{1x} denote the velocity of the tip of the support leg in x axis, and let Y_1 and V_{1y} denote displacement and velocity of the tip of the support leg in y axis. Denoting F_1 as $F_1 = (F_{1x}, F_{1y})$, the model of reaction forces from the terrain is given as follows.

$$F_{1x} = -dV_{1x}, F_{1y} = -kY_1 - dV_{1y} \quad (\text{if } Y_1 < 0 \text{ and } V_{1y} < 0) \tag{2.2}$$

$$F_{1x} = F_{1y} = 0 \quad (\text{otherwise}) \tag{2.3}$$

Similarly, reaction forces on the support leg is also modeled as

$$F_{2x} = -dV_{2x}, F_{2y} = -kY_2 - dV_{2y} \quad (\text{if } Y_2 < 0 \text{ and } V_{2y} < 0) \tag{2.4}$$

$$F_{2x} = F_{2y} = 0 \quad (\text{otherwise}) \tag{2.5}$$

where $V_{2x}, Y_2, V_{2y}, F_{2x}$ and F_{2y} are defined as in the support leg. Then, the relation between reaction forces from the terrain and generalized forces affecting to joints of the biped robot is following.

$$\boldsymbol{R} = \boldsymbol{J_1^T}(\boldsymbol{q})\boldsymbol{F_1} + \boldsymbol{J_2^T}(\boldsymbol{q})\boldsymbol{F_2} \tag{2.6}$$

where $\boldsymbol{J_1}$ and $\boldsymbol{J_2}$ are Jacobi matrices from the origin of inertia frame to the contact point of the support leg and the swing leg respectively.

3.Control Algorithm

In section 3.1, LIPM[4][5][6] is introduced shortly. After that, it is considered that the biped robot walks on flexible terrain in section 3.2 and controller is designed in section 3.3. In this case, reaction forces may disturb the value of orbital energy, therefore the algorithm extended in section 3.4 in order to make the biped robot be able to walk on the flexible terrain.

3.1 LIPM

In LIPM[4][5][6], the biped robot is modeled by mass point(mass: m) and mass less leg(length: l) as shown in Fig.2. Force(F) to expand and contract leg is given as input.

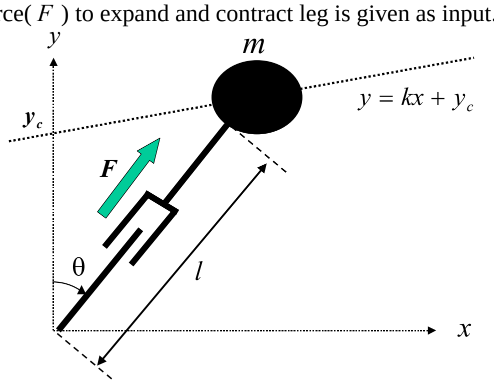

Fig.2: Linear Inverted Pendulum Model

When the mass m is constrained on the line $y = kx + y_c$, dynamics of the mass m become

$$\ddot{x} = \frac{g}{y_c} x \tag{3.1}$$

where g is the constant of gravitational acceleration. Consider the following function.

$$E = -\frac{mg}{2y_c} x^2 + \frac{1}{2} m\dot{x}^2 \tag{3.2}$$

where x is the position of the mass m and $\dot{x}$ is time derivative of x . This function is conservative and it is called "Orbital energy". E represents the motion of COM i.e. the mass is moving "forward" when $E > 0$, and it changes the direction of it's motion when $E < 0$. In this sense walking motion is able to be controlled by changing the value of E .

Assume following assumptions:

1. The swing leg and the support leg switches instantaneously.
2. The velocity of COM does not change by switching legs.

Then, the relation between the value of E at nth-step, denoted as E_n , and that at n+1th-step, denoted as E_{n+1} is written as follows.

$$E_{n+1} = E_n + \frac{mg}{2y_c} x^2 - \frac{mg}{2y_c} (L - x)^2 \tag{3.3}$$

where L is the length of the stride of the gait. Given L, E_{n+1} is equal to the desired value of E , denoted as E_d , if the biped robot switches legs when $x(t_f) = x_f$ where

$$x_f = \frac{y_c}{mgL}(E_d - E_n) + \frac{L}{2} \tag{3.4}$$

The x_f, v_f, and t_f are derived as follow, where v_f denotes $\dot{x}(t_f)$. Firstly, by solving (3.1) for $x(0) = -x_i, x(0) = v_i$,

$$x(t) = -x_i \cosh\left(\frac{t}{T_c}\right) + T_c v_i \sinh\left(\frac{t}{T_c}\right) \tag{3.5}$$

$$v(t) = \frac{-x_i}{T_c} \sinh\left(\frac{t}{T_c}\right) + v_i \cosh\left(\frac{t}{T_c}\right) \tag{3.6}$$

where $T_c = \sqrt{\frac{y_c}{g}}$.

Then, E_n, E_{n+1} are denoted by

$$E_n = -\frac{mg}{2L_{hd}} x_f^2 + \frac{m}{2} v_f^2 \tag{3.7}$$

$$E_{n+1} = -\frac{mg}{2y_c}(L - x_f)^2 + \frac{m}{2} v_f^2 \tag{3.8}$$

Let the value of E_{n+1} be given as desired orbital energy E_d which is given as a parameter. And eliminating v_f from (3.7) and (3.8), x_f is given as (3.4).

Since the orbital energy does not change through a walking cycle, E_n can also be denoted as the following.

$$E_n = -\frac{mg}{2y_c}(-x_i)^2 + \frac{m}{2} v_i^2 \tag{3.9}$$

From (3.4), (3.8) and (3.9), v_f is given as

$$v_f = \sqrt{\frac{2}{m} E_d + \frac{g}{y_c}(L - x_f)^2} \tag{3.10}$$

From (3.5), (3.6), (3.9), and (3,10)

$$t_f = T_c \ln\left(\frac{x_f + T_c v_f}{-x_i + T_c v_i}\right) \tag{3.11}$$

This means that the gait generating algorithm based on LIPM gives the moment of leg switching with respect to the position of COM. Therefore it is important that the biped robot switches legs at the adequate moment.

3.2 Reference trajectory of swing leg

In this subsection, the desired gait motion based on LIPM for 4 link biped robot is extended in order to take the flexible terrain into account. Boundary conditions at the beginning and the end of one walking cycle are shown in Fig.3 and Fig.4. The dashed circle of Fig.3 and Fig.4 denote the position of COM of the biped robot.

X_i, V_i, X_f, V_f and S_n corresponding to x_i, v_i, x_f, v_f and t_f and other parameters are defined in Table.1 which shows notations of parameters in figures. The desired motion of COM is same as in original LIPM[4][5][6]. The height of COM is constrained on a line as in Fig.2.

Table.1: Parameters of Fig3 and Fig4

ς : time since the swing leg took off
m : mass of the biped robot
L_h : height of COM when COM is above supporting point
X_i : position of COM at $\varsigma = 0$
V_i : velocity of COM at $\varsigma = 0$
δ_i : vertical displacement between the tip of swing leg and the tip of support leg
L : stride
S_n : the time when the robot switches legs with respect to ς.
X_f : position of COM at $\varsigma = S_n$
V_f : velocity of COM at $\varsigma = S_n$
δ_f : vertical displacement between the tip of swing leg and the tip of support leg

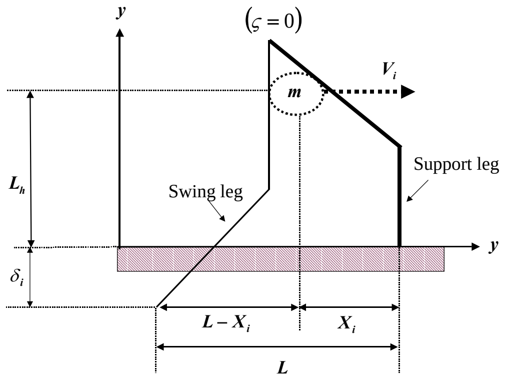

Fig.3:Beginning of a walking cycle

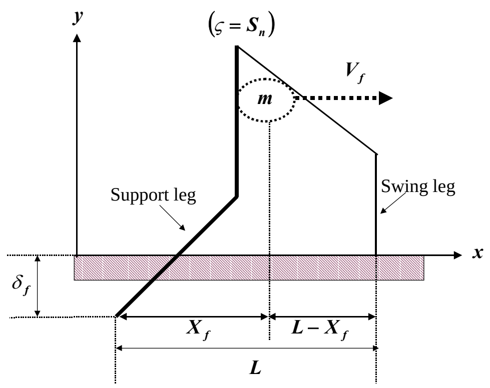

Fig.4:Ending of a walking cycle

Next, the desired trajectory of the swing leg is defined by using the parameters of boundary conditions. They are denoted as $U_{dx}(\varsigma)$ and $U_{dy}(\varsigma)$. $U_{dx}(\varsigma)$ denotes the desired x position from COM to the tip of the swing leg. $U_{dy}(\varsigma)$ denotes the desired y position from the tip of the support leg to the tip of the swing leg. They are required to satisfy boundary conditions shown in Fig.3 and Fig.4. Conditions that $U_{dx}(\varsigma)$ must satisfy are followings.

$$U_x(0) = -(L - X_i) \tag{3.12}$$

$$U_x(S) = L - X_f \tag{3.13}$$

$$\dot{U}_{dx}(0) = -V_i \tag{3.14}$$

$$\dot{U}_{dx}(S) = -V_f \tag{3.15}$$

(3.12) and (3.13) are conditions with respect to step length, and (3.14) and (3.15) are conditions with respect to velocity such that the relative velocity between the tip of the swing leg and terrain becomes zero. They mean that the tip of the swing leg performs smooth takeoff and landing in x axis. In this study, $U_{dx}(\varsigma)$ is designed by third order polynomial as follows.

$$U_{dx}(\varsigma) = \left\{ \frac{2(-2L + X_f + X_i)}{S^3} - \frac{V_i + V_f}{S^2} \right\} \varsigma^3 + \left\{ \frac{3(2L - X_f - X_i)}{S^2} + \frac{2V_i + V_f}{S} \right\} \varsigma^2 - V_i \varsigma - (L - X_i) \tag{3.16}$$

The objective of this study is to extend LIPM for biped robot to be able to walk on the flexible terrain. As expressed above, the timing of leg switching is important. When the terrain is soft or deformable, the biped robot sinks into the terrain. Hence, the swing leg contacts with the terrain earlier than the expected moment. This mismatch disturbs the system, and it may make the biped robot fall down. In order to overcome this difficulty, the effect of flexible terrain is taken into account. For simplicity it is assumed that the terrain is even, and flexibility of the terrain is uniform. The following is assumed.

$$\delta_f = \delta_i \tag{3.17}$$

Taking this into consideration, U_{dy} is required to satisfy following conditions.

$$U_{dy}(0) = -\delta_i \tag{3.18}$$

$$U_{dy}(S) = \delta_f = \delta_i \tag{3.19}$$

(3.18) and (3.19) are conditions of the position of the tip of the swing leg in y axis when the swing leg takes off or touches down. Since it is assumed that the biped robot does not have *a priori* information about the terrain such as its irregularity or hardness, it is difficult to make the tip of the swing leg contact the terrain at the desired moment exactly. When the desired velocity vanishes at the desired moment of the contact, the tip stays near the terrain before the contact and it may cause unexpected collision. To avoid this, $U_{dy}(\varsigma)$ is designed as the velocity of the swing leg does not vanish as the following.

$$U_{dy}(\varsigma) = \left\{ \frac{h}{1 - \cos\frac{3}{4}\pi} \right\} \left\{ -\cos\frac{3}{4}\pi + \cos\left[\frac{3\pi}{2}\left(\frac{2\varsigma}{S} - 0.5 \right) \right] \right\} - \delta_i \left(1 - \frac{2\varsigma}{S} \right) \tag{3.20}$$

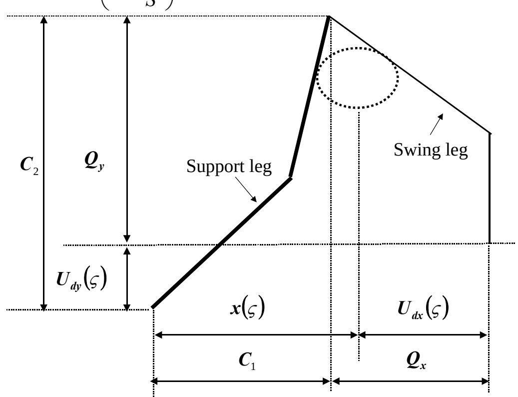

Fig.5:Relation of design parameters

3.3 Controller Design

Since the biped robot has dynamics (2.1), a controller is needed to compute torque $\boldsymbol{\tau}$ to make the biped robot follow the desired trajectory defined above. The desired position of the tip of the swing leg is defined as Q_x and Q_y in the frame which has its origin at the joint of lumber part of the biped robot. From Fig.5, following equations are given.

$$Q_x = U_{dx}(\varsigma) + x(\varsigma) - C_1 \tag{3.21}$$

$$Q_y = -U_{dy}(\varsigma) + C_2 \tag{3.22}$$

where C_1 and C_2 are parameters as shown in Fig.5. Solving inverse kinematics of (3.21) and (3.22), the desired values of θ_3 and θ_4 are given and they are denoted as θ_{d3} and θ_{d4} respectively.

$$\theta_{d4} = ATAN2\left[\sqrt{(2L_1L_2)^2 - \{(Q_x)^2 + (Q_y)^2 - (L_1^2 + L_2^2)\}^2}, (Q_x)^2 + (Q_y)^2 - (L_1^2 + L_2^2)\right] \tag{3.23}$$

$$\theta_{d3} = -\frac{\pi}{2} - (\theta_1 - \theta_2) - a\tan 2(Q_x, Q_y) + ATAN2(-L_1 \sin\theta_{d4}, L_2 + L_1\cos\theta_{d4}) \tag{3.24}$$

Next, denote the desired value of θ_2 such that the height of COM of the biped robot becomes desired value as θ_{d2}. By algebra(refer to Appendix)

$$\theta_{d2} = \phi + \cos^{-1}\left(\frac{\nu}{\sqrt{\mu^2 + \gamma^2}}\right) \tag{3.25}$$

where ϕ, μ, ν and γ are functions of θ_1, θ_3 and θ_4 (refer to Appendix).

Let

$$\boldsymbol{\theta}_D = \begin{bmatrix}\theta_1 & \theta_{d2} & \theta_{d3} & \theta_{d4}\end{bmatrix}^{\mathrm{T}}. \tag{3.26}$$

And τ is defined as the following feedback control law.

$$\boldsymbol{\tau} = -\boldsymbol{K}_1(\boldsymbol{\theta} - \boldsymbol{\theta}_D) - \boldsymbol{K}_2\dot{\theta}^{\mathrm{T}} \tag{3.27}$$

where $\boldsymbol{K_1}$ and $\boldsymbol{K_2}$ are gain matrices. Note that ankle torque is set to be zero in this control law because of LIPM[4][5][6]

3.4 Orbital energy adjustment

In the above section, the desired trajectory of the swing leg is designed in consideration of boundary conditions for flexible terrain. But it is not sufficient only to use walking algorithm proposed in preceding sections, because dynamics of biped robot that includes terrain model is significantly complex. For example, the swing leg cannot be controlled to track the desired trajectory perfectly, since the swing leg has inertia. Hence the motion of the swing leg is delayed to the desired motion of the swing leg and the biped robot can not make the swing leg touch down at the desired moment. Considering undesired collision between the swing leg and the terrain, it is not acceptable that the impulsive force effects to the motion of COM of the biped robot. It is common that high precision position control makes the robot stiff. In that case, the collision at the tip of the swing leg effects the motion of COM significantly. And big control inputs make the contacting point slip because it occurs moment of rotation. Hence control torques are required to be as small as possible. And, the control of the swing leg without delay is very difficult. The delay may lead that the value of orbital energy does not converge to the desired value. It means that it is difficult for the biped robot to achieve the desired steady gait.

For these reasons, we propose the way that makes the orbital energy converge to the desired value in prospect of landing time disagreement. The delay mentioned above is denoted as α and some quantities are defined as follow.

t_f^- : landing time of n-1th step , α_n^- : α of n-1th step

S^- : desired landing time of n-1th step

Since t_f^- and S^- are available at nth-step, α_n^- is given as the following.

$$\alpha_n^- = t_f^- - S^- \tag{3.28}$$

Since α_n^- is obtained at each step by (3.28), the value of α_n^- may change step by step. Therefore, let $\alpha(n)$ be given as the value of average from the first step to nth-step.

$$\alpha(n) = \frac{\sum \alpha^-}{n-1} \tag{3.29}$$

It is assumed that $\alpha(n)$ is constant through the nth walking cycle.

Next, when the landing time has the delay of $\alpha(n)$, the desired landing position changes to $U_{dx}(S + \alpha(n))$. Therefore, $U_{dx}(t)$ is multiplied by an appropriate coefficient to correct the desired trajectory of the swing leg in x axis direction. The coefficient is denoted by ε. Fig.6 shows the attitude of landing with the use of ε. The orbital energy before(nth-step) and after(n+1th-step) leg switching are given by followings.

$$E_n = -\frac{mg}{2L_{hd}}X_f^2 + \frac{m}{2}V_f^2 \tag{3.30}$$

$$E_{n+1} = -\frac{mg}{2L_{hd}}\{\varepsilon U_x(S + \alpha(n))\}^2 + \frac{m}{2}V_f^2 \tag{3.31}$$

From (3.33) and (3.34)

$$E_{n+1} - E_n = -\frac{mg}{2L_{hd}}\{\varepsilon^2 U_x^2(S + \alpha(n)) - X_f^2\} \tag{3.32}$$

With the use of (3.32), the following is given.

$$E_{n+1} - E_n = (E_{n+1} - E_n)(E_n - E_d) = -\frac{mg}{2L_{hd}}\{\varepsilon^2 U_x^2(S + \alpha(n)) - X_f^2\} + (E_n - E_d) \tag{3.33}$$

If the right hand side in (3.33) becomes zero, the orbital energy after leg switching becomes the desired value. Denote the value of ε which makes the right hand side in (3.33) zero as ε_0 and

$$\varepsilon_0 = \sqrt{\left\{\frac{X_f}{U_x(S + \alpha(n))}\right\}^2 + \frac{2L_{hd}}{mgU_x^2(S + \alpha(n))^2}(E_n - E_d)} \tag{3.34}$$

Therefore, the displacement of the tip of the swing leg in x axis direction is controlled in order to follow $\varepsilon_0 \times U_x(S + \alpha(n))$. From above, if $\varepsilon_0 > 1$, it means that the swing leg lands on farther side than the original landing point from the tip of the support leg. If $\varepsilon_0 = 1$, it means that the swing leg lands on the original landing point from the tip of the support leg. If $\varepsilon_0 < 1$, it means that the swing leg lands on nearer side than the original landing point.

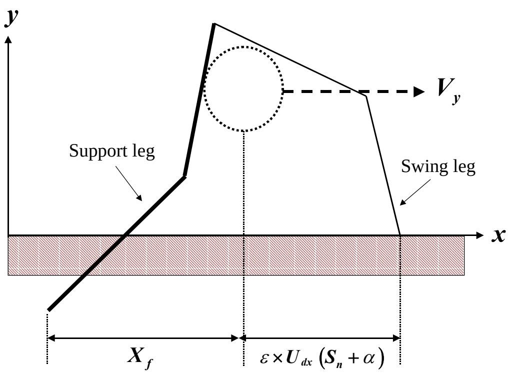

Fig.6:Parameters of landing with ε

4.Simulation

Simulation was executed by solving the differential equation (2.1). The design parameters are given Table.2.

Table.2: Simulation parameters

Simulated time : t	10[sec]
Desired orbital energy : E_d	0.5[J]
Desired height of COM	0.5[m]
Mass of link1 : m_1	1[kg]
Mass of link2 : m_2	10[kg]
Length of link1 : L_1	0.5[m]
Length of link2 : L_2	0.5[m]
Moment of inertia of link1	0.021[$\mathrm{kg}\cdot\mathrm{m}^2$]
Moment of inertia of link2	0.21[$\mathrm{kg}\cdot\mathrm{m}^2$]
Acceleration of gravity : g	9.81[$\mathrm{m}^2/\mathrm{sec}$]
Step length : L	0.15[m]
Maximum clearance of the swing leg from the terrain : h	0.07[m]
Spring coefficient of the terrain : k	9500[N/m]
Damper coefficient of the terrain : d	2200[$\mathrm{N}\cdot\mathrm{sec/m}$]
Gain K_1	$\mathrm{diag}(0,7000,3000,3000)$
Gain K_2	$\mathrm{diag}(0,500,50,50)$

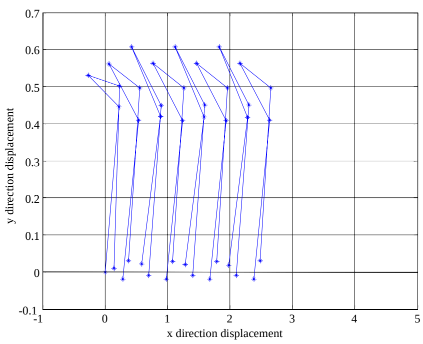

Fig.7 Stick diagram of the biped robot

Fig.7 shows that the biped robot can walk on flexible terrain. Although it is shown that the support leg sank into the terrain, it did not disturb the walk of the biped robot.

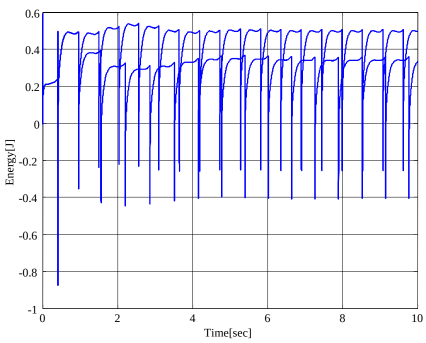

Fig.8 Orbital energy in walking

Fig.8 shows the orbital energy. The solid line shows the energy with ε , and the dashed line shows the energy without ε . From Fig.8, it is shown that the orbital energy converged to the desired value $E_d = 0.5$[J] by the proposed method.

In the interval 0-4[sec], since the value of $\alpha(n)$ varies, the orbital energy did not converge. After 4[sec], the law of adjustment of orbital energy was effective. Impulsive spikes of orbital energy are shown at every leg switching point since big impulsive forces occurred when the swing leg of the biped robot lands on the terrain.

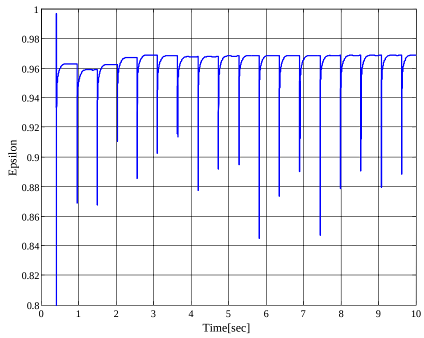

Fig.9 ε in walking

Fig.9 shows the value of ε . After 4[sec] ε converged to a constant value, about 0.97. It means that the swing leg landed on slightly nearer side than the original landing point.

5.Conlcusion

In this paper, we extended a walking algorithm based on LIPM in order that the biped robot can walk on the flexible terrain. In process of extending it, we also proposed a way that makes the orbital energy converge to the desired value in prospect of landing time disagreement. And the effectiveness of the proposed method is also shown in simulation.

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Appendix

The derivation of (3.28) is showed here. The biped robot system is defined in Fig10. Let θ_{d2} denote the desired value of θ_2 when the height of COM of biped robot corresponds to the desired height. Let y axis directional displacement of the tip of the support leg be y_0. Then, the desired height of COM of biped robot L_{hd} is given as

$$\begin{aligned} L_{hd} &= \frac{1}{2(m_1+m_2)}\{m_1\left[y_0+\frac{L_1}{2}\cos\theta_1\right] \\ &+ m_2\left[y_0+L_1\cos\theta+\frac{L_2}{2}\sin(\theta_1-\theta_{d2})\right] \\ &+ m_2\left[y_0+L_1\cos\theta+L_2\cos(\theta_1-\theta_{d2})-\frac{L_2}{2}\cos(\theta_1-\theta_{d2}+\theta_3)\right] \\ &+ m_1\begin{bmatrix} y_0+L_1\cos\theta_1+L_2\cos(\theta_1-\theta_{d2})-L_2\cos(\theta_1-\theta_{d2}+\theta_3) \\ -\frac{L_1}{2}\cos(\theta_1-\theta_{d2}+\theta_3+\theta_4) \end{bmatrix}\} \\ &- y_0 \end{aligned} \tag{A.1}$$

Transporting the terms that don't include θ_{d2} to left hand side in (A.1), the following equation is given.

$$\begin{aligned} &(2m_1+2m_2)(L_{hd}+y_0)-m_1\left(2y_0+\frac{3}{2}L_1\cos\theta_1\right)-m_2(2y_0+2L_1\cos\theta_1) \\ &= m_1\left\{L_2\cos(\theta_1-\theta_{d2})-L_2\cos(\theta_1-\theta_{d2}+\theta_3)-\frac{L_1}{2}\cos(\theta_1-\theta_{d2}+\theta_3+\theta_4)\right\} \\ &+ m_2\left\{\frac{3L_2}{2}\cos(\theta_1-\theta_{d2})-\frac{L_2}{2}\cos(\theta_1-\theta_{d2}+\theta_3)\right\} \end{aligned} \tag{A.2}$$

Denote the left hand side of (A.2) as ν. The right hand side of (A.2) shows displacement in y axis of the biped robot from the point M. Next, let M point be a pivot, and consider the part that consists of link2, link3 and link4 are rotated θ_{d2} in clockwise direction and let M be origin. The COM of the biped robot is defined μ and γ as follow.

$$\begin{aligned} \mu &= m_1\left\{L_2\sin\theta_1-L_2\sin(\theta_1+\theta_3)-\frac{L_1}{2}\sin(\theta_1+\theta_3+\theta_4)\right\} \\ &+ m_2\left\{\frac{3}{2}L_2\sin\theta_1-\frac{L_2}{2}\sin(\theta_1+\theta_3)\right\} \end{aligned} \tag{A.3}$$

$$\begin{aligned} \gamma &= m_1\left\{L_2\cos\theta_1-L_2\cos(\theta_1+\theta_3)-\frac{L_1}{2}\cos(\theta_1+\theta_3+\theta_4)\right\} \\ &+ m_2\left\{\frac{3}{2}L_2\cos\theta_1-\frac{L_2}{2}\cos(\theta_1+\theta_3)\right\} \end{aligned} \tag{A.4}$$

Fig.10 shows the above geometric relation.

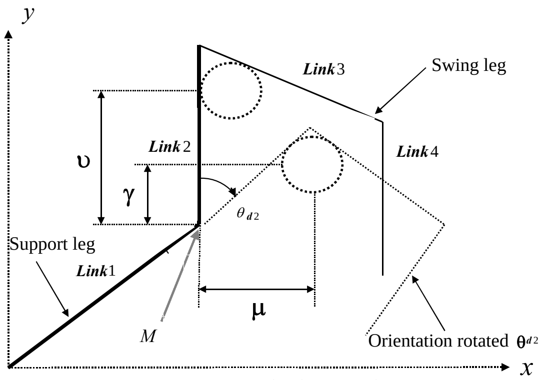

Fig.10:Geometric relation of before and after rotation

Dashed circles in Fig.10 and Fig.11 denote the position of COM of the biped robot. In order to get θ_{d2}, Let

$$\phi = ATAN2(\mu,\gamma) \tag{A.5}$$

Since the distance from point M to COM does not change by rotation θ_{d2}. Therefore, the following is given from geometric relation.

$$\frac{\nu}{\sqrt{\mu^2+\gamma^2}} = \sin\left(\theta_{d2}+\frac{1}{2}\pi-\phi\right) \tag{A.6}$$

From (A.6),

$$\theta_{d2} = \phi+\cos^{-1}\left(\frac{\nu}{\sqrt{\mu^2+\gamma^2}}\right) \tag{A.7}$$

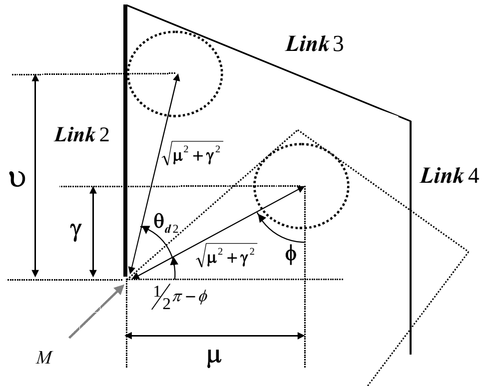

Fig.11:Details of geometric relation (Fig.10)