Adaptive Control for Tracking Trajectory of a Two-Wheeled Welding Mobile Robot with Unknown Parameters

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Abstract: This paper presents a method to design an adaptive controller for the kinematic model of a two-wheeled welding mobile robot (WMR) with unknown parameters. We propose a nonlinear controller based on the Lyapunov function to enhance the tracking properties of the WMR. The WMR can track any smooth curved welding path at a constant velocity of the welding point. The system has three degrees of freedom including two wheels and one torch slider. Torch slider motion is used for fast tracking. To design the tracking performance, the errors from WMR to steel wall is defined, and the controller is designed to drive the errors to zero as fast as possible. The effectiveness of the proposed controller is shown through simulation results.

Keywords: Welding Mobile Robot (WMR), kinematic model, tracking control, reference welding path, unknown parameter.

1. INTRODUCTION

Nowadays, robots become widely used especially in the tasks which are hazardous and difficult for human. In fact, the wheeled mobile robots are extensively used in several fields where transportation, inspection and operation task are required (industry, assembly, mining, safety, and so on). Today, the welding process is strongly encouraged for improvements in quality, productivity and labor conditions. In naval construction, the automation welding process is ultimately necessary, since the welding sites are spatially enclosed by floors and girders and the welders are exposed to severe working conditions. To solve this problem, some robotic welding systems have been developed such as the wheeled mobile robot. There are many works on tracking control method for the wheeled mobile robot in literatures. Fierro et al.[1] developed a combined kinetic/torque control law using a back-stepping approach. Sarkar et al.[6] proposed a nonlinear feedback that guarantees input-output stability and Lagrange stability for the overall system with reference paths of straight line and circular line. Yun et al.[8] focused on kinematics and control of a vehicle with two steerable wheels using a dynamic feedback linearization. But these papers did not consider the uncertainty of system parameters which always exist in mobile robot control problem. Fukao et al.[2] dealt with the adaptive tracking control of a two-wheeled mobile robot with unknown parameters.

The applications of the two-wheeled mobile robot for welding automation have been studied by Jeon and Kam[3-4]. Jeon[3] proposed a seam tracking and motion control of WMR for lattice type welding in which were three controllers for motion controls: straight locomotion, turning locomotion and torch slider. Kam[4] proposed a control algorithm for straight welding based on "trial and error" for each step time. Both controllers proposed by Jeon and Kam are used only for straight path tracking, not for smooth curved path tracking.

In this paper, the problem of tracking trajectory for the kinematic model of a two-wheeled welding mobile robot with unknown parameters is considered. We propose a nonlinear controller based on the Lyapunov function to enhance the tracking properties of the WMR. The distance from WMR's center to driving wheel, and the radius of the driving wheel are considered to be unknown parameters which are estimated using update laws in adaptive control scheme. To design the tracking performance, an error configuration is defined. And a simple method for measuring the errors using potentiometers is proposed. The controller is designed to drive the error to zero as fast as possible. The controlled system is stable in the sense of Lyapunov stability. The WMR has a geometrical property: the welding point is outside its wheels and is far from its center. This property leads to the slow convergence of tracking errors in the case of a fixed torch WMR. This disadvantage can be overcome by using a controlled torch slider. The effectiveness of the proposed controller is shown through simulation results with three cases of reference welding path were considered: straight path, circular path, and smooth curved path.

2. KINEMATIC MODEL OF A WMR

The model of a WMR is presented in Fig. 1.

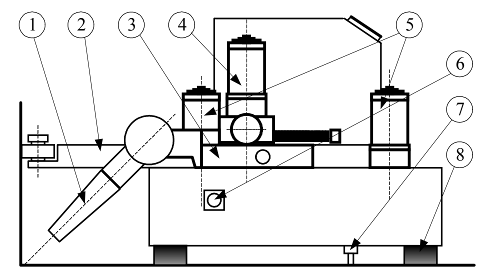

Fig. 1 Configuration of the WMR.

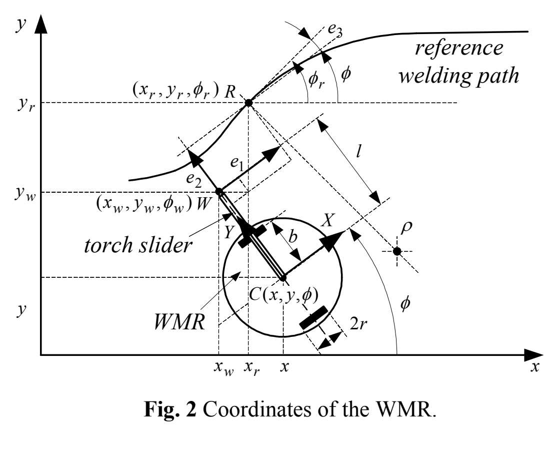

Fig. 2 Coordinates of the WMR.

The relation of the WMR's coordinates with its reference welding path are shown in Fig. 2. There are three controlled motions in this model: two driving wheels and one torch slider

It is assumed that the wheels roll and do not slip. That is, the center point velocity $C(x, y)$ of WMR must be in the direction of the axis of symmetry and the wheels must not slip.

These constraints are present as follows

$$\begin{cases} \dot{y}\cos\phi - \dot{x}\sin\phi = 0 \\ \dot{x}\cos\phi + \dot{y}\sin\phi + b\dot{\phi} = r\omega_{rw} \\ \dot{x}\cos\phi + \dot{y}\sin\phi - b\dot{\phi} = r\omega_{lw} \end{cases} \quad (1)$$

where,

$C(x, y)$: Cartesian coordinate of WMR's center,

ϕ : the heading angle of the WMR,

ω_{rw}, ω_{lw} : the angular velocities of the right and left wheels,

b : the distance from WMR's center to driving wheel,

r : driving wheel radius.

The ordinary form of a mobile robot with two actuated wheels can be derived as follows

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos\phi & 0 \\ \sin\phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (2)$$

where v and ω are the straight and angular velocities of the WMR's center, respectively.

The relationship between v, ω and ω_{rw}, ω_{lw} is

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{b}{r} \\ \frac{1}{r} & -\frac{b}{r} \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (3)$$

The welding point coordinates, $W(x_w, y_w)$, and the heading angle, ϕ_w, can be calculated from the WMR's center point:

$$\begin{cases} x_w = x - l\sin\phi \\ y_w = y + l\cos\phi \\ \phi_w = \phi \end{cases} \quad (4)$$

The welding point dynamics can be expressed as follows

$$\begin{cases} \dot{x}_w = v\cos\phi - l\omega\cos\phi - \dot{l}\sin\phi \\ \dot{y}_w = v\sin\phi - l\omega\sin\phi + \dot{l}\cos\phi \\ \dot{\phi}_w = \omega_w \end{cases} \quad (5)$$

A reference point, R, moving with the constant velocity of v_r on the reference path has the coordinates (x_r, y_r), and the heading angle, ϕ_r, satisfies the dynamic equation

$$\begin{cases} \dot{x}_r = v_r \cos\phi_r \\ \dot{y}_r = v_r \sin\phi_r \\ \dot{\phi}_r = \omega_r \end{cases} \quad (6)$$

where ϕ_r is defined as the angle between $\vec{v}_r$ and x coordinates and ω_r is the rate of change of $\vec{v}_r$ direction.

3. ADAPTIVE CONTROLLER DESIGN

3.1 The parameters r, b are known

The objective is to design a controller so that the welding point W tracks to the reference point R at a constant velocity v_r. We define the tracking errors $e = [e_1, e_2, e_3]^T$ as shown in Fig. 2.

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x_w \\ y_r - y_w \\ \phi_r - \phi_w \end{bmatrix} \quad (7)$$

A controller is designed to achieve $e_i \to 0$ when $t \to \infty$; as the result, the welding point W tracks to the reference point R. The torch is adjusted during welding, that is the torch length l is changeable. The linear velocity of the torch slider is an additional system input. The configuration of the torch slider is given in Fig. 3. From Eq. (5), the dynamics of errors can be expressed as follows

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (8)$$

The chosen Lyapunov function and its derivative are given as

$$V_0 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 \geq 0 \quad (9)$$

$$\begin{aligned} \dot{V}_0 &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 \\ &= e_1(-v + l\omega + v_r\cos e_3) + e_2(v_r \sin e_3 - \dot{l}) + e_3(-\omega + \omega_r) \end{aligned} \quad (10)$$

To achieve the negativity of $\dot{V}_0$, we choose (v, ω) as

$$\begin{cases} v = l\,(\omega_r + k_3 e_3) + v_r \cos e_3 + k_1 e_1 \\ \omega = \omega_r + k_3 e_3 \\ \dot{l} = v_r \sin e_3 + k_2 e_2 \end{cases} \quad (11)$$

where k_1, k_2, k_3 are positive values.

The parameters r, b are known, from Eq. (11) and Eq. (3), we can calculate the necessary velocities of the two driving wheels ω_{rw}, ω_{lw}.

3.2 The parameters r, b are unknown (Adaptive control)

When r, b are unknown, we design an adaptive controller to attain the control objective by using the estimates of r, b.

From Eq. (8) and Eq. (3), we obtain

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 - \dot{l} \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2b} & -\frac{r}{2b} \end{bmatrix} \begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} \quad (12)$$

Define $\quad a_1 = \dfrac{1}{r}; \quad a_2 = \dfrac{b}{r}$ (13)

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 \\ a_1 & -a_2 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{14}$$

Equation (12) becomes

$$\begin{bmatrix} \dot e_1 \\ \dot e_2 \\ \dot e_3 \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 - \dot l \\ \omega_r \end{bmatrix} + \frac{1}{2} \begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \dfrac{1}{a_1} & \dfrac{1}{a_1} \\ \dfrac{1}{a_2} & -\dfrac{1}{a_2} \end{bmatrix} \begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} \tag{15}$$

Because r, b are unknown, so

$$\begin{bmatrix} \omega_{rw} \\ \omega_{lw} \end{bmatrix} = \begin{bmatrix} \hat a_1 & \hat a_2 \\ \hat a_1 & -\hat a_2 \end{bmatrix} \begin{bmatrix} v_d \\ \omega_d \end{bmatrix} \tag{16}$$

where $v_d = v$, $\omega_d = \omega$,
$\hat a_1$, $\hat a_2$ are estimated values of a_1, a_2, respectively.

$$\begin{bmatrix} \dot e_1 \\ \dot e_2 \\ \dot e_3 \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 - \dot l \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \dfrac{\hat a_1}{a_1} & 0 \\ 0 & \dfrac{\hat a_2}{a_2} \end{bmatrix} \begin{bmatrix} v_d \\ \omega_d \end{bmatrix} \tag{17}$$

Define estimation error $\quad \begin{cases} \tilde a_1 \equiv a_1 - \hat a_1 \\ \tilde a_2 \equiv a_2 - \hat a_2 \end{cases}$ (18)

Equation (17) becomes

$$\begin{bmatrix} \dot e_1 \\ \dot e_2 \\ \dot e_3 \end{bmatrix} = \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 - \dot l \\ \omega_r \end{bmatrix} + \begin{bmatrix} -1 & e_2 + l \\ 0 & -e_1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 - \dfrac{\tilde a_1}{a_1} & 0 \\ 0 & 1 - \dfrac{\tilde a_2}{a_2} \end{bmatrix} \begin{bmatrix} v_d \\ \omega_d \end{bmatrix} \tag{19}$$

The Lyapunov function is chosen as

$$V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 + \frac{1}{2} e_3^2 + \frac{1}{2\gamma_1 a_1} \tilde a_1^2 + \frac{1}{2\gamma_2 a_2} \tilde a_2^2 \tag{20}$$

and its derivative yields

$$\dot V_1 = e_1 \dot e_1 + e_2 \dot e_2 + e_3 \dot e_3 + \frac{1}{\gamma_1} \frac{\tilde a_1}{a_1} \dot{\tilde a}_1 + \frac{1}{\gamma_2} \frac{\tilde a_2}{a_2} \dot{\tilde a}_2$$

$$= e_1 \left[-v_d \left(1 - \frac{\tilde a_1}{a_1} \right) + e_2 \omega_d \left(1 - \frac{\tilde a_2}{a_2} \right) + l \omega_d \left(1 - \frac{\tilde a_2}{a_2} \right) + v_r \cos e_3 \right]$$

$$+ e_2 \left[-e_1 \omega \left(1 - \frac{\tilde a_2}{a_2} \right) + v_r \sin e_3 - \dot l \right]$$

$$+ e_3 \left[-\omega_d \left(1 - \frac{\tilde a_2}{a_2} \right) + \omega_r \right] - \frac{1}{\gamma_1} \frac{\tilde a_1}{a_1} \dot{\hat a}_1 - \frac{1}{\gamma_2} \frac{\tilde a_2}{a_2} \dot{\hat a}_2$$

$$= e_1 \left[-v_d \left(1 - \frac{\tilde a_1}{a_1} \right) + l \omega_d \left(1 - \frac{\tilde a_2}{a_2} \right) + v_r \cos e_3 \right]$$

$$+ e_2 (v_r \sin e_3 - \dot l) + e_3 \left[-\omega_d \left(1 - \frac{\tilde a_2}{a_2} \right) + \omega_r \right] - \frac{1}{\gamma_1} \frac{\tilde a_1}{a_1} \dot{\hat a}_1 - \frac{1}{\gamma_2} \frac{\tilde a_2}{a_2} \dot{\hat a}_2$$

$$= \dot V_0 + e_1 v_d \frac{\tilde a_1}{a_1} + (e_3 - e_1 l) \omega_d \frac{\tilde a_2}{a_2} - \frac{1}{\gamma_1} \frac{\tilde a_1}{a_1} \dot{\hat a}_1 - \frac{1}{\gamma_2} \frac{\tilde a_2}{a_2} \dot{\hat a}_2$$

$$= \dot V_0 - \frac{\tilde a_1}{\gamma_1 a_1} (\dot{\hat a}_1 - \gamma_1 e_1 v_d) - \frac{\tilde a_2}{\gamma_2 a_2} [\dot{\hat a}_2 - \gamma_2 (e_3 - e_1 l) \omega_d] \tag{21}$$

The controller is still the same (11), but there is two update laws for unknown parameters

$$\begin{cases} v_d = l(\omega_r + k_3 e_3) + v_r \cos e_3 + k_1 e_1 \\ \omega_d = \omega_r + k_3 e_3 \\ \dot l = v_r \sin e_3 + k_2 e_2 \\ \dot{\hat a}_1 = \gamma_1 e_1 v_d \\ \dot{\hat a}_2 = \gamma_2 (e_3 - e_1 l)\, \omega_d \end{cases} \tag{22}$$

3.2 Measurement of the errors

To attain the controllers (11) or (22), the errors e_1, e_2, e_3 must be measured. We propose a simple measurement scheme using potentiometers to obtain these values as shown in Fig. 3. Two rollers are placed at points O_1 and O_2. The distance between the two rollers, O_1, O_2, is chosen according to the curve radius of the reference welding path at contact point $R(x_r, y_r)$ such as $\vec v_r \,//\, \overrightarrow{O_1 O_2}$.

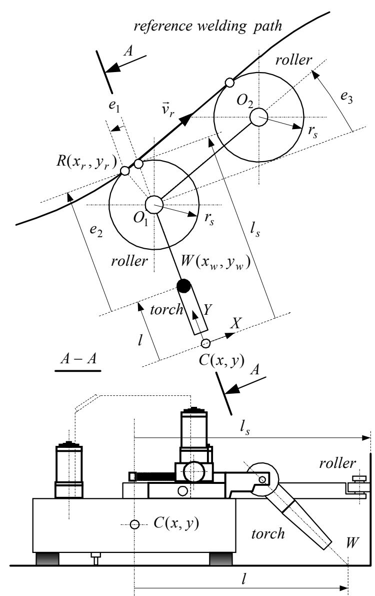

Fig. 3 Error measurement scheme

The errors as shown in Fig. 3 can be expressed by

$$\begin{cases} e_1 = -r_s \sin e_3 \\ e_2 = (l_s - l) - r_s (1 - \cos e_3) \\ e_3 = \angle(O_1 C, O_1 O_2) - \pi / 2 \end{cases} \tag{23}$$

where r_s is the radius of the roller, and l_s is the length of the sensor. Hence, we need two sensors for measuring the errors, that is, one linear sensor for measuring $(l_s - l)$ and one rotating sensor for measuring the angle between the X coordinate of the WMR and $\vec{v}_r$.

4. SIMULATION RESULTS

To verify the effectiveness of the proposed modeling and controller, simulations have been done for a WMR with three cases of reference welding path were considered: straight path, circular path, and smooth curve path.

The physical and designed parameters of the WMR are chosen as follows $b = 0.105\ m$, $r = 0.025\ m$, $k_1 = 14.2$, $k_2 = 7.5$, $k_3 = 3.5$. The welding velocity is $7.5\ mm/s$.

Table 1. The numerical values and initial values

Parameter	Circular path	Smooth curved path
x_r $[m]$	0.290	0.270
y_r $[m]$	0.500	0.500
ϕ_r $[deg]$	-90	0
x_w $[m]$	0.280	0.265
y_w $[m]$	0.520	0.495
ϕ_w $[deg]$	-75	15
v $[mm/s]$	0	0
ω $[rad/s]$	0	0
ω_r $[rad/s]$	0	0
l $[m]$	0.15	0.15

The first simulation was done for the WMR to tracks the circular path with radius $R = 210\,mm$. Simulation results are given in Figs. 4-9. At beginning, the WMR adjusts its position very fast to reduce the initial errors. As shown in Fig. 4, the errors converge to zero after about 1.5 seconds.

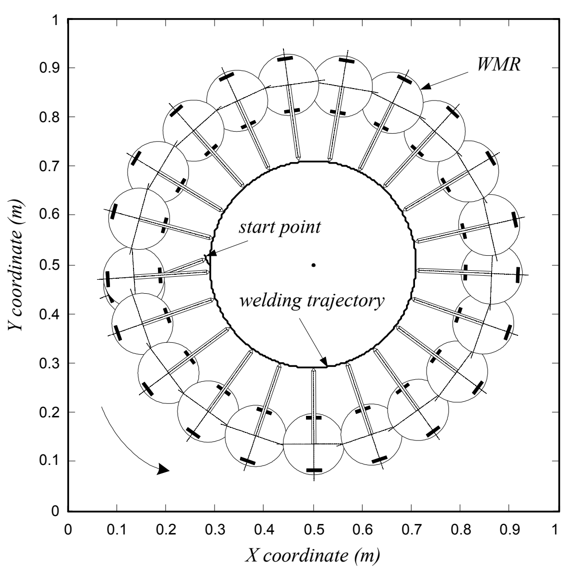

Fig. 4 Movement of the WMR when tracking circular path.

The velocities of WMR's center and welding point are shown in Fig. 6. The control inputs are given in Fig. 7.

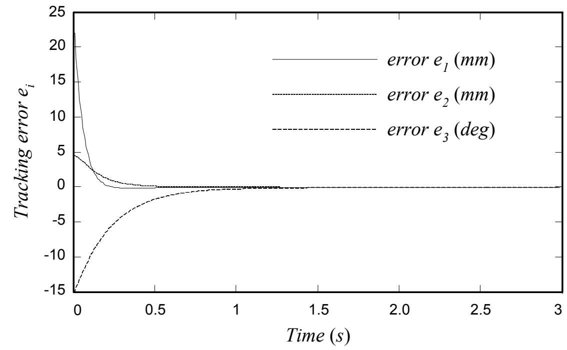

Fig. 5 Tracking errors at beginning.

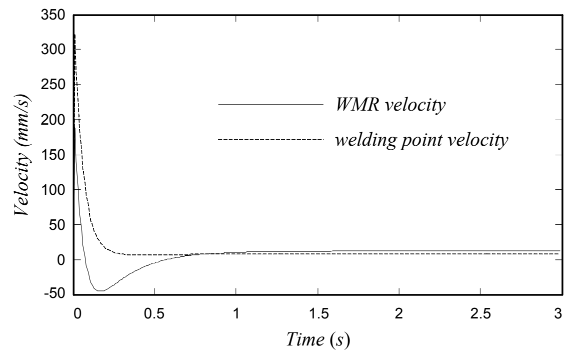

Fig. 6 Velocities of the WMR and the welding point.

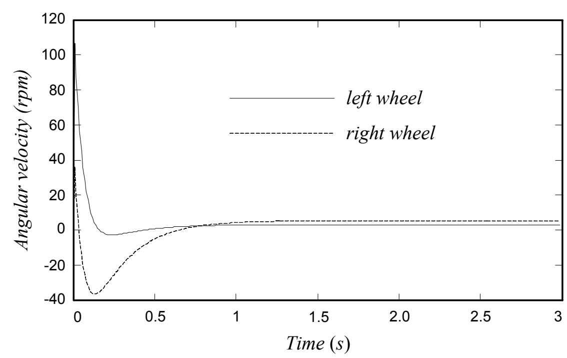

Fig. 7 Control input: angular velocities of the WMR wheels.

Torch length is given in Fig. 8. Fig. 9 shown the estimation errors of parameters. The WMR can track circular welding path with good performances.

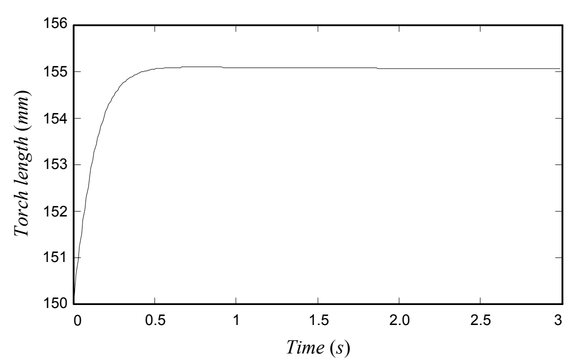

Fig. 8 Torch length.

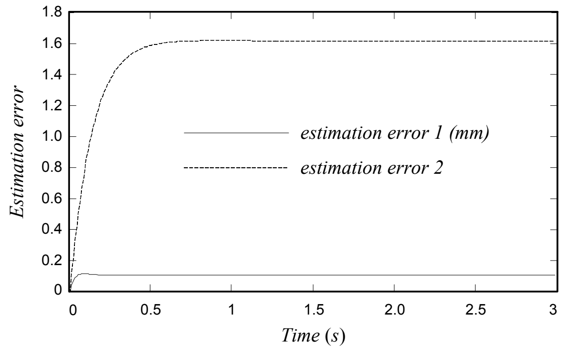

Fig. 9 Estimation errors.

Figs. 10-20 shown the performances of the WMR when tracking the smooth curved path. The case WMR tracks straight path is included in this case. At beginning, the convergence of the errors is very fast as shown in Fig. 12. The errors go to nearly zero after 1.5 seconds. From straight line to curved line, there is a sudden change of ω_r (from zero to a constant); therefore, there are errors as shown in Fig. 13.

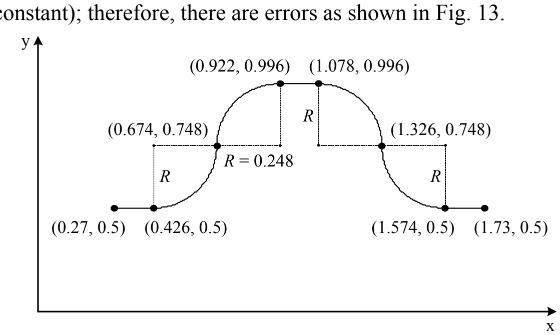

Fig. 10 Smooth curved welding path.

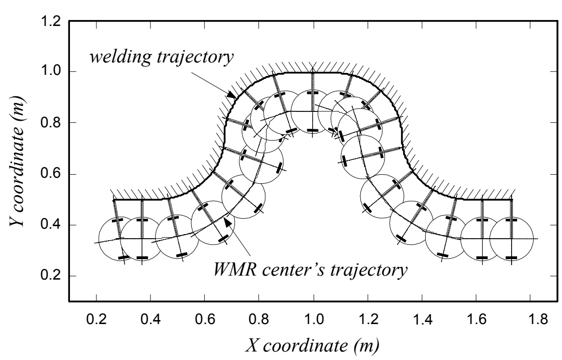

Fig. 11 Movement of the WMR.

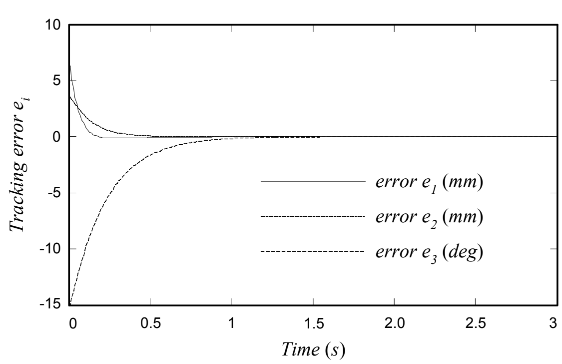

Fig. 12 Tracking errors at beginning.

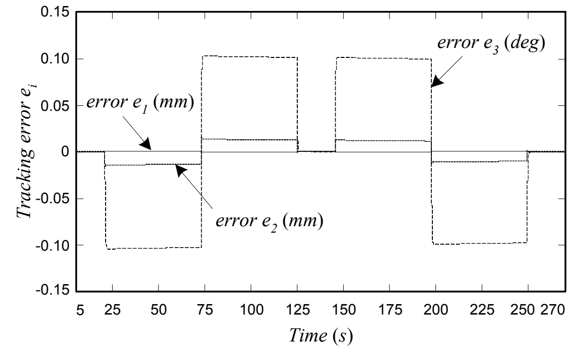

Fig. 13 Tracking errors.

The velocities of the WMR and the welding point are given in Figs. 14-15. The welding velocity is unaffected. The control inputs are given in Figs. 16-17. The torch length is changed as shown in Figs. 18-19. Fig. 20 shown the estimation errors.

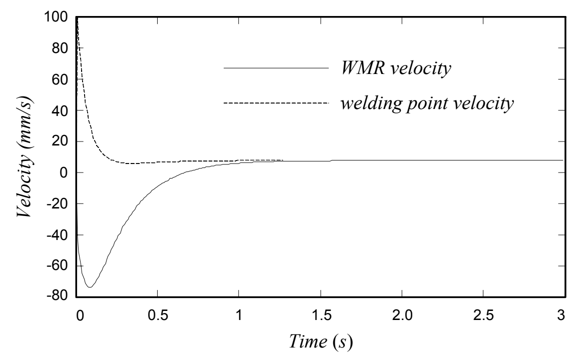

Fig. 14 Velocities of the WMR and the welding point.

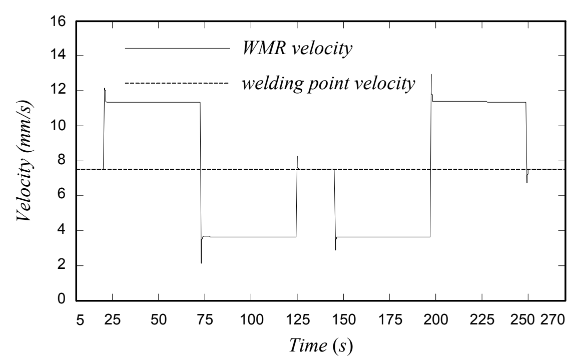

Fig. 15 Velocities of the WMR and the welding point.

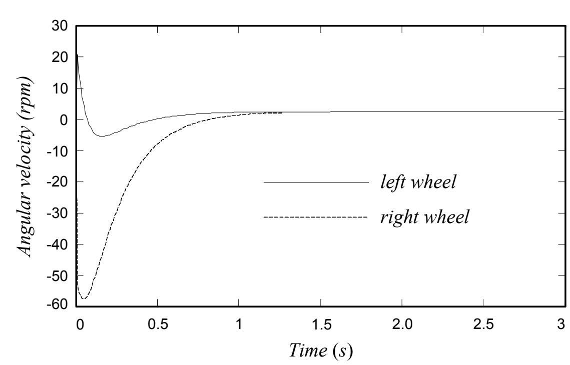

Fig. 16 Control input: angular velocities of the WMR wheels.

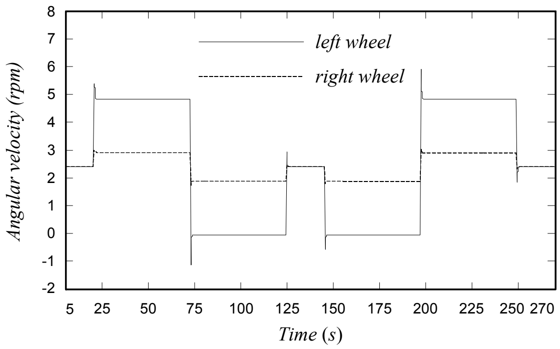

Fig. 17 Control input: angular velocities of the WMR wheels.

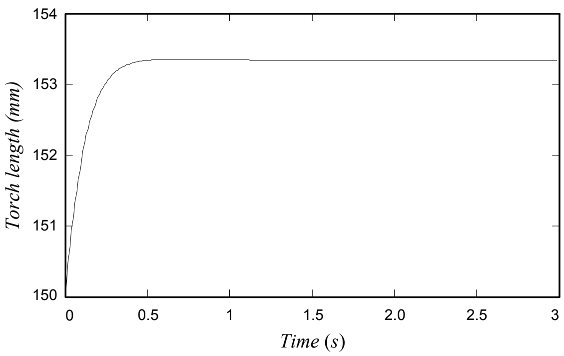

Fig. 18 Torch length.

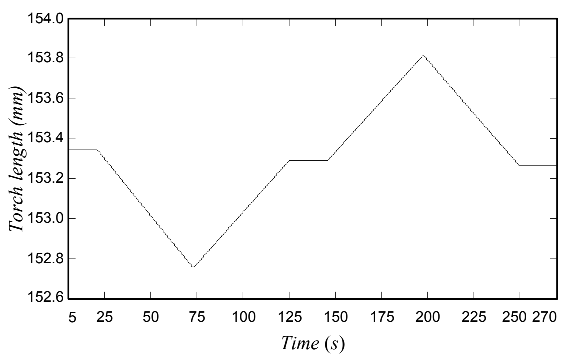

Fig. 19 Torch length.

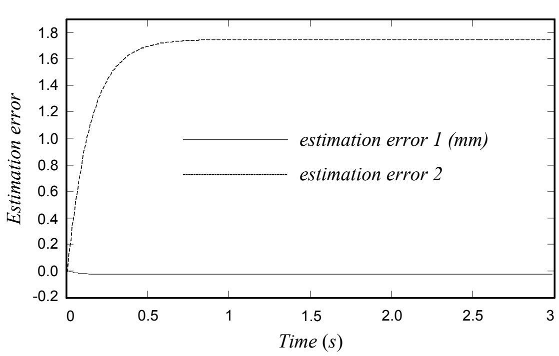

Fig. 20 Estimation errors.

Through the above simulation results, we can see that the tracking errors have small oscillation, and the welding velocity can tracks the reference velocity. This is acceptable for the WMR application.

5. CONCLUSION

In this paper, the problem of tracking trajectory for the kinematic model of a two-wheeled welding mobile robot with unknown parameters is considered. We proposed a nonlinear controller based on the Lyapunov control function to enhance the tracking properties of the WMR. The distance from WMR's center to driving wheel, and the radius of the driving wheel are considered to be unknown parameters which are estimated using update laws in adaptive control scheme. The controlled system is stable in the sense of Lyapunov stability and the controller is flexible with three adjustable parameters. From the simulation results, we can conclusion that the WMR with proposed controller can track its reference and can be used for tracking any smooth curved path with acceptable small errors.

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