

# Optimal path planning for the capturing of a moving object

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**Abstract:** In this paper, we propose an algorithm for planning an optimal path to capture a moving object by a mobile robot in real-time. The direction and rotational angular velocity of the moving object are estimated using the Kalman filter, a state estimator. It is demonstrated that the moving object is tracked by using a 2-DOF active camera mounted on the mobile robot and then captured by a mobile manipulator. The optimal path to capture the moving object is dependent on the initial conditions of the mobile robot, and the real-time planning of the robot trajectory is definitely required for the successful capturing of the moving object. Therefore the algorithm that determines the optimal path to capture a moving object depending on the initial conditions of the mobile robot and the conditions of a moving object is proposed in this paper. For real-time implementation, the optimal representative blocks have been utilized for the experiments to show the effectiveness of the proposed algorithm.

**Keywords:** Path planning, Kalman filter, capture, trajectory, initial condition

## 1. INTRODUCTION

Recently, there has been a lot of interest in the intelligent mobile robot. The mobile robot affects many factors in many and various situations. Robots have been substituted for people in extreme situations and environments to perform difficult work. Also, as FA(Factory Automation) and the flexible production system increase, a mobile robot with adaptability in changing industrial environments and a wide work radius is increasingly required. And so position estimation of the robot, path planning for capturing a moving object, and a control technique using a sensor.

Path planning aims for the to move in the optimal path from its position to a target in work situations. Therefore, to achieve the aim, the robot is tasked, recognizing the environment, and then it must work through the optimal path. Path planning is required for the more intelligent operation of the robot. In other words, because the optimal path is varied as the state of mobile robot for capturing the object, the path, as the robot's state must continuously be defined.

In this paper, we propose an algorithm for planning an optimal path to capture a moving object by a mobile robot in real-time, according to the initial position and direction of the robot. The object position is determined by a active camera mounted on the mobile robot, and the linear and angular velocity of the moving object are estimated using a state estimator, the Kalman filter. The trajectory of the mobile robot is estimated from the linear and angular velocity, and the optimal path is planned according to the initial position and direction of the mobile robot.

In Section 2, we discuss the system of the mobile robot. Section 3 deals with the position estimate of the moving object and the Kalman filter, and Section 4 deals with the algorithm for planning an optimal path to capture a moving object. In Section 5, we confirm the validity of the proposed method through experimentation. Section 6 presents conclusions drawn from this study.

## 2. SYSTEM OF MOBILE ROBOT

The vision sensor system makes possible the comparatively accurate information extraction for the tracking object through image information and also obstacle recognition. Recently, with well utilized for increasing processing rates, processor have been position recognition and the control of robot posture.

In this paper, the mobile robot used a 2-DOF active camera system with pan and tilt capabilities. The active vision sensor can control the visual domain independently of the posture and direction of the robot and extract image information by moving the camera actively. In connection to the control of the mobile robot for tracking and capturing the moving object, this section deals with the kinematics of the mobile robot.

### 2.1 KINEMATICS OF MOBILE ROBOT

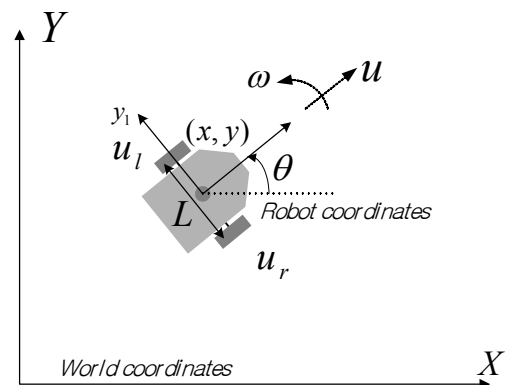


Fig. 1. A mobile robot

The state of the robot is represented as the vector  $\vec{P} = [x, y, \theta]^T$  which has a position such as those direction represented in Fig. 1. In general, the motion of the mobile robot is expressed as Eq.(1), (2) which indicate the linear velocity and the angular velocity.

$$u = \frac{1}{2}(u_r + u_l) \quad (1)$$

$$\omega = \frac{1}{L}(u_r - u_l) \quad (2)$$

And Eq.(3) and (4) are represented by using the Jacobian matrix. Here,  $u$  is the linear velocity of the mobile robot and  $\omega$  is the angular velocity of the robot's axis.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ \omega \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} u \\ \omega \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \quad (4)$$

Also, to represent Eq.(3) and (4) in relation to the angular velocity, Eq.(5) is used.

$$\begin{bmatrix} u \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{L} & -\frac{1}{L} \end{bmatrix} \begin{bmatrix} u_r \\ u_l \end{bmatrix} \quad (5)$$

Here,  $L$  is the distance between the two wheels. The robot position is calculated by using the encoder of each wheel. In the control of the robot, linear and angular velocity are transformed by the inverse kinematics of each wheel's velocity. Fig. 2 is represented the mobile robot actually.

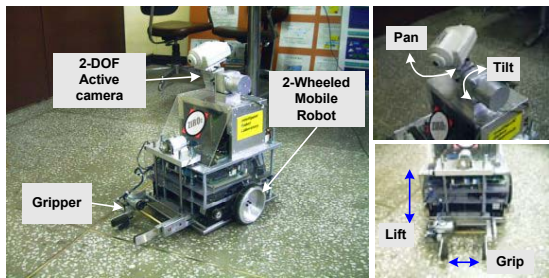


Fig. 2. ZIRO

### 3. STATE ESTIMATION OF MOBILE ROBOT

This section concerns motion analysis and position estimation to track a moving object by use of an active camera. The motion of the moving object is analyzed according to the movement and angular velocity to estimate the state of the moving object in coordinate space. And the state of the moving object is estimated by using the Kalman filter, a state estimator.

#### 3.1 POSITION ESTIMATION OF MOVING OBJECT

If the velocity and acceleration of each direction can be estimated, the next object position can be predicted as follows

$$\hat{P}_{x+\delta} = \hat{P}_x + \hat{V}_x \delta + \frac{1}{2} \hat{A}_x \delta^2 \quad (6)$$

$$\hat{P}_{y+\delta} = \hat{P}_y + \hat{V}_y \delta + \frac{1}{2} \hat{A}_y \delta^2 \quad (7)$$

$$\hat{P}_{z+\delta} = \hat{P}_z + \hat{V}_z \delta + \frac{1}{2} \hat{A}_z \delta^2 \quad (8)$$

where  $\delta t$  is the sampling time, and  $(P_x P_y P_z)$ ,  $(V_x V_y V_z)$  and  $(A_x A_y A_z)$  are the current Cartesian coordinate estimates of the object position, velocity and acceleration, respectively. According to the X-Y coordinates, the movement of the object can be decomposed into the velocity element and the angular velocity element, as following Eq.(9)-(11).

$$\delta x_{k+\delta} = v_k \delta \cos \theta_k + \frac{1}{2} \omega_k \delta^2 \approx v_k \cos(\theta_k) \delta - \frac{1}{2} \omega_k v_k \sin(\theta_k) \delta^2 \quad (9)$$

$$\delta y_{k+\delta} = v_k \delta \sin \theta_k + \frac{1}{2} \omega_k \delta^2 \approx v_k \sin(\theta_k) \delta + \frac{1}{2} \omega_k v_k \cos(\theta_k) \delta^2 \quad (10)$$

$$\delta \theta_{k+\delta} = \omega_k \delta \quad (11)$$

$$\delta v_{k+\delta} = \xi_v \quad (12)$$

$$\delta \omega_{k+\delta} = \xi_w \quad (13)$$

where  $v_k$  and  $w_k$  are the linear velocity and angular velocity of the moving object, and  $\xi_v$  and  $\xi_w$  are the variations of linear velocity and angular velocity, respectively.

#### 3.2 STATE ESTIMATION AND ERROR CORRECTION

To apply the state estimation of a moving object to the Kalman filter, Eq.(14) and (15) of the state transition matrix are required. The Kalman filter minimizes the estimation error by modifying the state transition model based on the error between the estimated vectors and the measured vectors, with an appropriate filter gain. The state vector which consists of a position on the x-y plane, the direction, linear and angular velocity can be estimated using the measured vectors representing the position of the moving object on the image plane.

$$x_k = \Phi_k x_{k-1} + w_{k-1} \quad (14)$$

$$z_k = H_k x_k + v_k \quad (15)$$

The Kalman filter is a recursive algorithm to determine  $\hat{x}_k$ , the optimal estimation value of state vector  $x_k$ , in a linear dynamic system. Here,  $k$  indicates time. Kalman filtering is divided into the three steps of prediction, measurement, and correction.

In the prediction step, the next state vector  $x_{k+1(-)}$  and the covariance matrix of the estimated error  $P_{k+1(-)}$  are predicted. The symbol (-) means that the values don't correct through measurement. The covariance matrix of the estimated error is just like Eq.(16).

$$P_{k+1(-)} = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \quad (16)$$

The projected estimates of the covariance matrix of the estimated error and the state vector in the prediction step are represented as

$$\hat{x}_{k+1(-)} = \Phi_k \hat{x}_{k(+)} + \omega_k \quad (17)$$

$$P_{k+1(-)} = P_{k(+)} + Q_k \quad (18)$$

where  $\Phi_k$  is the state transition matrix of  $\hat{x}_{k(+)}$ ,  $\omega_k$  is the model noise of the system, where  $Q_k$  is the covariance matrix of  $\omega_k$ . The measurement step is represented as

$$z_k = H_k \cdot \hat{x}_k + v_k \quad (19)$$

where  $z_k$  is the measurement vector,  $H_k$  represents the relationship between the measurement and the state vector, and  $v_k$  is the measurement error.

In the final correction step, the state vector and the estimate error are corrected to a new value based on the measurement value of the measurement step. The formula is represented as

$$K_k = P_{k(-)} H_k^T [H_k P_{k(-)} H_k^T + R_k]^{-1} \quad (20)$$

$$\hat{x}_{k(+)} = \hat{x}_{k(-)} + K_k [z_k - H_k \hat{x}_{k(-)}] \quad (21)$$

$$P_{k(+)} = [I - K_k H_k] P_{k(-)} \quad (22)$$

where  $R_k$  is the covariance matrix of the measurement noise, and  $K_k$  represents the Kalman gain. The optimal filter gain  $K_k$  minimizes the estimate errors by the covariance matrix of the estimate error  $P_{k(-)}$ , the measurement matrix  $H_k$ , and the covariance matrix of measurement noise  $R_k$  in Eq.(20). Next time, the estimate of the state vector  $\hat{x}_{k(+)}$  from the measurement  $z_k$  is expressed as Eq.(21). The Kalman gain functions as the weighting between the measurement and the estimate value when the state vector  $x_k$  is corrected. In the end, as in Eq.(22), the covariance matrix of the estimated error is corrected.

#### 4. OPTIMAL PATH PLANNING OF MOBILE ROBOT FOR CAPTURING MOVING OBJECT

##### 4.1 POSITION AND DIRECTION OF MOBILE ROBOT

The robot path is dependent on the initial conditions of the mobile robot, position and direction. The position of the mobile robot is defined by two perpendicular coordinates, and the direction is defined by the orientation that the robot faces. That is, the position is the x and y coordinates and the direction is  $\theta$ . The position and direction of the mobile robot using the robot's kinematics are defined as

$$x_r(k+1) = x_r(k) + T \frac{u_r(k) + u_l(k)}{2} \cos \theta_r(k) \quad (23)$$

$$y_r(k+1) = y_r(k) + T \frac{u_r(k) + u_l(k)}{2} \sin \theta_r(k) \quad (24)$$

$$\theta_r(k+1) = \theta_r(k) + T \frac{u_r(k) - u_l(k)}{l} \quad (25)$$

Here,  $u_r$  and  $u_l$  are the linear velocity of the right and left wheels, and  $l$  is the distance between the two wheels. The path to track the moving object is varied and real-time plan of varied path is required for successful capturing of the moving object. If the object is located in the initial direction of the robot, it can be tracked on a straight line. But, otherwise, after the robot is rotated to the object's direction, the robot must be tracked. So, the path is varied according to the initial state.

##### 4.2 VELOCITY AND ACCELERATION OF MOBILE ROBOT

The mobile robot has many possible paths. But, the paths are limited from a shortest-time point of view. First of all, to plan the path, the maximum values of the mobile robot's velocity and acceleration are limited as

$$|v| \leq v_{\max} \quad (26)$$

$$|a| \leq a_{\max} \quad (27)$$

Here,  $v_{\max}$  is the maximum velocity and  $a_{\max}$  is the maximum acceleration. We consider that when the target is given, the mobile robot is vectored to the target's position according to the robot's present position and velocity in the shortest-time, satisfying the given constraint of the mobile robot.

##### 4.3 OPTIMAL PATH PLANNING

###### 4.3.1 PATH PLANNING FOR CURVED LINE

When the mobile robot tracks an object in a curved line, the robot rotates. So, we must consider a curvature. The curvature is the curve's crooked rate, when any curve is given on a plane. And, the curvature radius is the inverse number of the curvature. So, the mobile robot moving to the curve at a rotation angle of  $d\theta$ , having the curvature radius of  $r$ , we define as

$$r = \frac{1}{\gamma} \quad (28)$$

$$d\theta = \gamma dS \quad (29)$$

And if the mobile robot rotates at high acceleration, it deviates from the desired path. So, we assume that the mobile robot moves in a uniform velocity in the curved line. And the curvature of the curved line is defined as the maximum curvature that the robot can rotate in, and we determine the direction  $\theta$  in which the robot faces is in the direction of the next position of the moving object.

$$\gamma \leq \frac{1}{r} \quad (30)$$

$$\theta = \theta_0 + d\theta \quad (31)$$

Here, we determine that the mobile robot is rotated to  $\theta$  that the  $\theta_0$  of the object's initial angle is  $d\theta$  of the change value. Therefore, we determine that the robot can track after its rotation.

4.3.2 PATH PLANNING FOR STRAIGHT LINE

To plan the path of the straight line, we define a velocity vector.

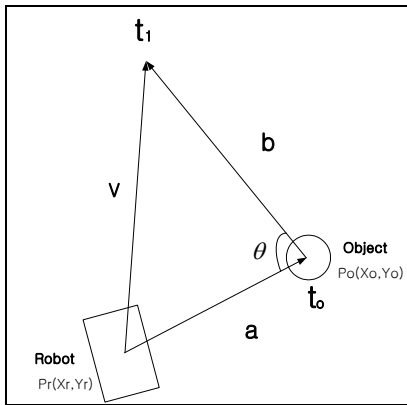


Fig. 3. A definition of velocity vector

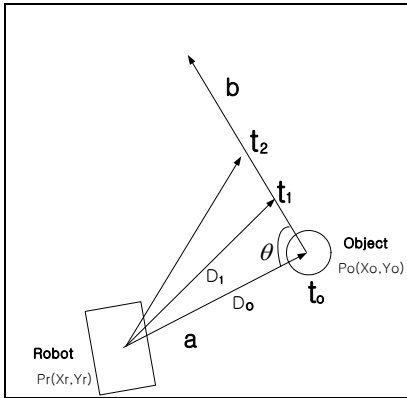


Fig. 4. The velocity vector as distance difference

$$V^2 = a^2 + b^2 - 2ab \cos \theta \tag{32}$$

Above all, we define that the present position of the robot is  $P_r$ , the present velocity is  $V_r$ , the present position of the object is  $P_o$ , and that the present velocity is  $V_o$ . We consider that the velocity vector is determined in order to plan the path of the straight line, as in Fig. 3. The distance between the robot and the object can be determined, as in Fig. 4. The distance difference between the present distance and distance at the next position is divided by the time that the object moves, and then we can determine the velocity vector between the robot and the object. This is defined as vector  $a$ . The next position of the object can be predicted by using the Kalman filter; therefore, we can determine the distance between the present position and the predicted next position. Hence, we can determine the velocity by using this distance. This velocity vector is defined as vector  $b$ . We can know the velocity from the present position of the mobile robot to the object, and the velocity from the present position of the object to the next object position, and so then the velocity in which the mobile robot has to move is determined by using the Second Cosine Law. In the end, we can predict the velocity  $V$  in which the present robot must move in the direction of time  $t_1$  of the predicted object, in Fig. 3. Eq.(32) is determined by using the Second Cosine Law.

4.3.3 PATH PLANNING FOR CAPTURING

Finally, to capture the moving object, the object must be rotated inside of the definite distance that the robot can move. In other words, the mobile robot captures the object when the next position of the object lies within the area that the mobile robot can move during the sampling time. And the mobile robot captures the object at a right angle to the moving object and 5 cm in front of it. The angle change to capture the moving object is represented as

$$\theta_n = \hat{\omega}_{n+1} - 90^\circ \text{ (If } \omega \text{ increases)} \tag{33}$$

$$\theta_n = \hat{\omega}_{n+1} + 90^\circ \text{ (If } \omega \text{ decreases)} \tag{34}$$

Here,  $\theta_n$  is represented as the angle that the robot has to move and  $\omega_n$  is represented as the object angle in x-axis coordinates. As in Eq.(33), if the angle of the next object increases, the robot's direction is controlled by subtracting 90 degrees from the predicted angle. And as in Eq.(34), if the angle of the next object decreases, the robot's direction is controlled by adding 90 degrees to the predicted angle. In conclusion, the mobile robot moves in the direction in which the object moves, and eventually captures it.

5. EXPERIMENTATION AND CONSIDERATIONS

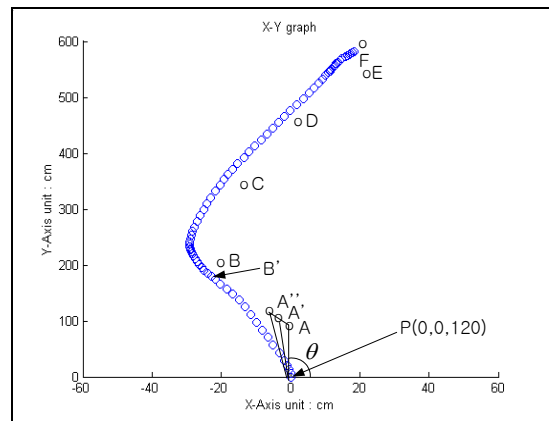


Fig. 5. Trajectory of mobile robot

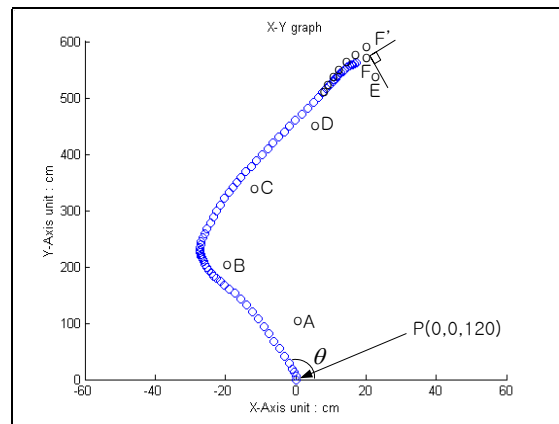


Fig. 6. Trajectory correction of mobile robot

We form the optimal path to capture the moving object from the position and direction of the mobile robot. The moving object moves between 0 degrees and 180 degrees in of x-axis coordinates. And the 2-DOF camera is rotated between 0

degrees and 180 degrees. We suppose that the camera stares at the initial object. It is restricted by the 20 cm/s maximum velocity of the mobile robot, the 16 cm/s maximum velocity of the moving object, and the 15 cm/s<sup>2</sup> maximum acceleration of the mobile robot.

Fig. 5 shows the trajectory of the mobile robot. We establish that the initial position of the robot is P(0,0) and the initial direction is  $\theta = 120^\circ$ . The moving object moves to the points ABCDEF. To confirm the velocity vector, when we look at the triangle OAA' of Fig. 5, the velocity vector of OA', dividing the distance difference between OA and OA' by the time, is 8 cm/s. And the velocity vector from A' to A" is 16 cm/s. So, in conclusion, the velocity in which the robot must move is determined to be 18.8 cm/s by using the Second Cosine Law. When the mobile robot moves in the curved line in B'(-20,90), the velocity is 10 cm/s, and the curvature rate is 0.025.

Fig. 5 is the path that predicted where the object moves initially. Fig. 6 shows that the initially predicted path is corrected. The position that is predicted initially is F'(20,590). The position that is predicted subsequently is F(20,581). It corrected the error difference between F and F'. Because the predicted angle is 118 degrees, the mobile robot moves in the direction of 28 degrees and captures the object. In the end, when the object is captured, the mobile robot predicts the next object's position, and then the object is optimally captured 5 cm in front of the next object's position.

The method that recognizes the object is varied, but in this place, we used the optimal representative blocks to capture the objects efficiently in real-time. Fig. 7 shows that the object is matched by using the representative blocks.

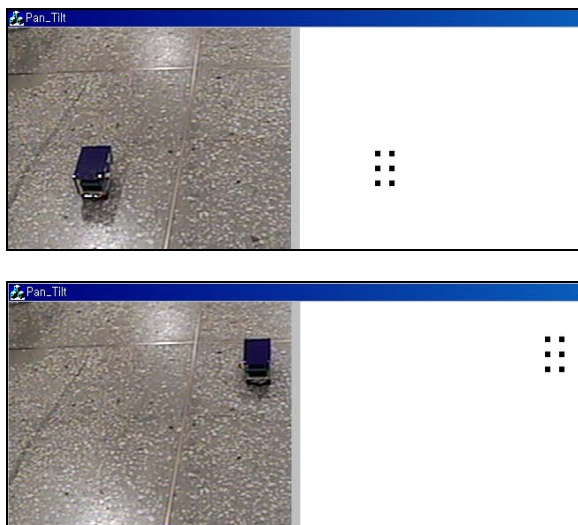


Fig. 7. Representative blocks.

## 6. CONCLUSIONS

In this paper, we proposed an algorithm for planning an optimal path to capture a moving object by a mobile robot in real-time. The direction and rotational angular velocity of the moving object were estimated using the Kalman filter, a state estimator. It was demonstrated that the moving object was tracked using a 2-DOF active camera mounted on the mobile robot and then captured by the mobile manipulator. The initial state of the robot was defined for path planning, and the moving object could be tracked by the velocity vector in real-time. And, by using the Kalman filter, the moving object's was estimated. The mobile robot moved according to the

estimated path, and then, determining the object's state, the error was corrected. To perform in real-time, the optimal representative blocks were used.

## REFERENCES

- [1] Eric T. Baumgartner and Steven B. Skaar, "An Autonomous Vision-Based Mobile Robot," *IEEE Trans. on Automatic Control*, vol. 39, no. 3, pp. 493-502. 1994
- [2] B.H. Kim, D.K. Roh, Jang M. Lee, M.H. Lee, K. Son, M.C. Lee, J.W. Choi, and S.H. Han, "Localization of a Mobile Robot using Image of a Moving Target," *Proceedings of the 2001 IEEE International Conference on Robotics & Automation*, May, 2001.
- [3] Mark W. Spong, M. Vidyasagar, *Robot Dynamics and Control*, John Wiley & Sons, Inc. 1989.
- [4] Jinwoo Park, J.H. Park, K.S. Yun, and Jang-Myung Lee, "Tracking and Capturing a Moving Object using Active Camera Mounted on a Mobile Robot," *Journal of Control, Automation and System Engineering*, vol. 7, no. 9, pp. 741-748, 2001.
- [5] M. S. Grewal and A. P. Andrews, *Kalman Filtering Theory and Practice Using MATLAB*, Wiley Interscience, 2001.
- [6] H. W. Sorenson, "Kalman filtering techniques," *Advances in Control Systems Theory and Applications*, vol. 3, pp. 219-292, 1966.
- [7] W. Wu, H. Chen, and P.Y. Woo, "Optimal Motion Planning for a Wheeled Mobile Robot," *Proceedings of the 1999 IEEE International Conference on Robotics & Automation*, vol. 1, pp. 41-46, May, 1999.
- [8] Y. Kanayama and B.I. Hartman, "Smooth Local Path Planning for Autonomous Vehicle," *Proceedings of the 1989 IEEE International Conference on Robotics & Automation*, vol. 3, pp. 1265-1270, May, 1989.
- [9] J. Su and Y. Xi, "Path Planning for Robotic Hand/Eye System to Intercept Moving Object," *Proceedings of the 38<sup>th</sup> Conference on Decision & Control*, December 1999, pp. 2963-2968.
- [10] Wan C. Kim, Cheol H. Hwang, Su H. Choi and Jang M. Lee, "Efficient Tracking a Moving Object Using Optimal Representative Blocks," *ICCAS 2002, International conference on Control, Automation and Systems*, Muju Resort, Jeonbuk, Korea, 16-19, October 2002, pp.264-269.