Natural Resolution of DOF Redundancy in Execution of Robot Tasks: Stability on a Constraint Manifold

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Abstract: In order to enhance dexterity in execution of robot tasks, a redundant number of degrees-of-freedom (DOF) is adopted for design of robotic mechanisms like robot arms and multi-fingered robot hands. Associated with such redundancy in the number of DOFs relative to the number of physical variables necessary and sufficient for description of a given task, an extra performance index is introduced for controlling such a redundant robot in order to avoid arising of an ill-posed problem of inverse kinematics from the task space to the joint space. This paper shows that such an ill-posedness of DOF redundancy can be resolved in a natural way by using a novel concept named "stability on a manifold". To show this, two illustrative robot tasks 1) robotic handwriting and 2) control of an object posture via rolling contact by a multi-DOF finger are analyzed in details.

Keywords: Redundancy Resolution, Robot Task, Redundant Robot, Stability on a Manifold, Constraint Manifold

1. INTRODUCTION

If a robot is designed so as to mimic human limb then its mechanism must be kinematically redundant, that is, its total degrees of freedom is higher than a number of independent physical variables required for description of a given motion task. This kinematic redundancy contributes to enhancement of dexterity and versatility in execution of robot tasks as discussed in a variety of literatures and books [1-7]. However, such redundancy of DOFs usually increases the complexity of robot dynamics and therefore makes control problems for execution of given tasks more difficult. It is emphasized in particular that in such a case the inverse kinematics from the operational space (task-description space) to the robot joint space becomes ill-posed. In order to avoid this ill-posedness, many methods have been proposed, most of which are based on an idea of introducing some extra performance criterion for determining uniquely an appropriate joint space trajectory by minimizing the criterion. This paper proposes a novel method for resolving such an ill-posedness problem related to redundancy of DOFs by a natural way without introducing any extra performance criterion. Instead, a novel concept named "stability on a manifold" is introduced [8-10] and it is shown that there exists a sensory feedback based on measurements of physical variables of task description and this sensory feedback signal enables the overall system naturally and coordinately to converge to a lower-dimensional constraint manifold that describes a set of joint states fulfilling a target given task. The original idea of "stability on a manifold" was first introduced in control of multi-fingered hands for stable grasping and object-manipulation. In this paper, two typical robotic tasks 1) robotic handwriting and 2) control of an object posture via rolling contact by a multi-DOF finger are treated. It is shown that in both cases proposed feedback control signals are of a simpler form than those obtained by conventional methods of using extra performance criteria and result in human-like distribution of joint motions.

2. DYNAMICS OF HAND-WRITING ROBOTS WITH DOF REDUNDANCY

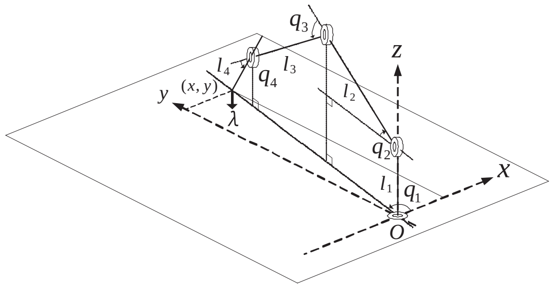

Fig. 1: A handwriting robot with four joints (four degrees of freedom).

Robot dynamics like a handwriting robot as shown in Fig.1 can be described by the following Lagrange equation with a holonomic constraint (for example, see [11]).

$$H(q)\ddot{q} + \left\{ \frac{1}{2}\dot{H}(q) + S(q,\dot{q}) + C_0 \right\} \dot{q} + g(q) - \left(\frac{\partial \phi}{\partial q} \right)^{\mathrm{T}} \lambda = u \quad (1)$$

where $q = (q_1, \cdots, q_4)^{\mathrm{T}}$ denotes the vector of joint angles, u a control input vector, $\phi(q) = 0$ the constraint, λ a corresponding Lagrange multiplier (constraint force), $H(q)$ the 4×4 inertia matrix, C_0 a positive definite diagonal matrix whose diagonal components express damping coefficients in corresponding joint actuators, $g(q)$ the gravity term expressing $(\partial P/\partial q)^{\mathrm{T}} = g(q)$, where $P(q)$ denotes potential energy of the system. Note that $S(q, \dot{q})$ is skew-symmetric, the angular velocity vector $\dot{q}$ is orthogonal to $(\partial\phi/\partial q)^{\mathrm{T}}$ because of $\dot{\phi} = (\partial\phi/\partial q) = 0$, there are positive constants h_M and h_m such that $h_M I \geq H(q) \geq h_m I$ for all q. Hence, it follows that taking an inner product between $\dot{q}$ and both sides of eq.(1) yields

$$\dot{q}^{\mathrm{T}} u = \frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{2}\dot{q}^{\mathrm{T}} H(q)\dot{q} + P(q)\right\} + \dot{q}^{\mathrm{T}} C_0 \dot{q}. \tag{2}$$

In the case of a handwriting robot shown in Fig. 1 the constraint is described as

$$\begin{aligned}\phi(q) &= z(q)\\ &= l_1 + l_2\cos q_2 + l_3\cos(q_2+q_3)\\ &\quad + l_4\cos(q_2+q_3+q_4).\end{aligned} \tag{3}$$

Hence, the gradient of ϕ in q is described as

$$\frac{\partial\phi}{\partial q} = \left(0,\ \begin{array}{c}-l_2\sin q_2 - l_3\sin(q_2+q_3)\\ -l_4\sin(q_2+q_3+q_4)\end{array},\ \begin{array}{c}-l_3\sin(q_2+q_3)\\ -l_4\sin(q_2+q_3+q_4)\end{array},\ -l_4\sin(q_2+q_3+q_4)\right). \tag{4}$$

At first it is assumed that the gravity term $g(q)$ is known or can be computed in real-time based on measurement data by optical encoders of joint actuators. The case when $g(q)$ is uncertain will be treated later. Then, consider a control signal composed of four terms of gravity compensation, angular velocity feedback for damping shaping, feedforward signal of desired pressing force, and position feedback from measured position error in the xy-plane as described in the following:

$$u = g(q) - C_1\dot{q} - J_{\boldsymbol{x}}^{\mathrm{T}} K(\boldsymbol{x} - \boldsymbol{x}_d) - \left(\frac{\partial\phi}{\partial q}\right)^{\mathrm{T}} \lambda_d \tag{5}$$

where $\boldsymbol{x} = (x, y)^{\mathrm{T}}, \boldsymbol{x}_d = (x_d, y_d)^{\mathrm{T}}$ denotes a given desired position on the xy-plane, $J_{\boldsymbol{x}}(q) = \partial\boldsymbol{x}/\partial q$ the 2×4 Jacobian matrix of q in $\boldsymbol{x}$, and $\lambda_d > 0$ a desired pressing force. Substituting this into eq.(1) yields

$$H(q)\ddot{q} + \left\{\frac{1}{2}\dot{H}(q) + S(q,\dot{q}) + C\right\}\dot{q} + J_{\boldsymbol{x}}^{\mathrm{T}}(q) K\Delta\boldsymbol{x} - \left(\frac{\partial\phi}{\partial q}\right)^{\mathrm{T}}\Delta\lambda = 0 \tag{6}$$

where $C = C_0 + C_1, \Delta\lambda = \lambda - \lambda_d$, and $\Delta\boldsymbol{x} = \boldsymbol{x} - \boldsymbol{x}_d$. Then, taking an inner product of eq.(6) with $\dot{q}$ leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}E = -\dot{q}^{\mathrm{T}} C\dot{q} \tag{7}$$

where

$$E = \frac{1}{2}\dot{q}^{\mathrm{T}} H(q)\dot{q} + \frac{1}{2}\Delta\boldsymbol{x}^{\mathrm{T}} K\Delta\boldsymbol{x}. \tag{8}$$

It is important to note that E is neither positive definite in the joint space $R^8 = \{(q, \dot{q})\}$ nor positive definite in the constraint 6-dimensional manifold

$$M_6 = \left\{(q,\dot{q}) : \phi(q) = 0 \quad \text{and} \quad \dot{\phi} = \frac{\partial\phi}{\partial q}\dot{q} = 0\right\} \tag{9}$$

because E includes a quadratic term of only two position variables x and y. Therefore the scalar quantity E can not be regarded as a Lyapunov function in verification of stability of the closed-loop dynamics of eq.(6). However, the quantity E plays an important role in the proof of convergence of a solution to the equation (6). To show this, it is necessary to introduce the following one-dimensional manifold

$$M_1 = \{(q, \dot{q} = 0) : \phi(q) = 0, x(q) = x_d, y(q) = y_d\}. \tag{10}$$

Note that on this manifold the equality $\lambda = \lambda_d$ follows from eq.(6) automatically as far as $\partial\phi/\partial q$ is non-zero. Then, consider a state $(q^0, \dot{q}^0 = 0)$ that belongs to M_1 and call it the reference state. At the same time, define a neighborhood $N^8(r)$ of the fixed reference state $(q^0, 0)$ with radius r in such a way that

$$N^8(r) = \left\{(q,\dot{q}) : \frac{1}{2}\dot{q}^{\mathrm{T}} H(q)\dot{q} + \frac{1}{2}\Delta q^{\mathrm{T}} H(q)\Delta q \leq r^2\right\} \tag{11}$$

where $\Delta q = q - q^0$ and assume that there exists some positive value $r_0 > 0$ such that for any $(q, \dot{q})$ in $N^8(r_0)$ three Jacobian vectors $\partial\phi/\partial q$ $(= \partial z/\partial q)$, $\partial x/\partial q$, $\partial y/\partial q$ are independent, and hence $J_{\boldsymbol{x}}(q)$ is of full rank (non-degenerate), and therefore $J_{\boldsymbol{x}}(q) P_\phi J_{\boldsymbol{x}}^{\mathrm{T}}(q) > 0$, where $J_\phi = \partial\phi/\partial q$,

$$P_\phi = I - J_\phi^{+} J_\phi, \quad \text{and} \quad J_\phi^{+} = J_\phi^{\mathrm{T}}(J_\phi J_\phi^{\mathrm{T}})^{-1}. \tag{12}$$

As seen from eq.(4), the Jacobian vector J_ϕ becomes of zero-vector if and only if all q_i $(i = 2, 3, 4)$ are equal to zero or $\pm\pi$, provided that $0 \geq q_2 \geq -\pi$, $|q_3| \leq \pi$, and $-\pi \leq q_4 \leq \pi$. Hence it is reasonable to assume that r_0 can be chosen not so small and must be of order of the square root of the largest eigenvalue of the inertia matrix $H(q)$ over all q. It is also necessary to introduce a family of neighborhoods of the reference state $(q^0, 0)$ on the constraint manifold M_6 in such a manner that

$$N_6(\varepsilon) = \left\{(q,\dot{q}) : \phi(q) = 0, J_\phi(q)\dot{q} = 0, \quad \text{and} \quad E(\Delta\boldsymbol{x}(q), \dot{q}) \leq \varepsilon^2\right\} \tag{13}$$

where E stands for the quadratic function of $\dot{q}$ and $\Delta\boldsymbol{x}(q)$ $(= \boldsymbol{x}(q) - \boldsymbol{x}_d)$. Note that any state lying on $M_1 \cap N^8(r_0)$ is included in $N_6(\varepsilon) \cap N^8(r_0)$ for any $\varepsilon \geq 0$. Now, it is important to introduce the following two concepts:

Definition (Stability on a manifold) If for any $\varepsilon > 0$ there exist $\delta(\varepsilon) > 0$ depending on $\varepsilon > 0$ and another constant $r_1 > 0$ being less than r_0 and independent of ε such that any solution to eq.(6) starting from an arbitrary initial state lying in $N_6(\delta(\varepsilon)) \cap N^8(r_1)$ remains in $N_6(\varepsilon) \cap N^8(r_0)$ for any $t > 0$, then the reference state $(q^0, 0)$ is said to be stable on a manifold (see Fig.2).

Definition (Asymptotically transferable to a manifold) If there exist positive values $\delta_1 > 0$ and $r_1 > 0$ less than r_0 such that any solution starting from an arbitrary initial state in $N_6(\delta_1) \cap N^8(r_1)$ remains in $M_6 \cap N^8(r_0)$ and converges asymptotically to some state in the set $M_1 \cap N^8(r_0)$ as $t \to \infty$, then the neighborhood $N_6(\delta_1) \cap N^8(r_1)$ together with the reference state $(q^0, 0)$ is asymptotically transferable to $M_1 \cap N^8(r_0)$ (see Fig.3).

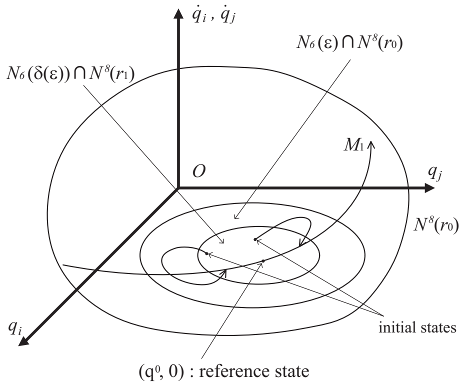

Fig. 2: Stability on a manifold.

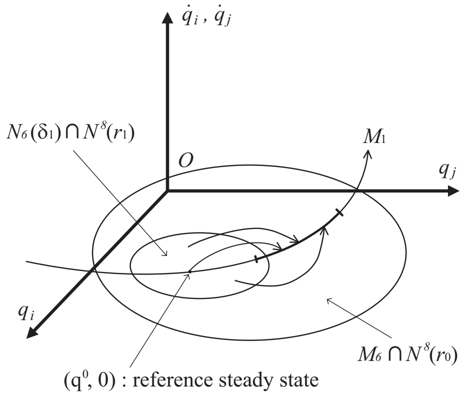

Fig. 3: Any solution trajectory starting from $N_6(\delta_1) \cap N^8(r_1)$ converges asymptotically to a submanifold of M_1.

3. DYNAMICS OF POSTURE CONTROL OF A PIVOTED OBJECT BY A 3 DOF PLANER FINGER ROBOT

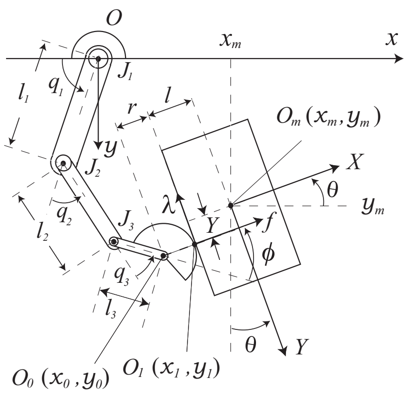

Fig. 4: A single finger robot with 3 DOFs controlling posture of a planer object pivoted at O_m.

Next consider the problem of posture control of a pivoted object by means of a redundant multi-DOF robot finger as shown in Fig.4. The object with a flat surface is pivoted at the fixed point $O_m(x_m, y_m)$ and hence only a rotational motion around O_m in the xy-plane is permitted. The problem is to control the rotational angle θ toward the desired value θ_d by a 3DOF planer robot finger. Hence the overall motion of both the robot and the object is confined to the xy-plane and the gravity force can be ignored. Then, the kinetic energy of the system can be expressed as

$$K = \frac{1}{2}\dot{q}^{\mathrm{T}} H(q)\dot{q} + \frac{1}{2} I\dot{\theta}^2 \tag{14}$$

where $q = (q_1, q_2, q_3)^{\mathrm{T}}$, $H(q)$ and I denote the inertia matrix of the finger and the inertia moment of the object around O_m. Since the finger-end hemisphere contacts with the surface of the object, the following constraint equation follows:

$$Q = -(r + l) + (x_m - x_0)\cos\theta - (y_m - y_0)\sin\theta = 0. \tag{15}$$

On the other hand, the rolling contact without splipping induces the constraint that two speeds of the contact point $O_1(x_1, y_1)$ relative to ϕr and Y must be concident, that is,

$$\frac{\mathrm{d}}{\mathrm{d}t}(\phi r) = -\frac{\mathrm{d}}{\mathrm{d}t} Y \tag{16}$$

where

$$\left\{ \begin{array}{l} Y = (x_0 - x_m)\sin\theta + (y_0 - y_m)\cos\theta \\ \phi = \pi + \theta - q_1 - q_2 - q_3 = \pi + \theta - q^{\mathrm{T}} e \end{array} \right. \tag{17}$$

with $e = (1, 1, 1)^{\mathrm{T}}$. Since eq.(16) can be easily integrated in such a way that

$$R = c_0 + Y + \phi r = 0 \tag{18}$$

where c_0 is an integration constant, it is possible to write the Lagrangian by introducing Lagrange multipliers f and λ correspondingly to the quantities Q and R in the following way:

$$L = K + fQ + \lambda R \tag{19}$$

Thus, by applying Hamilton's principle to the Lagrangian it is possible to obtain Lagrange's equation of motion for this system, which are described as follows:

$$\left\{ H(q)\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{2}\dot{H}(q) + S(q, \dot{q}) \right\} \dot{q} - \left(\frac{\partial Q}{\partial q}\right)^{\mathrm{T}} f - \left(\frac{\partial R}{\partial q}\right) \lambda = u, \tag{20}$$

$$I\ddot{\theta} - \frac{\partial Q}{\partial \theta} f - \frac{\partial R}{\partial \theta}\lambda = 0. \tag{21}$$

More in details, Jacobian vectors in eqs.(20) and (21) can be calculated as

$$\left\{ \begin{array}{l} \partial Q/\partial q = -(\cos\theta, \; -\sin\theta) J(q) \\ \partial R/\partial q = (\sin\theta, \; \cos\theta) J(q) - re^{\mathrm{T}} \\ \partial Q/\partial \theta = Y, \quad \partial R/\partial \theta = -l \end{array} \right. \tag{22}$$

where $J(q) = \partial(x_0, y_0)^{\mathrm{T}} / \partial(q_1, q_2, q_3)$, the Jacobian matrix of $(x_0, y_0)^{\mathrm{T}}$ (the cartesian coordinates of the center of curvature O_0) with respect to the joint coordinates

Table 1: Link parameters

Parameter		Constant	Value		Unit
Link1	Length	l_1	0.050		[m]
	Mass	m_1	40.15×10^{-3}		[kg]
	Inertia moment	I_1	$\begin{bmatrix} 0.870 & 0 & 0 \\ 0 & 0.067 & 0 \\ 0 & 0 & 0.870 \end{bmatrix}$	$\times 10^{-5}$	[kgm^2]
Link2	Length	l_2	0.080		[m]
	Mass	m_2	64.24×10^{-3}		[kg]
	Inertia moment	I_2	$\begin{bmatrix} 0.107 & 0 & 0 \\ 0 & 3.480 & 0 \\ 0 & 0 & 3.480 \end{bmatrix}$	$\times 10^{-5}$	[kgm^2]
Link3	Length	l_3	0.050		[m]
	Mass	m_3	40.15×10^{-3}		[kg]
	Inertia moment	I_3	$\begin{bmatrix} 0.067 & 0 & 0 \\ 0 & 0.870 & 0 \\ 0 & 0 & 0.870 \end{bmatrix}$	$\times 10^{-5}$	[kgm^2]
Link4	Length	l_4	0.030		[m]
	Mass	m_4	24.09×10^{-3}		[kg]
	Inertia moment	I_4	$\begin{bmatrix} 0.040 & 0 & 0 \\ 0 & 0.201 & 0 \\ 0 & 0 & 0.201 \end{bmatrix}$	$\times 10^{-5}$	[kgm^2]

$(q_1,\, q_2,\, q_3)$. Hence, the equation of motion of the object can be written in details as follows:

$$I\ddot{\theta} - Yf + l\lambda = 0. \tag{23}$$

Now, consider the control problem of maneuvering the object toward a specified rotational angle θ_d with a desired pushing force f_d. If a vision sensor can detect the rotation angle $\theta(t)$ of the object, it is possible to define a control signal described as

$$u = -C\dot{q} - \left(\frac{\partial Q}{\partial q}\right)^{\mathrm{T}} f_d - f_d Y e - \left(\frac{\partial R}{\partial q}\right)^{\mathrm{T}} \beta\Delta\theta \tag{24}$$

where $\Delta\theta = \theta - \theta_d$. Substituting this control into eq.(20) yields the closed-loop dynamics

$$\left\{H(q)\frac{\mathrm{d}}{\mathrm{d}t} + \frac{1}{2}\dot{H}(q) + S(q,\, \dot{q}) + C\right\}\dot{q}$$
$$-\left(\frac{\partial Q}{\partial q}\right)^{\mathrm{T}}\Delta f - \left(\frac{\partial R}{\partial q}\right)^{\mathrm{T}}(\lambda - \beta\Delta\theta) + Y f_d e = 0. \tag{25}$$

The dynamics of the object expressed by eq.(23) does not change because they do not have any direct control input, but it is convenient to rewrite them in the following way:

$$I\ddot{\theta} - Y\Delta f + l(\lambda - \beta\Delta\theta) - Y f_d + \beta l \Delta\theta = 0. \tag{26}$$

Finally, it is possible to show the passivity relation concerning the closed loop dynamics of eqs.(25) and (26) as follows:

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{d}t}V = -\dot{q}^{\mathrm{T}}C\dot{q} \\ V = \dfrac{1}{2}\left\{\dot{q}^{\mathrm{T}}H(q)\dot{q} + I\dot{\theta}^2 + \beta l\Delta\theta^2 + \dfrac{f_d}{r}Y^2\right\} \end{cases} \tag{27}$$

The scalar quantity V is not positive definite in the eight-dimensional state space $(q,\, \theta,\, \dot{q},\, \dot{\theta})$ but positive definite in the four-dimensional constraint space

$$M_4 = \left\{(q^{\mathrm{T}}, \theta, \dot{q}^{\mathrm{T}}, \dot{\theta})^{\mathrm{T}} : Q = 0, R = 0, \dot{Q} = 0, \dot{R} = 0\right\} \tag{28}$$

because V includes quadratic terms of two positional variables $\Delta\theta$ and Y. Hence, the well-known theorem of LaSalle can be applied to eq.(27). However, in the case that the control objective is only to stop motion of the object by using a feedback signal

$$u = -C\dot{q} - \left(\frac{\partial Q}{\partial q}\right)^{\mathrm{T}} f_d - f_d Y e, \tag{29}$$

the scalar function V becomes of the form

$$V_0 = \frac{1}{2}\left\{\dot{q}^{\mathrm{T}}H(q)\dot{q} + I\dot{\theta}^2 + \frac{f_d}{r}Y^2\right\}. \tag{30}$$

This quantity is no more positive definite in the constraint manifold M_6 defined by eq.(28).

4. THEOREMS AND SIMULATION RESULTS ON REDUNDANCY RESOLUTION

In the case of handwriting robots, if feedback gain matrices K and C_1 are adequately chosen relatively to kinematic parameters of the robot, then it is possible to prove the following theorem:

Theorem 1 If the handwriting robot has physical parameters likely as in Table 1 and feedback gain matrices C_1 and K and the desired constraint force λ_d are chosen in the vicinity of corresponding values given in

Table 2: Choice for gains

Constant [Unit]	Value
k[N/m]	15
c[smN]	0.02
α[1/smN]	$\alpha = 1/c$, 50
γ	500
ϵ_0	0.20

Table 3: Initial conditions

Parameter	Variable	Value	unit
Position	x_0	0	[m]
	y_0	9.12×10^{-2}	[m]
Force	λ_0	0.20	[N]

Table 2, then the reference state $(q^0,\,0) \in M_1$ is stable on a manifold. Furthermore, there exist $\delta_1 > 0$ and $r_1 > 0$ such that a neighborhood $N_6(\delta_1) \cap N^8(r_1)$ of the reference state $(q^0,\,0)$ is asymptotically transferable to $M_1 \cap N^8(r_0)$.

In the case of stopping motion of the object by rolling contact with a robot finger as shown in Fig.4, an anologous theorem can be derived.

In order to show the validity of Theorem 1, computer simulations based on physical parameters of the robot and feedback gain parameters as shown in Tables 1 and 2 and initial and desired conditions as shown in Table 3 are carried out. Numerical solutions of the differential equations under the geometric constraint are obtained by using Baumgarte's method [11], in which a coefficient γ_f for highly over-damped second order differential equations corresponding to the constraint $z = 0$, that is,

$$\ddot{Q} + \gamma_f \dot{Q} + (\gamma_f^2/4)Q = 0 \tag{31}$$

is chosen as $\gamma_f = 1000$. In the first simulation when the control of eq.(5) is used, a better choice for the damping gain $C = cI$ with $c = 0.02$ is chosen firstly after several trials of simulation by changing values of c together with k for $K = kI$. Then, for a fixed $c = 0.02$ [msN] transient responses of x, y, and constraint force λ are shown in Figs.5 to 7 with changing the position feedback gain k. According to the figures, the best choice for k is around $k = 15$ [N/m]. It is quite interesting to note that in trasient behaviours of robot motion the projected value of q to the subspace orthogonal to the image space of $J^{\mathrm{T}}(q) = (J_{\boldsymbol{x}}^{\mathrm{T}},\, J_{\phi}^{\mathrm{T}})$ is almost fixed as shown in Fig.8, where Δq is defined as

$$\Delta q = \left\{ I - J^{+}(q)J(q) \right\} q. \tag{32}$$

This shows that the control signal defined by eq.(5) does little affect joint motion of the robot irrelevant to movement of the endpoint of the robot in the xy-plane and control of the constraint force, which looks like a human-like skilled motion.

Table 4: Desired conditions

Parameter	Variable	Value	unit
Desired position	x_d	x_0+0.03	[m]
	y_d	y_0+0.03	[m]
Desired force	λ_d	0.50	[N]

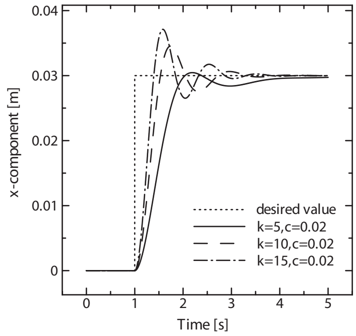

Fig. 5: Transient responses of x-component of the robot endpoint position for various values of k.

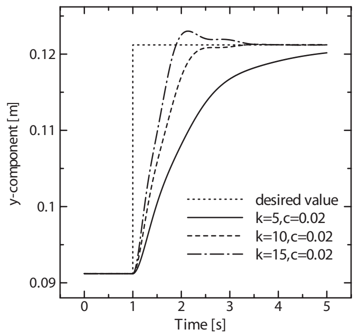

Fig. 6: Transient responses of y-component of the robot endpoint position for various values of k.

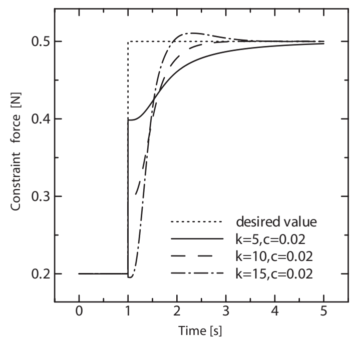

Fig. 7: Transient responses of contraint force λ for various values of k.

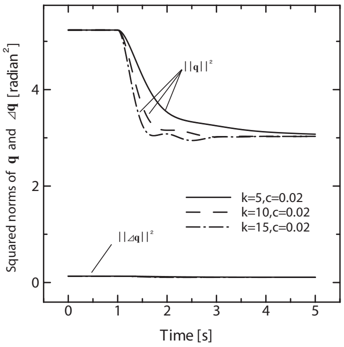

Fig. 8: Transient responses of the magnitudes of squared norms $\|q\|^2$ and $\|\Delta q\|^2$ where $\Delta q = (I - J^{+}J)q$.

5. GRAVITY COMPENSATION BY MEANS OF REGRESSOR

When physical parameters appearing in the gravity torque are uncertain, the gravity term $g(q)$ in eq.(1) can not be compensated exactly. In this case it is possible to use a regressor $Z(q)$ together with an estimator $\hat{\Theta}$ of uncertain parameters for approximating $\hat{g}(q) = Z(q)\hat{\Theta}$, where

$$Z(q) = g\begin{pmatrix} 0 & 0 & 0 \\ \cos q_2 & \cos(q_2+q_3) & \cos(q_2+q_3+q_4) \\ 0 & \cos(q_2+q_3) & \cos(q_2+q_3+q_4) \\ 0 & 0 & \cos(q_2+q_3+q_4) \end{pmatrix}, \quad (33)$$

$$\Theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} m_2s_2 + m_3l_2 + m_4l_2 \\ m_3s_3 + m_4l_3 \\ m_4s_4 \end{pmatrix} \quad (34)$$

and the gravity term can be expressed as

$$g(q) = Z(q)\Theta. \quad (35)$$

Then, the control signal can be composed of the form

$$u = Z(q)\hat{\Theta} - C\dot{q} - J_{\boldsymbol{x}}^{\mathrm{T}}K\Delta\boldsymbol{x} - J_{\phi}^{\mathrm{T}}\lambda_d \quad (36)$$

and the estimator $\hat{\Theta}$ can be updated by the form

$$\hat{\Theta}(t) = \hat{\Theta}(0) - \int_0^t \Gamma^{-1}Z^{\mathrm{T}}(q(\tau))\left\{\dot{q}(\tau) + \alpha P_{\phi}J_{\boldsymbol{x}}^{\mathrm{T}}K\Delta\boldsymbol{x}(\tau)\right\}\mathrm{d}\tau. \quad (37)$$

6. CONCLUSIONS

A problem of redundancy resolution is resolved in a natural way based on the new concept "stability on a manifold" in the cases of tasks of a handwriting robot and posture control of an object by means of a robot finger. It is expected that such difficulties in controlling systems with many DOFs under ill-posedness of inverse kinematics can be coped in a similar way to that proposed in this paper.

REFERENCES

[1] D.E. Whitney, "Resolved motion rate control of manipulators and human protheses", *IEEE Trans. Man-Machine Syst.*, Vol. MMS-10, No. 2, pp. 47–53 (1969).

[2] A. Liegeois, "Automatic supervisory control of the configuration and behavior of multibody mechanism", *IEEE Trans. Systems, Man, and Cybern.*, Vol. SMC-7, No. 12, pp. 868–871 (1977).

[3] M. Vukobratoric and M. Kircanski, *Kinematics and Trajectory Synthesis of Manipulatin Robots*, Springer-Verlag, Berlin, Germany (1986).

[4] L. Sciavicco and B. Siciliano, "A solution algorithm to the inverse kinematic problem for redundant manipulators", *IEEE J. of Robotics and Automation*, Vol. 4, No. 4, pp. 403–410 (1988).

[5] M. Takegaki and S. Arimoto, "A new feedback method for dynamic control of manipulators", *ASME J. of Dynamic Systems, Measurement, and Control*, Vol. 103, pp. 119–125 (1981).

[6] J.M. Hollerbach and K.C. Suh, "Redundancy resolution of manipulation through torque optimization", *IEEE J. of Robotics and Automation*, Vol. 3, No. 4, pp. 308–316 (1987).

[7] V. Potkonjak, M. Popovic, M. Lazarevic, and J. Sinanovic, "Redundancy problem in writing: from human to authropomorphic robot arm", *IEEE Trans. Systems, Man, and Cybernetics*, Vol. 28, No. 6, pp. 790–805 (1998).

[8] S. Arimoto, J.-H. Bae, and K. Tahara, "Dynamic stable pinching by a pair of robot fingers", *Proc. of the 2nd IFAC Conf. on Mechatronic Systems*, Berkeley, Cal., USA, pp. 731–736 (2002).

[9] S. Arimoto, K. Tahara, J.-H. Bae, and M. Yoshica, "A stability theory of a manifold: concurrent realization of grasp and orientation control of an object by a pair of robot fingers", *Robotica*, Vol. 21, No. 2, pp. 163–178 (2003).

[10] S. Arimoto, M. Yoshida, J.-H. Bae, and K. Tahara, "Dynamic force/torque balance of 2D polygonal objects by a pair of rolling contacts and sensory-motor coodination", to be published in *J. of Robotic Systems*, Vol. 20, No. 9 (2003).

[11] J. Baumgarte, "Stabilization of constraints and integrals of motion in dynamical systems", *Comput. Math. Appl. Mech. Eng.*, Vol. 1, No. 1, pp. 1–16 (1972).