Front Points Tracking in the Region of Interest with Neural Network in Electrical Impedance Tomography

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Abstract: In the conventional boundary estimation in EIT (Electrical Impedance Tomography), the interface between anomalies and background is expressed in usual as Fourier series and the boundary is reconstructed by obtaining the Fourier coefficients. This paper proposes a method for the boundary estimation, where the boundary of anomaly is approximated as the interpolation of front points located discretely along the boundary and is imaged by tracking the points in the region of interest. In the solution to the inverse problem to estimate the front points, the multi-layer neural network is introduced. For the verification of the proposed method, numerical experiments are conducted and the results indicate a good performance.

Keywords: Electrical impedance tomography, Boundary estimation, Front point tracking, Neural network.

1. INTRODUCTION

In electrical impedance tomography (EIT) an array of electrodes is attached around the object and small alternating currents are injected via these electrodes and the resulting voltages are measured. Based on the imposed currents and the measured voltages, an approximation for the internal impedance distribution is computed. Usually the object is discretized into a number of small pixels, in each of which impedance is assumed to be constant. The image reconstruction in EIT will be the inverse problem to estimate the impedance of each pixel.

Inexpensive hard-wear, negligible or no health consideration, and high time resolution of EIT could imply a good possibility in the practical applications to many areas including medical diagnosis, engineering process monitoring, non-destructive detection and so on. Due to the well-known ill-posed nature of EIT, however, the reconstructed image is very susceptible to the noise in the imposed and measured electrical signals and regularization should be introduced to mitigate the ill-posedness. This results in poor spatial resolution and the boundary of anomalies is hardly achieved. Hence, some researchers are interested in the direct estimation of the boundary rather than the impedance distribution when anomalies and background have different but constant impedance values [1-5].

In most papers dealing with the boundary estimation, the boundary is expressed as Fourier series and the inverse problem is to find the Fourier coefficients instead of the impedances of pixels. If the anomalies are nearly circular or elliptic, in the Fourier expression a few lowest modes will be dominant and higher modes will be negligible. When the anomalies are deformed severely from the simple shapes, however, higher Fourier modes may prevail over lower modes. Since the sensitivity that is the rate of change of the boundary voltage with respect to the Fourier coefficient tends to be smaller as the Fourier mode becomes higher, the Fourier decomposition method may show poor performance for the boundary estimation of deformed anomalies.

In this paper, we propose a new method of boundary estimation in EIT, in which the boundary of anomaly is approximated as the interpolation of front points located discretely along the boundary and is imaged by tracking the points in the region of interest (ROI). Especially, we are considering the reconstruction of an anomaly whose boundary is partly known and partly unknown. In tracking the front points, the multi-layer neural network is adopted since it is easily expanded to dynamic imaging.

2. METHOD

2.1 Forward problem

In the forward problem of EIT we calculate the potentials on the boundary of an object given the resistivity distribution and injected currents on the boundary. When electrical currents $I_l\,(l=1,2,\cdots,L)$ are injected into the object $\Omega\in\Re^2$ through the electrodes $e_l\,(l=1,2,\cdots,L)$ attached on the boundary and the resistivity distribution $\rho(x,y)$ is known for the Ω, the corresponding electrical potential $u(x,y)$ on the Ω can be determined uniquely from the following partial differential equation, which can be derived from the Maxwell equations:

$$\nabla\cdot(\rho^{-1}\nabla u)=0 \quad \text{in } \Omega. \tag{1}$$

subject to the boundary conditions

$$\int_{e_l}\rho^{-1}\frac{\partial u}{\partial n}dS=I_l, \quad l=1,2,\cdots,L \tag{2}$$

$$\rho^{-1}\frac{\partial u}{\partial n}=0 \quad \text{on } \partial\Omega\setminus\bigcup_{l=1}^{L}e_l. \tag{3}$$

From the complete electrode model (CEM), the boundary potential at the electrodes is expressed as:

$$u + z_l \rho^{-1} \frac{\partial u}{\partial n} = U_l \quad \text{on} \quad e_l, \quad l = 1,2,\cdots,L \tag{4}$$

where z_l is the effective contact impedance between l-th electrode and object, U_l is the potential on the l-th electrode, e_l is l-th electrode, n is outward unit normal, and L is the number of electrodes. The CEM takes into account the shunting effect (i.e. the voltage U_l is constant over the electrode e_l) and the additional voltage drop due to the contact impedance.

In addition, we must ensure the following two constraints for the injected currents and the measured voltages from the conservation of electrical charge and the uniqueness of the solution, respectively.

$$\sum_{l=1}^{L} I_l = 0\,, \tag{5}$$

$$\sum_{l=1}^{L} U_l = 0\,. \tag{6}$$

Since the forward problem cannot be solved analytically for arbitrary geometries we have to resort to the numerical method. In this paper, we used the finite element method (FEM) to obtain the numerical method. In the FEM, the object area is discretized into small elements having a node at each corner. It is assumed that the resistivity distribution is constant within an element. The potential at each node is calculated by discretizing Eq. (1) into $Yv = c$, where $Y \in \Re^{N \times N}$ is the admittance matrix that is a function of resistivity and c represents the current injected into the object. The number of FEM nodes is denoted by N [6].

2.2 Boundary approximation with front points

We approximate the boundary as an interpolation with front points located discretely along the boundary instead of Fourier series which was used in most previous works for the boundary estimation. If we choose M points spacing equivalently along the polar angle in the frame of polar coordinate, the angle position will be

$$\theta_k = 2\pi(k-1)/M, \quad k = 1,2,\cdots,M\,. \tag{7}$$

The set of front points, R, is defined as follows:

$$\begin{aligned} R &= \{r_k \mid k = 1,2,\cdots,M\} \\ &= R^{ref} + R^{change} \end{aligned} \tag{8}$$

where r_k is the distance of the k-th front point measured from the center of the reference coordinate. R^{ref} is the reference points and R^{change} denotes the deviation from the reference. If we define the ROI set Q whose elements are the indices of the front points included in the ROI, the deviation will be

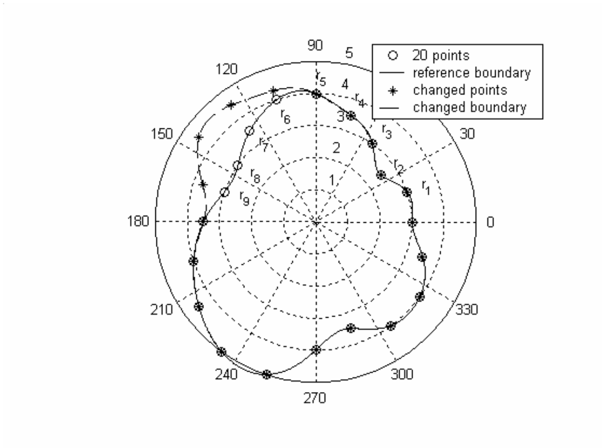

Fig. 1. Reference boundary and part of the boundary changed in ROI

$$R^{change} = \begin{cases} r_j^{change} & , j \in Q \\ 0 & , otherwise \end{cases} \tag{9}$$

To express the boundary with the discrete front points, an interpolation is required. In this paper Fourier interpolation is used. Figure 1 shows an example of boundary approximation with front-point concept. In this, 20 front points are considered and the ROI set is assumed to be $Q = \{6,7,8,9\}$. If we formulate a boundary estimation problem for the above example based on Fourier series, higher mode Fourier coefficients (more than tenth) are required. If the boundary is deformed in part as in the case of the example, the Fourier coefficients should be altered totally. However, in the proposed algorithm, only the front points belonging to ROI set needs to be modified.

2.3 The neural network

For the determination of the front points, we use a multi-layer neural network as shown Fig. 2. The weight matrices W_1 and W_2 are calculated by training the processes of the neural network. The neural network trains W_1 and W_2 using the change of the voltages on the electrodes with respect to the change of R^{ref}.

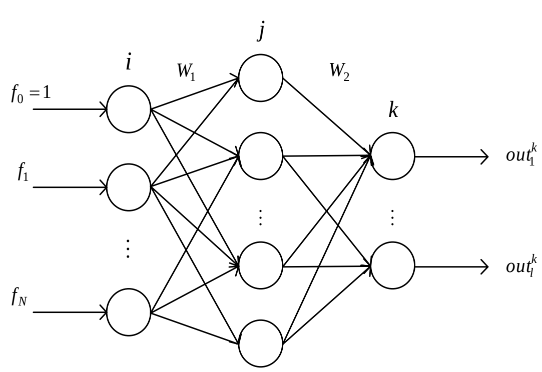

Fig. 2. The Multi-layer neural network.

The measurement vector, f_n, is defined as the ratio of the change of the voltage before and after R^{change} change in ROI [7].

$$f_n = \frac{v_n^i - v_n^{ref}}{v_n^i + v_n^{ref}}, \quad \begin{array}{l} i = 1,2,\cdots,S \\ n = 1,2,\cdots,N, \end{array} \tag{10}$$

where v_n denotes the *n*-th voltage from the measured voltage, U_l, at each frame, S is the number of front points in ROI, N is the number of the measured voltages at each frame, and v^{ref} is the measured voltage for R^{ref}.

Let's consider half of the deviation of R from R^{ref}, ΔR_{known}, which is known during the training process.

$$\Delta R_{known} = \frac{R^i - R^{ref}}{2} = \frac{(R^{change})^i}{2}, \quad i = 1,2,\cdots,S, \tag{11}$$

Using the parameters f_n and ΔR_{known}, we update the weight matrix as follows.

a) The input neuron.

$$Out^i = f_0, f_1, \cdots, f_n, \tag{12}$$

b) The input and the output at the hidden layer.

$$net^j = \sum_{i=1}^{n} (W_1 \cdot Out^i), \quad Out^j = g(net^j), \tag{13}$$

c) The input and the output at the output layer.

$$net^k = \sum_{j=1}^{J} (W_2 \cdot Out^j), \quad Out^k = g(net^k). \tag{14}$$

where Out^i, Out^j, and Out^k are the output of the neuron at the input layer *i*, the hidden layer *j*, and the output layer *k*, respectively. Also net^i, net^j, and net^k are the input of the neuron at the each layer. The function $g(x)$ is defined as follows.

$$g(x) = \frac{1}{1 + e^{-bx}}, \tag{15}$$

where *b* is called the gradient parameter and in this work is set to 1.

The performance index error, E, is calculated by the following equation.

$$E = \sum (\Delta R_{known} - Out^k)^2. \tag{16}$$

We perform a), b), and c) orderly. So if E is sufficiently small, we stop updating the weight matrices W_1 and W_2. Otherwise, we perform a), b), and c) again after we update W_1 and W_2 as the following d) and e).

d) Weight matrix W_2 is updated as follows.

$$\begin{aligned} &\delta^k = (\Delta R_{known} - Out^k) \cdot f'(net^k) \\ &\Delta W_2 = \eta \cdot \delta^k \cdot Out^j \\ &W_2(k+1) = W_2(k) + \Delta W_2 + \alpha \cdot (W_2(k) - W_2(k-1)), \end{aligned} \tag{17}$$

e) Weight matrix W_1 is updated as follows.

$$\begin{aligned} &\delta^j = f'(net^j) \cdot \sum_k (\delta^k \cdot W_2) \\ &\Delta W_1 = \eta \cdot \delta^j \cdot Out^i \\ &W_1(k+1) = W_1(k) + \Delta W_1 + \alpha \cdot (W_1(k) - W_1(k-1)). \end{aligned} \tag{18}$$

We estimate ΔR using the change of the voltage and the updated weight matrices W_1 and W_2:

$$\Delta \hat{R} = g(W_2 \cdot g(W_1 \cdot \tilde{f}_n + b) + b) \tag{19}$$

where $\tilde{f}_n$ is the change of the voltage in the actual system.

3. SIMULATIONS

For the verification of the proposed model, we conducted numerical experiments. In the simulation, we estimate r_j^{change}, which have not been used for teaching during the pre-process to obtain the weight matrices. We consider a circular object of radius $14cm$, which has 16 electrodes along the boundary. The domain is discretized into 1968 triangular elements in the finite element calculation. The resistivity values of the anomaly and the background are set to $600\Omega cm$ and $300\Omega cm$, respectively. As for the current injection pattern, the opposite method is used.

In order to implement the multi-layer neural network to the inverse solution, we should train the weight matrices with boundary voltage data corresponding to possible variations of anomaly boundary in the ROI. In the present problem, the inputs of the multi-layer network are the change of the voltages, f_n, at each frame and ΔR_{known}.

We approximate the boundary with 30 front points and assume 7 points are in the ROI. The number of the measured voltages is set to 128 and the number of the sampled cases to train is 1459. The number of the units in the hidden layer is 48. Fourier series of order 10 is used for the interpolation.

The assumed reference front points R^{ref} and ROI set Q are

$$\begin{aligned} R^{ref} = \{&2\ 3\ 5\ 7\ 8\ 7.5\ 7\ 6.5\ 6\ 6\ 5\ 5\ 5\ 5\ 5 \cdots \\ &5\ 5\ 5.5\ 6\ 7\ 8.5\ 9\ 9\ 8\ 7\ 4\ 2.5\ 2\ 1.5\ 1.5\}, \end{aligned} \tag{18}$$

$$Q = \{1,2,3,4,5,6,30\}. \tag{19}$$

To evaluate the estimation of r_j^{change} in the ROI, RMS error is defined as:

$$RMS\ error = \sum (R^{change} - \hat{R}^{change})^2. \tag{20}$$

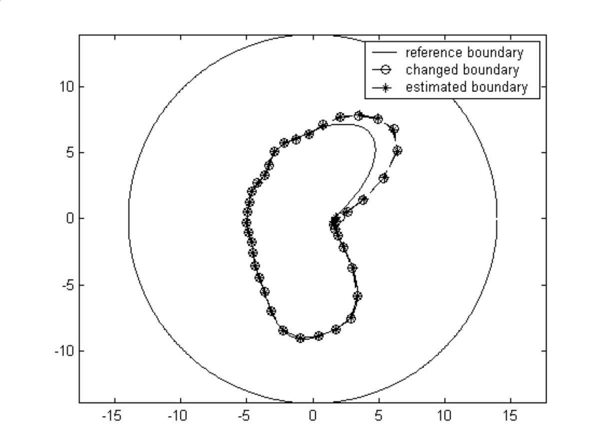

Fig. 3. Reference, target, and estimated boundary (error free).

After training with the sampled cases, the multi-layer neural network is applied to the synthesized voltage data for r_j^{change}

$$r_j^{change} = \{0.8 \quad 1.5 \quad 3 \quad 2 \quad 1 \quad 0.8 \quad 0.5\},$$

which have not been used for teaching. The reference and the target boundary are shown in Fig. 3.

When no measurement error is assumed, the estimated boundary $\hat{r}_j^{change}$ is obtained as

$$\hat{r}_j^{change} = \{0.7038, 1.7138, 2.9602, 1.8780, 1.0992, 0.7400, 0.2400\}$$

$$RMS\ error = 0.3905\ .$$

As can be seen in Fig. 3, the estimation is quite excellent. For the same example, now, we assume 1% random noise in the measured voltages. The neural network generates the estimated front points in the ROI as

$$\hat{r}_j^{change} = \{0.5021, 1.2066, 3.0108, 1.8394, 1.1646, 0.8046, 0.2249\}$$

$$RMS\ error = 0.5509\ .$$

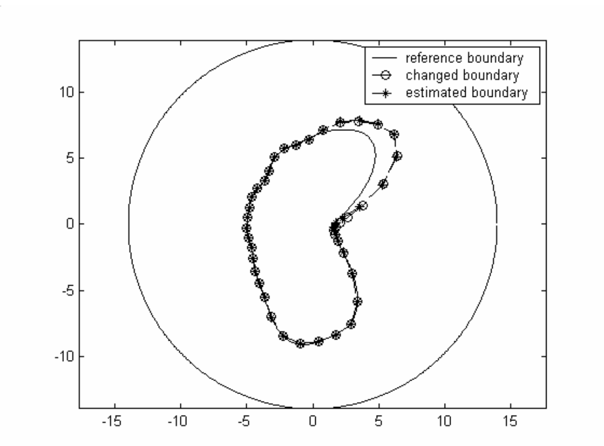

Fig. 4. Reference, target, and estimated boundary (1% error).

The estimate is compared with the target in Fig. 4, which implies that the proposed algorithm has a good performance in the boundary estimation even with noisy data.

4. CONCLUSIONS

In this paper, we propose a new method to estimate the boundary of anomaly in the electrical impedance imaging. Contrary to previous methods where the boundary is expressed by Fourier series and the Fourier coefficients are unknowns to be estimated, in the present method, the boundary is approximated as an interpolation of front points along the boundary and front points in the ROI are unknowns to be determined. The unknown front points are tracked by adopting with the multi-layer neural network.

For the verification of the proposed method, we perform numerical experiments with synthesized voltage data with no error and 1% random error. The results show excellent agreements with the true boundary.

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