Robust NN Controller for Autonomous Diving Control of an AUV

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Abstract: In general, the dynamics of autonomous underwater vehicles(AUVs) are highly nonlinear and time-varying, and the hydrodynamic coefficients of vehicles are hard to estimate accurately because of the variations of these coefficients with different navigation conditions. For this reason, in this paper, the control gain function is assumed to be unknown and the exogenous input term is assumed to be unbounded, although it still satisfies certain restrict condition. And these two kinds of wild assumptions have been seldom handled simultaneously in one system because of the difficulty of stability analysis. Under the above two relaxed assumptions, a robust neural network control scheme is presented for autonomous diving control of an AUV, and can guarantee that all the signals in the closed-loop system are UUB (uniformly ultimately bounded). Some practical features of the proposed control law are also discussed.

Keywords: Robust adaptive control, neural network, uncertainties, AUV

1. INTRODUCTION

In recent two decades, robust adaptive control has become a powerful methodology in the control problem of nonlinear uncertain systems, and several correspondent control methods have been developed. By using the adaptive backstepping design procedure, Kanellakopoulos *et al*. [1] have presented a systematic design of globally stable and asymptotically tracking adaptive controllers for a class of nonlinear systems transformable to a parametric strict-feedback form. The over-parameterization problem was soon overcome by Krstic *et al*. [2] by introducing the concept of tuning function. Krstic *et al*. [3] also introduced the nonlinear damping to improve the transient performance of the control system. In these design procedures, the only handled uncertainties were that could be linearly parameterized by unknown constant parameters.

Due to the approximation capacities of neural networks for nonlinear mappings [4,5] and their learning characteristics, there have been a considerable interests in exploring the applications of neural networks to the control of nonlinear uncertain systems [6-11]. The main characteristic of the neural network adaptive control scheme is just to approximate the smooth uncertainties of the systems using neural networks, and this can relax the restrict conditions in the previous adaptive control scheme where the uncertainties were assumed to be linearly parameterized. For the remained uncertainties, such as exogenous inputs and neural networks' reconstruction errors, there are various wild assumptions on them to progress the stability analysis. In [6,9], exogenous input terms were assumed to be bounded by known constants, and in [7,8,10], they were bounded by unknown constants, and they also assumed to be bounded by known function [11]. Recently, there were another kinds of various wild assumptions on the control gain functions of NN adaptive control scheme. In [7,8], they were assumed to be known, and in [6,11], they were assumed to be unknown and approximated by neural networks, in [9], the control gain function was assumed to be bounded by known functions.

In recent two decades, underwater robotic vehicles (URVs) have become an intense area of oceanic research because of their emerging applications. However, unfortunately, URVs' dynamics are highly nonlinear and time-varying, and the hydrodynamic coefficients of vehicles are difficult to be identified exactly *in priori* because of the variations of these coefficients with the vehicle's different navigation conditions. For this reason, in general, URVs' dynamics often include unknown exogenous input terms, and the control gain functions also could not be known exactly. In this paper, the exogenous input term of the AUV's dynamics is assumed to be unbounded, although it still satisfies certain restricting condition - Lipschitz condition with unknown Lipschitz constant. And the gain function of control is assumed to be unknown with known sign. Under these two relaxed wild assumptions, presented robust NN control scheme can guarantee that all the signals in the closed-loop system satisfy to be UUB. Some practical features of the proposed control law are also discussed.

2. PROBLEM STATEMENTS

The dynamic behaviour of an AUV can be described in common way [13] through a 6DOF nonlinear equation

$$M(\eta)\ddot{\eta} + C_D(\eta,\dot{\eta})\dot{\eta} + g(\eta) + d = \tau, \tag{2.1}$$

where $\eta = [x, y, z, \phi, \theta, \psi]^T$ is the position and orientation vector, $M(\eta) \in \Re^{6\times 6}$ is inertia matrix (including added mass), $C_D(\eta,\dot{\eta}) \in \Re^{6\times 6}$ is matrix of Coriolis, centripetal and damping term, $g(\eta) \in \Re^6$ is gravitational forces and moments vector, d denotes the exogenous input vector, and τ is the input torque vector. Further, we define $\dot{\eta} = [u, v, w, p, q, r]^T$ be the velocity and angular rate vector.

The diving equations of the AUV should include the heave velocity w, the angular velocity in pitch q, the pitch angle θ, the depth z and the stern plane deflection and/or thrust force of the propellers. Restricting the vehicle in the constant forward motion and, for simplicity, assume that the heave velocity during diving is small and negligible. This is quite realistic since most small underwater vehicles move slowly in the vertical direction. Further, in general, underwater vehicles are designed to have symmetric structures and it is reasonable to assume that the body fixed coordinate is located at the center of gravity with the gravity force equal to the buoyancy force of the vehicle. Consequently, the pitch and depth motion of the vehicle during diving can be expressed as following, which is a certain modified expression from [13,14]

$$\begin{aligned} \dot{z} &= -u_0\theta + \Delta f_z, \\ \dot{\theta} &= q, \\ \dot{q} &= f_q + b\tau_\theta, \end{aligned} \tag{2.2}$$

where u_0 is a known constant forward speed, Δf_z denotes the uncertainty of the vehicle, f_q and b are defined as $f_q := \zeta_1(M, C_D, g)$, $b := \zeta_2(M)$ with $\zeta_1(\cdot)$ and $\zeta_2(\cdot)$ smooth functions.

Due to the highly nonlinear dynamics of URVs and the unpredictable environments of the vehicles, in the most applications of AUVs, it is hard to determine the exact values of M, C_D and g in the equation (2.1) *in priori*. For this reason, we make the following assumptions on equation (2.2).

Assumption 1: Exogenous input term Δf_z satisfies that $\Delta f_z = \Delta \cdot \zeta_3(z)$, where $|\Delta| \le c_1$ and c_1 is an unknown constant. Further, unknown function $\zeta_3(z)$ satisfies certain Lipschitz condition with unknown Lipschitz constant c_2, such that $|\zeta_3(z)| \le c_2 |z| + c_3$, where c_3 is also an unknown constant.

Assumption 2: f_q and b are smooth unknown functions, and b is known sign and nonzero. Without any loss of generality, we assume that $b \ge c_4 > 0$ with c_4 unknown constant. Further, the derivative of b^{-1} is assumed to be bounded by unknown constant c_5 such that $d(b^{-1})/dt \le c_5$.

Remark 1: In general, most of URVs are designed to move slowly in the deepsea environment. In this case, the control gain function b varies slowly, and this make the Assumption 2 be reasonable.

Using above Assumption 2, the final equation of (2.2) can be rewritten as

$$b^{-1}\dot{q} = b^{-1} f_q - \tau_\theta. \tag{2.3}$$

Here we want to approximate the first terms of the right sides of the above equation using neural network. According to Assumption 2, the smooth term $b^{-1} f_q$ can be written in the parametric form [8]

$$b^{-1} f_q = W^{*T} \phi^*(\eta, \dot{\eta}, \ddot{\eta}), \tag{2.4}$$

where $W^* \in \Re^{N^*}$ is a constant vector and $\phi^*(\eta, \dot{\eta}, \ddot{\eta}) \in \Re^{N^*}$ is a basis function vector of $b^{-1} f_q$. If the basis of a function is exactly known, then the functional approximation problem can be converted to the well-known parameter estimation problem. However, in practice, we could not exactly know the basis of an unknown function *in priori*, and there always remains network's reconstruction error. Consequently, (2.4) can be expressed as

$$b^{-1} f_q = W^T \phi(\eta, \dot{\eta}, \ddot{\eta}) + \varepsilon(\eta, \dot{\eta}, \ddot{\eta}), \tag{2.5}$$

where $W \in \Re^N$ is the optimal weight vector of the constructed neural network, $\phi(\cdot) \in \Re^N$ is the constructed basis function vector, and $\varepsilon(\cdot)$ is the network's reconstruction error. In [6-8], authors gave some practical selection methods of the basis function vectors according to the physical properties of the target plants.

The optimal weight vector W in (2.5) is an "artificial" quantity required only for analytical purposes. Typically, W is chosen as the value of W' that minimizes $\varepsilon(\cdot)$ for all $\eta, \dot{\eta}, \ddot{\eta} \in \Omega$, where $\Omega \subset \Re^6$ is a compact region, i.e., [8]

$$W := \arg\min_{W \in \Re^N} \left\{ \sup_{\eta, \dot{\eta}, \ddot{\eta} \in \Omega} |b^{-1} f_q - W'^T \phi(\eta, \dot{\eta}, \ddot{\eta})| \right\}. \tag{2.6}$$

We make the following assumption on the network's reconstruction error

Assumption 3: On a certain compact region $\Omega \subset \Re^6$

$$|\varepsilon(\eta, \dot{\eta}, \ddot{\eta})| \le c_6 \qquad \eta, \dot{\eta}, \ddot{\eta} \in \Omega, \tag{2.7}$$

where $c_6 \ge 0$ is an unknown constant.

3. ROBUST NN CONTROLLER DESIGN

The obgective of the autonomous diving control of an AUV can be expressed as: consider the nonlinear uncertain system (2.2) with a given desired trajectory z_d, design a control input torque τ_θ such that the tracking error $e_z = z - z_d$ and all other signals in the closed-loop are guaranteed to be UUB.

Step 1

First equation of (2.2) can be rewritten in the tracking error form as following

$$\dot{e}_z = -\dot{z}_d - u_0\theta + \Delta f_z. \tag{3.1}$$

Lemma: Consider the dynamical equation (3.1) with Assumption 1, where pitch angle θ considered as control input. If the control law is chosen as

$$\theta = u_0^{-1}[-\dot{z}_d + k_1 e_z + \hat{L}_1 e_z + \hat{L}_2 \cdot \tanh(e_z/\sigma_1)], \tag{3.2}$$

where $k_1 > 0$ is a design parameter, $\hat{L}_1$ and $\hat{L}_2$ are the estimations of certain unknown constants L_1 and L_2, which will be defined later, σ_1 is a certain strictly positive definite design parameter, and $\tanh(\cdot)$ denotes hyperbolic function. And the parameter update laws are chosen as

$$\begin{aligned} \dot{\hat{L}}_1 &= \gamma_1[e_z^2 + \alpha_1(\hat{L}_1 - L_{10})], \\ \dot{\hat{L}}_2 &= \gamma_2[e_z \cdot \tanh(e_z/\sigma_1) + \alpha_2(\hat{L}_2 - L_{20})], \end{aligned} \tag{3.3}$$

where $\gamma_1, \gamma_2, \alpha_1, \alpha_2$ are certain strictly positive definite weghting factors, and L_{10}, L_{20} are certain design parameters. Then, all the signals in the closed-loop system are guaranteed to be UUB.

Proof: See Appendix.

According to Lemma, we choose the stabilizing function θ_d as following

$$\theta_d = u_0^{-1}[-\dot{z}_d + k_1 e_z + \hat{L}_1 e_z + \hat{L}_2 \cdot \tanh(e_z/\sigma_1)]. \tag{3.4}$$

With a new error variable $e_\theta = \theta - \theta_d$ and equation (3.4), (3.1) can be expressed as

$$\begin{aligned} \dot{e}_z &= -\dot{z}_d - u_0(e_\theta + \theta_d) + \Delta f_z \\ &= -k_1 e_z - u_0 e_\theta - \hat{L}_1 e_z - \hat{L}_2 z_d \cdot \tanh(e_z/\sigma_1) + \Delta \cdot \zeta_3(z). \end{aligned} \tag{3.5}$$

Consider the Lyapunov function candidate as

$$V = 1/2(e_z^2 + \gamma_1^{-1}\tilde{L}_1^2 + \gamma_2^{-1}\tilde{L}_2^2). \tag{3.6}$$

Using (3.5), and similar to the expansion procedure in Appendix, the derivative of equation (8) can be expressed as

$$\begin{aligned} \dot{V}_1 \le& -k_1 e_z^2 - u_0 e_\theta e_z - (\alpha_1/2)\tilde{L}_1^2 - (\alpha_2/2)\tilde{L}_2^2 + L_2\kappa\sigma_1 \\ &+ (\alpha_1/2)(L_1 - L_{10})^2 + (\alpha_2/2)(L_2 - L_{20})^2. \end{aligned} \tag{3.7}$$

Step 2

Define another new error variable $e_q = q - q_d$, where q_d

is a stabilizing function for the second equation in (2.2), then we the following expression

$$\dot{e}_\theta = q - \dot{\theta}_d = e_q + q_d - \dot{\theta}_d . \quad (3.8)$$

Similar to the previous expansion, select the stabilizing function q_d as

$$q_d = u_0 e_z - k_2 e_\theta + \dot{\theta}_d , \quad (3.9)$$

where $k_2 > 0$ is a certain design parameter.
Consider the following Lyapunov function candidate

$$V_2 = V_1 + 1/2 e_\theta^{\ 2} . \quad (3.10)$$

Using (3.7)~(3.9), the derivative of (3.10) can be expressed as

$$\dot{V}_1 \le -k_1 e_z^{\ 2} - k_2 e_\theta^{\ 2} + e_\theta e_q - (\alpha_1/2)\tilde{L}_1^{\ 2} - (\alpha_2/2)\tilde{L}_2^{\ 2} + L_2 \kappa \sigma_1$$
$$+ (\alpha_1/2)(L_1 - L_{10})^2 + (\alpha_2/2)(L_2 - L_{20})^2 . \quad (3.11)$$

Step 3

Consider the final equation of (2.2) with $e_q = q - q_d$, we have

$$\dot{e}_q = f_q + b\tau_\theta - \dot{q}_d . \quad (3.12)$$

Using Assumption 2, above equation can be rewritten as

$$b^{-1}\dot{e}_q = b^{-1} f_q + \tau_\theta - b^{-1}\dot{q}_d . \quad (3.13)$$

Using equation (2.5), above equation can be rewritten as

$$b^{-1}\dot{e}_q = W^T \phi + \tau_\theta - b^{-1}\dot{q}_d + \varepsilon . \quad (3.14)$$

We choose the control law as

$$\tau_\theta = -k_3 e_q - e_\theta - \hat{W}^T \phi - 1/2 \hat{L}_3 e_q$$
$$- \hat{L}_4 \dot{q}_d \cdot \tanh(e_q \dot{q}_d / \sigma_2) - \hat{L}_5 \cdot \tanh(e_q / \sigma_3) \quad (3.15)$$

where $\hat{W}$ is the estimation of W, $\hat{L}_3$, $\hat{L}_4$ and $\hat{L}_5$ are the estimations of certain unknown constants L_3, L_4 and L_5, which will be defined later, and σ_2, σ_3 are certain strictly positive definite design parameters.
Substituting (3.15) into (3.14), we have

$$b^{-1}\dot{e}_q = W^T\phi - k_3 e_q - e_\theta - \hat{W}^T\phi - \hat{L}_4 \cdot \tanh(e_q \dot{q}_d / \sigma_2)$$
$$- 1/2 \hat{L}_3 e_q - \hat{L}_5 \cdot \tanh(e_q/\sigma_3) - b^{-1}\dot{q}_d + \varepsilon . \quad (3.16)$$

Theorem: Consider the autonomous diving model of AUV expressed as (2.2) with Assumption 1~3. If we take the control laws as (3.4), (3.9) and (3.15), and the parameter adaptation laws are chosen as (3.3) and

$$\dot{\hat{L}}_3 = \gamma_3 [1/2 e_q^{\ 2} + \alpha_3(\hat{L}_3 - L_{30})],$$
$$\dot{\hat{L}}_4 = \gamma_4 [e_q \cdot \tanh(e_q/\sigma_4) + \alpha_4(\hat{L}_4 - L_{40})],$$
$$\dot{\hat{L}}_5 = \gamma_5 [e_q \dot{q}_d \cdot \tanh(\dot{q}_d e_q / \sigma_5) + \alpha_5(\hat{L}_5 - L_{50})],$$
$$\dot{\hat{W}} = \Gamma_1 [e_q \phi + \Gamma_2(\hat{W} - W_0)], \quad (3.17)$$

where γ_i, α_i, $i = 3,4,5$ are certain strictly positive definite weghting factors, L_{j0}, $j = 3,4,5$ and W_0 are certain design parameters, and Γ_1, Γ_2 are strictly positive definite matrix. Then, all the signals in the closed-loop system are guaranteed to be UUB.

Proof: Consider the following Lyapunov function candidate

$$V_3 = V_2 + 1/2 b^{-1} e_q^{\ 2} + 1/2 \tilde{W}^T \Gamma_1^{-1} \tilde{W} + 1/2 \sum_{i=3}^{5} \gamma_i^{-1} \tilde{L}_i^{\ 2} . \quad (3.18)$$

Differentiating (3.18) and substituting (3.16) and (3.17) into it, we have

$$\dot{V}_3 = \dot{V}_2 + e_q b^{-1} \dot{e}_q + 1/2[d(b^{-1})/dt] e_q^{\ 2} + \tilde{W}^T \Gamma_1^{-1} \dot{\tilde{W}}$$
$$+ \sum_{i=3}^{5} \gamma_i^{-1} \tilde{L}_i \dot{\tilde{L}}_i$$
$$\le \dot{V}_2 + e_q \tilde{W}^T \phi - k_3 e_q^{\ 2} - 1/2 \hat{L}_3 e_q^{\ 2} - \hat{L}_4 \dot{q}_d e_q \cdot \tanh(\dot{q}_d e_q / \sigma_2)$$
$$- \hat{L}_5 e_q \cdot \tanh(e_q/\sigma_3) - b^{-1}\dot{q}_d e_q + \varepsilon e_q + 1/2 c_5 e_q^{\ 2} - e_q e_\theta$$
$$+ \tilde{W}^T \Gamma_1^{-1} \dot{\tilde{W}} + \sum_{i=3}^{5} \gamma_i^{-1} \tilde{L}_i \dot{\tilde{L}}_i$$
$$\le \dot{V}_2 - k_3 e_q^{\ 2} + 1/2(c_5 - L_3) e_q^{\ 2} + c_6 |e_q| - L_5 e_q \cdot \tanh(e_q/\sigma_3)$$
$$+ c_4^{-1} |\dot{q}_d e_q| - L_4 \dot{q}_d e_q \cdot \tanh(\dot{q}_d e_q / \sigma_2) - e_q e_\theta$$
$$+ \tilde{W}^T \Gamma_2 (\hat{W} - W_0) + \sum_{i=3}^{5} \alpha_i \tilde{L}_i (\hat{L}_i - L_{i0}) . \quad (3.19)$$

Here we define the constants L_i, $i = 3,4,5$ as

$$L_3 = c_5,\ L_4 = c_4^{\ -1},\ L_5 = c_6 . \quad (3.20)$$

Then, combining with (3.11), (3.19) can be rewritten as

$$\dot{V}_3 \le -k_1 e_z^{\ 2} - k_2 e_\theta^{\ 2} - k_3 e_q^{\ 2} - \sum_{i=1}^{5} (\alpha_i/2) \tilde{L}_i^{\ 2} - \tilde{W}^T \Gamma_2 \tilde{W}$$
$$+ L_2 \kappa \sigma_1 + L_4 \kappa \sigma_2 + L_5 \kappa \sigma_3 + \sum_{i=3}^{5} (\alpha_i/2)(L_i - L_{i0})^2$$
$$+ (W - W_0)^T \Gamma_2 (W - W_0) . \quad (3.21)$$

From (3.21) we have

$$\dot{V}_3 \le -\lambda_3 V_3 + \rho_3 , \quad (3.22)$$

where λ_3 and ρ_3 are positive constants defined by

$$\lambda_3 := \min\left\{2 \cdot \min_{i=1,2,3}\{k_i\}, \min_{j=1,\ldots,5}\{1/(\alpha_j \gamma_j)\}, 2/(\lambda_{\max}(\Gamma_1)\lambda_{\max}(\Gamma_2))\right\} \quad (3.23)$$

$$\rho_3 := L_2 \kappa \sigma_1 + L_4 \kappa \sigma_2 + L_5 \kappa \sigma_3 + \sum_{i=3}^{5} (\alpha_i/2)(L_i - L_{i0})^2$$
$$+ (W - W_0)^T \Gamma_2 (W - W_0) . \quad (3.24)$$

where $\lambda_{\max}(\cdot)$ denotes the maximum singular value of the given matrix.

If we let $\mu_3 := \rho_3 / \lambda_3$, then (3.22) satisfies

$$0 \le V_3(t) \le \mu_3 + [V_3(0) - \mu_3] e^{-\lambda_3 t} . \quad (3.25)$$

Therefore e_z, e_θ, e_q and $\tilde{L}_i$, $i = 1, \cdots, 5$ are all satisfied to be UUB. Since $z = e_z + z_d$, combined with the assumption that the desired trajectory is bound, we can conclude that z is bound also. Further, from (3.4) and (3.9), it is obvious that the stabilizing functions are bound, from which we can know that θ and q are bound too. Consequently, all signals in the closed-loop system are guaranteed to be UUB. □

Remark 2: In many practical applications, given a control plant, the constructed neural network's optimal weight vector W and the bounding parameter L_i, $i = 1, \cdots, 5$ may not be completely unknown. Instead, we may have rough estimations of them through off-line identification or other useful schemes. In this case, the design parameters W_0 and $L_{i0}, i = 1, \cdots, 5$

are considered as the initial estimation values of W and $L_i, i = 1,\cdots,5$. From (3.25), we can see that the accurate initial estimations of these parameters may results in smaller tracking error, respectively.

Remark 3: From (3.25), we can see that large k_i or small σ_i, $i = 1,2,3$ may result in smaller tracking error. However, increasing k_i may cause certain high-gain control problems and the small values of σ_i could result in certain infinite frequency efforts. Therefore, these parameters should be chosen carefully in practice.

Remark 4: Design parameters $\alpha_i, i = 1,\cdots,5$ present certain trade-off between the tracking performance and the robustness of the proposed control scheme. In particular, if the basis function vectors $\phi(\eta,\dot{\eta},\ddot{\eta})$ satisfy the persistency excitation conditions, then $\alpha_i = 0$ could result in the exact estimation of W. However, the persistency excitation conditions are hard to be satisfied in many practical applications, and $\alpha_i > 0$ could keep the parameter estimations from being divergent.

4. SIMULATION STUDIES

The proposed robust NN controller is applied to the autonomous diving control of an AUV. 6 DOF dynamical equation of REMUS AUV [15] is employed in this simulation studies.

In general, exogenous input terms were seldom considered in the control problem of the underwater vehicles, even if they were considered, only known bound terms were handled [13, 14]. To discuss the advantage of the presented control scheme, here we consider the exogenous input term Δf_z in (2.2) as: $\Delta f_z = 0.01z$, which is unbound and assume to be unknown.

The basis function vector of the constructed neural network is as following

$$\phi(\eta,\dot{\eta},\ddot{\eta}) = [\ddot{\eta}^T \quad \dot{\eta}^T \quad (\dot{\eta}\otimes\dot{\eta})^T \quad G^T \quad (G\otimes G)^T]^T, \qquad (4.1)$$

where $G = [\sin\phi \;\; \sin\theta \;\; \cos\phi \;\; \cos\theta]^T$. For more details on the construction of neural network, refer to [16].

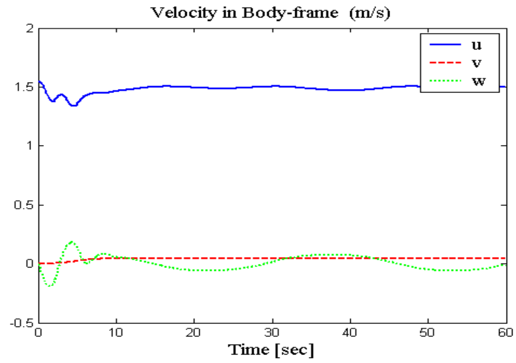
The desired trajectory for this simulation is taken as

$$z_d = 10 + 3\sin(0.2t), \quad \dot{z}_d = 0.6\cos(0.2t). \qquad (4.2)$$

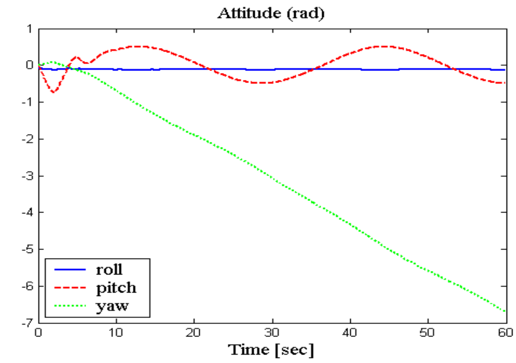
The constant forward speed u_0 is taken as $u_0 = 1.54 m/s$, and other parameters used in this simulation are taken as

$$\begin{aligned} &k_1 = 2.0,\; k_2 = 3.0,\; k_3 = 2.0,\\ &\sigma_1 = \sigma_2 = \sigma_3 = 1.0, \\ &\alpha_i = 0.005,\; \gamma_i = 0.02,\; L_{i0} = 0, \quad i = 1,\cdots,5. \end{aligned} \qquad (4.3)$$

The simulation results are depicted in Fig 1~4. In the process of simulation, we set both the rudder angle and pitch fin angle as zeroes. And from Fig 2, we can see that the vehicle turns to left by approximately 6degrees/second rate while keeping constant forward speed ($1.54 m/s$). This kind of phenomenon is caused by the characteristic of REMUS model [15]. Fig 3 showes that the constructed neural network has a certain approximation capacity for the given nonlinear uncertainty, and as discussed in section 3, Fig 4 showes that the network's weights estimation values do not divergence under the adaptation law (3.17). In general, constant forward speed motion of vehicles does not cause all the nonlinear dynamical terms to be exciting, in other words, the persistency excitation conditions are hard to be satisfied in the constant



(a) Vehicle's velocities in body-frame



(b) Vehicle's Euler angles in earth-frame

Fig. 1 Vehicle's motion in the simulation

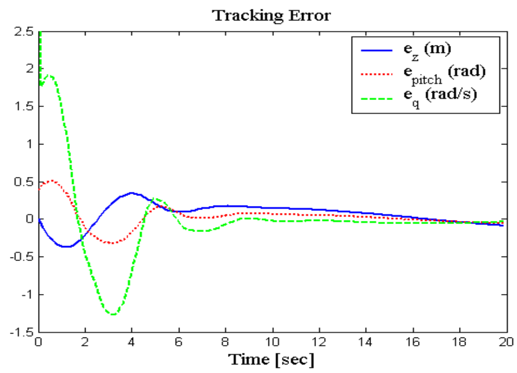


Fig. 2 Tracking error in depth, pitch and pitch angle rate

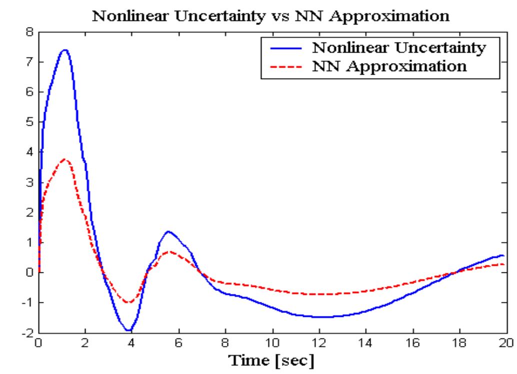


Fig. 3 NN's approximation capacity

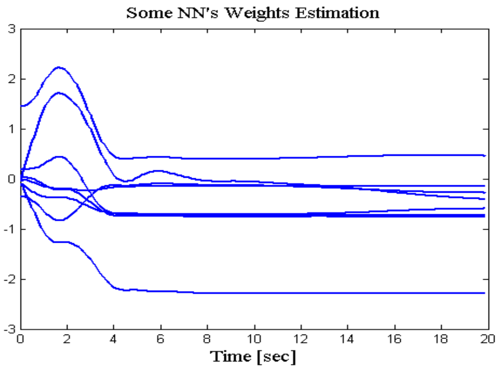


Fig .4 NN's weights estimation values

forward speed motion of underwater vehicles. If the constructed basis function vector $\phi(\eta,\dot{\eta},\ddot{\eta})$ does not satisfy the persistency excitation condition, then we should select the design parameter $\lambda_{\min}(\Gamma_2)>0$ such that the network's weights estimations to be bound. Fig 2 showes the tracking errors of z_d, θ_d and q_d respectively, from which we can conclude that the proposed control scheme can get certain satisfactory performance under the relaxed assumptions on the nonlinear uncertainties.

5. CONCLUSIONS

This paper presents a robust NN controller for autonomous diving control of an AUV. The exogenous input term of the vehicle's dynamics is assumed to be unbound, although it still satisfies certain restricting condition, and the gain function of control is assumed to be unknown with known sign. Under these two relaxed wild assumptions, presented robust NN control scheme can guarantee that all the signals in the closed-loop system satisfy to be UUB. Some practical features of the proposed control law are also discussed.

6. ACKNOWLEDGMENT

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APPENDIX

Proof of Lemma. Substituting (4) into (3) yields

$$\dot{e}_z = -k_1 e_z - \hat{L}_1 e_z - \hat{L}_2 z_d \cdot \tanh(e_z/\sigma) + \Delta\cdot\zeta_3(z). \tag{A.1}$$

Consider the Lyapunov function candidate as

$$V_1 = 1/2(e_z^{\ 2} + \gamma_1^{\ -1}\tilde{L}_1^{\ 2} + \gamma_2^{\ -1}\tilde{L}_2^{\ 2}). \tag{A.2}$$

Differentiating (A.2) and substituting (A.1) into it yields

$$\begin{aligned}\dot{V}_1 &= e_z\dot{e}_z + \gamma_1^{\ -1}\tilde{L}_1\dot{\tilde{L}}_1 + \gamma_2^{\ -1}\tilde{L}_2\dot{\tilde{L}}_2\\ &= -k_1 e_z^{\ 2} - \hat{L}_1 e_z^{\ 2} - \hat{L}_2 e_z\cdot\tanh(e_z/\sigma) + e_z\Delta\cdot\zeta_3(z)\\ &\quad + \gamma_1^{\ -1}\tilde{L}_1\dot{\tilde{L}}_1 + \gamma_2^{\ -1}\tilde{L}_2\dot{\tilde{L}}_2\\ &\le -k_1 e_z^{\ 2} - \hat{L}_1 e_z^{\ 2} - \hat{L}_2 e_z\cdot\tanh(e_z/\sigma) + \gamma_1^{\ -1}\tilde{L}_1\dot{\tilde{L}}_1\\ &\quad + \gamma_2^{\ -1}\tilde{L}_2\dot{\tilde{L}}_2 + |e_z| c_1(c_2|z| + c_3)\\ &\le -k_1 e_z^{\ 2} - \hat{L}_1 e_z^{\ 2} - \hat{L}_2 e_z\cdot\tanh(e_z/\sigma) + \gamma_1^{\ -1}\tilde{L}_1\dot{\tilde{L}}_1\\ &\quad + \gamma_2^{\ -1}\tilde{L}_2\dot{\tilde{L}}_2 + |e_z| c_1(c_2|e_z| + c_2|z_d| + c_3).\end{aligned} \tag{A.3}$$

Here we need the following Lemma.

Lemma A.1. The following inequality holds for any $\sigma>0$ and for any $e_z\in\Re$

$$0 \le |e_z| - e_z\cdot\tanh(e_z/\sigma) \le \kappa\sigma$$

where κ is a constant that satisfies $\kappa = e^{-(\kappa+1)}$.

Proof: Refer to [8].

Using Lemma A.1, (A.3) can be rewritten as

$$\begin{aligned}\dot{V}_1 &\le -k_1 e_z^{\ 2} - \hat{L}_1 e_z^{\ 2} - \hat{L}_2 e_z\cdot\tanh(e_z/\sigma) + \gamma_1^{\ -1}\tilde{L}_1\dot{\tilde{L}}_1 + \gamma_2^{\ -1}\tilde{L}_2\dot{\tilde{L}}_2\\ &\quad + c_1c_2 e_z^{\ 2} + (c_1c_2|z_d| + c_1c_3)|e_z|\\ &\le -k_1 e_z^{\ 2} + (c_1c_2 - \hat{L}_1)e_z^{\ 2} - \hat{L}_2 e_z\cdot\tanh(e_z/\sigma) + \gamma_1^{\ -1}\tilde{L}_1\dot{\tilde{L}}_1\\ &\quad + \gamma_2^{\ -1}\tilde{L}_2\dot{\tilde{L}}_2 + (c_1c_2c_4 + c_1c_3)[e_z\cdot\tanh(e_z/\sigma) + \kappa\sigma]\\ &= -k_1 e_z^{\ 2} + (c_1c_2 - \hat{L}_1)e_z^{\ 2} + \gamma_1^{\ -1}\tilde{L}_1\dot{\tilde{L}}_1 + \gamma_2^{\ -1}\tilde{L}_2\dot{\tilde{L}}_2\\ &\quad + [(c_1c_2c_4 + c_1c_3) - \hat{L}_2]e_z\cdot\tanh(e_z/\sigma)\\ &\quad + (c_1c_2c_4 + c_1c_3)\kappa\sigma.\end{aligned} \tag{A.4}$$

In above expansion, we use the assumption that the desired trajectory is in a bounded domain, such that $|z_d| \le c_4,\ t\ge 0$.

Here we define the constants L_1 and L_2 as: $L_1 := c_1c_2$, $L_2 := c_1c_2c_4 + c_1c_3$. Then, (A.4) can be rewritten as

$$\begin{aligned}\dot{V}_1 &\le -k_1 e_z^{\ 2} + L_2\kappa\sigma + \tilde{L}_1 e_z^{\ 2} + \tilde{L}_2 e_z\cdot\tanh(e_z/\sigma)\\ &\quad + \gamma_1^{\ -1}\tilde{L}_1\dot{\tilde{L}}_1 + \gamma_2^{\ -1}\tilde{L}_2\dot{\tilde{L}}_2.\end{aligned} \tag{A.5}$$

Substituting the parameters' update laws (5) into above equation, then we can get

$$\begin{aligned}\dot{V}_1 &\le -k_1 e_z^{\ 2} + L_2\kappa\sigma + \alpha_1\tilde{L}_1(\hat{L}_1 - L_{10}) + \alpha_2\tilde{L}_2(\hat{L}_2 - L_{20})\\ &= -k_1 e_z^{\ 2} + L_2\kappa\sigma - \alpha_1\tilde{L}_1^{\ 2} + \alpha_1(L_1 - L_{10})\tilde{L}_1 - \alpha_2\tilde{L}_2^{\ 2}\\ &\quad + \alpha_2(L_2 - L_{20})\tilde{L}_2\\ &\le -k_1 e_z^{\ 2} + L_2\kappa\sigma - \alpha_1\tilde{L}_1^{\ 2} + (\alpha_1/2)\tilde{L}_1^{\ 2} + (\alpha_1/2)(L_1 - L_{10})^2\\ &\quad - \alpha_2\tilde{L}_2^{\ 2} + (\alpha_2/2)\tilde{L}_2^{\ 2} + (\alpha_2/2)(L_2 - L_{20})^2\\ &= -[k_1 e_z^{\ 2} + (\alpha_1/2)\tilde{L}_1^{\ 2} + (\alpha_2/2)\tilde{L}_2^{\ 2}] + L_2\kappa\sigma\\ &\quad + (\alpha_1/2)(L_1 - L_{10})^2 + (\alpha_2/2)(L_2 - L_{20})^2.\end{aligned} \tag{A.6}$$

From (A.6) we have

$$\dot{V}_1 \le -\lambda_1 V_1 + \rho_1, \tag{A.7}$$

where λ_1 and ρ_1 are positive constants defined by

$$\begin{aligned}\lambda_1 &:= \min\{2k_1,\ 1/(\alpha_1\gamma_1),\ 1/(\alpha_2\gamma_2)\},\\ \rho_1 &:= L_2\kappa\sigma + (\alpha_1/2)(L_1 - L_{10})^2 + (\alpha_2/2)(L_2 - L_{20})^2.\end{aligned} \tag{A.8}$$

If we let $\mu_1 := \rho_1/\lambda_1$, then (A.7) satisfies

$$0 \le V_1(t) \le \mu_1 + [V_1(0) - \mu_1]e^{-\lambda t}. \tag{A.9}$$

Therefore e_z, $\tilde{L}_1$, $\tilde{L}_2$ are all satisfied to be UUB. Further, since $z = e_z + z_d$, combined with the assumption that the desired trajectory is bound, we can conclude that z is bound also. Consequently, all signals in the closed-loop system are guaranteed to be UUB. □

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