Direct Learning Control For Linear Feedback Systems

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Abstract: In this paper, a DLC method is proposed for linear feedback systems to improve the tracking performance when the task of the system is repetitive. DLC can generate the desired control input directly from the previously learned control inputs corresponding to other output trajectories. It is assumed that all the desired output functions considered in this paper have some relations called *proportionality* and it is shown by mathematical analysis that DLC can be utilized to generate additional control efforts for the perfect tracking. To show the validity and tracking performance of the proposed method, some simulations are performed for the tracking control of a linear system with a PI controller.

Keywords: Direct learning control, linear feedback systems, iterative learning control, tracking control.

1. INTRODUCTION

Iterative Learning Control(ILC) is a technique for improving the transient response and tracking performance processes, machines, equipments, or systems that execute the same trajectory of motion or operation. It is apparently a new but effective technique for particular class of control problems and overcomes some of the traditional difficulties associated with performance design of control systems [1]-[6].

However, there still exists a number of problems which prohibit practical application and extension of ILC schemes. The major obstacle may be the requirement for the desired trajectory being repeatable over operations. Even if a small change occurs in the desired trajectory, the learning control system has to be resumed from the beginning and the previously learned control input profiles can no longer be used.

Xu et. al [7],[8] defined non-repeatable control problems handled in learning control: non-repeatable motion task and non-repeatability of a process. The non-repeatable motion task could be shown through an example of an XY-table drawing several circles with the same period but different radii. The non-repeatability of a process could be seen in the systems such as welding different parts in a manufacturing line. From the practical point of view, non-repeatable learning control is more important an indispensable.

In order to deal with non-repeatable learning control problems, we need to explore the inherent relations of different motion trajectory patterns. We can note that, in spite of the variations in the trajectory patterns, the underlying dynamic properties of the controlled system remain the same. Hence, previously learned control inputs are obviously correlated and contain a lot of important information about the system itself. To effectively use these prior control knowledge, *Direct Learning Control*(DLC) schemes were suggested [7].

DLC is defined as the direct generation of the desired control input profile from existing control inputs without any repetitive learning process. The objective of DLC is to fully utilize all pre-stored control profiles and eliminate the time consuming iteration process thoroughly, even though these control input profiles may correspond to different motion patterns and be obtained using different control methods.

Originally developed was DLC for open-loop control systems like ILC but it can be extended for feedback control systems where ILC is being used to modify the control input to obtain a better control performance. It was shown by simulation results that DLC can be combined with PD controller to improve the control system performance. However, mathematical analysis for the feedback system with DLC was not performed but tracking performance was shown to be improved [1].

In this paper, it is shown by mathematical analysis that DLC method can be effectively used to improve the tracking performance in feedback systems where the desired output is repetitive. It is also illustrated by simulation results that the control input to the plant, which is suitably modified by the proposed method without any iterative process, yields a good tracking performance for a new desired output trajectory which has not been previously learned.

2. PROBLEM STATEMENTS AND ASSUMPTIONS

The convergence of the iterative learning control system is guaranteed under a sufficient condition but the required control performance is satisfied after a lot of trials. However, the transient response cannot be controlled and hence, we may often have large overshoot and long settling time. Actually, most control systems have already feedback controller and thus, ILC has often been used together with feedback controller like PID controller.

The basic configuration of the ILC for the feedback control system is as shown in Fig. 1 where the desired output is periodic and the feedback controller is designed to satisfy the stability and the control specifications [9]. Whenever a new desired output is given to the system, a lot of iterations should be performed to get u_{ILC} which satisfies the tolerance error bound for the output error. Even when the newly given desired output is different from previous ones only in magnitude scales or in time scales, the iterative process cannot be avoided.

In order to remove the iterative process for learning and to obtain a precise tracking performance, it is required such a method that yields the additional control input modification quantity directly using the information on the desired output as shown in Fig. 2.

Another configuration of ILC for feedback systems is the reference input modification type. In this configuration the reference input to the feedback system is modified based on the output error function and the given reference input [10].

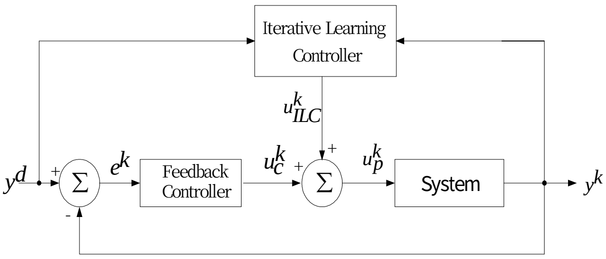

Fig. 1. The configuration of ILC for feedback systems.

The control problem in this paper is to find the required additional control efforts (u_{DLC}) in the feedback control system as shown in Fig. 2 for the perfect tracking. The modified control input to the system is generated to guarantee the precise output tracking for the new desired output which is *proportional* to previous ones. The definition of the "*proportionality*" and assumptions are as the followings.

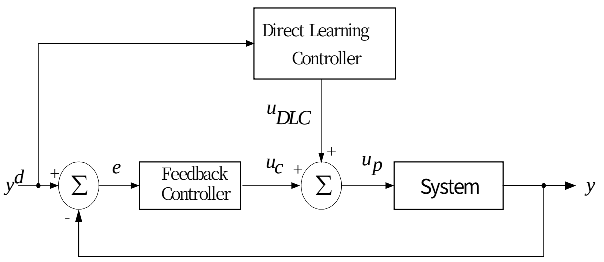

Fig. 2. The control input modification using DLC.

Definition 1: Trajectory $\mathbf{y}_i(t_i)$ $t_i \in [0, T_i]$ is said to be *proportional* to another trajectory $\mathbf{y}(t)$, $t \in [0, T]$ in *time scales* if and only if $\mathbf{y}_i(t_i) = \mathbf{y}(t)$, where $\rho_i(t) = t_i = p_i t$ is the time scaling factor satisfying $\rho_i(0) = 0$ and $\rho_i(T) = T_i$.

Assumption 1: The product of the control input matrix and the output matrix in the system description is nonsingular.

Assumption 2: There are $l (l \geq 2)$ prestored trajectories $\mathbf{y}_i(t_i), t_i \in [0, T_i]$. The corresponding control input profiles $\mathbf{u}_i(t_i)$ have already been obtained *a priori* through iterative learning process. For any prestored trajectories $\mathbf{y}_i$ and $\mathbf{y}_j$ $(i \neq j)$, it should be $p_i \neq 0$, $p_j \neq 0$ and $p_i \neq p_j$ for $i, j = 1, \cdots, N$.

3. DLC FOR CONTROL INPUT MODIFICATION

Consider a class of linear systems described by

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{b}u_p(t) \\ y(t) &= \mathbf{c}^T\mathbf{x}(t) \end{aligned} \tag{1}$$

where $\mathbf{x}(t) \in R^n$ is the state vector, $u(t)$ and $y(t)$ are the scalar input and the scalar output respectively. If we use PI controller as a kind of feedback controllers and let the state of the controller be $\mathbf{x}_c(t)$, then the state-space representation of the system in Fig. 2 can be written as the following.

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{x}}_c \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{b}K_P\mathbf{c}^T & \mathbf{b}K_I \\ -\mathbf{c}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_c \end{bmatrix} + \begin{bmatrix} \mathbf{b} \\ 0 \end{bmatrix} u_{DLC} + \begin{bmatrix} \mathbf{b}K_P \\ 1 \end{bmatrix} y_d \tag{2}$$

$$y = \begin{bmatrix} \mathbf{c}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_c \end{bmatrix}$$

Define some matrices with appropriate dimensions and let $u = u_{DLC}$, then we can rewrite (2) into (3).

$$\begin{aligned} \dot{\mathbf{z}} &= \overline{\mathbf{A}}\mathbf{z} + \overline{\mathbf{b}}u + \overline{\mathbf{d}}y_d \\ y &= \overline{\mathbf{c}}^T\mathbf{z} \end{aligned} \tag{3}$$

where $\mathbf{z}(t) = \begin{bmatrix} \mathbf{x} & \mathbf{x}_c \end{bmatrix}^T$ is the augmented state vector, $u(t)$ is the output of the direct learning controller and $y(t)$ is the system output. In this configuration of feedback control systems, additional control efforts may be inserted into the control input to the system.

In the following Theorem, it is assumed that the additional control efforts are generated by DLC and it is used as in the form of Fig. 2.

Theorem 1: For a class of linear feedback system in (3) the desired output of which is *proportional* to the previous outputs, the required additional control efforts $\mathbf{u}_d(t_d)$ for the perfect tracking, which yields the $\mathbf{y}_d(t_d)$, $t_d \in [0, T_d]$, can be directly generated from the prestored control efforts $\mathbf{u}_i(t_i)$ as follows:

$$\mathbf{u}_d(t_d) = [I \quad I]\mathbf{W}^{\#}\overline{\mathbf{u}}_l \tag{4}$$

where

$$\mathbf{W}^{\#} = (\mathbf{W}^T \ \mathbf{W})^{-1}\mathbf{W}^T \ , \ \ \overline{\mathbf{u}}_l = [\mathbf{u}_1^T(t_1), \cdots, \mathbf{u}_l^T(t_l)]^T \ ,$$

and

$$\mathbf{W} = \begin{bmatrix} p_1^{-1} & 1 \\ p_2^{-1} & 1 \\ \vdots & \vdots \\ p_l^{-1} & 1 \end{bmatrix}.$$

Proof: For the feedback system (3), the desired control input can be found as follows:

$$u(t) = \left(\overline{\mathbf{c}}^T\overline{\mathbf{b}}\right)^{-1}\left[\dot{y} - \overline{\mathbf{c}}^T\overline{\mathbf{A}}\mathbf{z} - \overline{\mathbf{c}}^T\overline{\mathbf{d}}y_d\right], \ \ t \in [0, T] \tag{5}$$

For the new desired output $\mathbf{y}_d(t_d),\ t_d \in [0, T_d]$, we can have

$$u_d(t_d) = \left(\bar{\mathbf{c}}^T\bar{\mathbf{b}}\right)^{-1}\left[\frac{dy_d(t_d)}{dt_d} - \bar{\mathbf{c}}^T\bar{\mathbf{A}}\mathbf{z}_d(t_d) - \bar{\mathbf{c}}^T\bar{\mathbf{d}}y_d(t_d)\right]$$
$$= \left(\bar{\mathbf{c}}^T\bar{\mathbf{b}}\right)^{-1}\left[\frac{dy_d(t_d)}{dt_d} - \bar{\mathbf{c}}^T(\bar{\mathbf{A}} + \bar{\mathbf{d}}\bar{\mathbf{c}}^T)\mathbf{z}_d(t_d)\right]. \quad (6)$$

Note that $\mathbf{u}_d(t_d)$ is not available directly in terms of above formula due to the existence of system uncertainties in $\bar{\mathbf{A}}$, $\bar{\mathbf{b}}$, $\bar{\mathbf{d}}$, and $\bar{\mathbf{c}}$. Now choose prestored output trajectories $\mathbf{y}_i(t_i), t_i \in [0, T_i]$, $i = 1,2,...,l$ in which the corresponding additional control efforts have been obtained *a priori* i.e.,

$$u_i(t_i) = \left(\bar{\mathbf{c}}^T\bar{\mathbf{b}}\right)^{-1}\left[\frac{dy_i(t_i)}{dt_i} - \bar{\mathbf{c}}^T(\bar{\mathbf{A}} + \bar{\mathbf{d}}\bar{\mathbf{c}}^T)\mathbf{z}_i(t_i)\right] \quad (7)$$
$$, t_i \in [0, T_i]$$

Recall that $t_i = \rho_i(t_d)$ and differentiating $\mathbf{y}_d(t_d)$ with respect to t_d,

$$\frac{dy_d(t_d)}{dt_d} = \frac{d}{dt_i}(y_i(t_i))\frac{d\rho_i(t_d)}{dt_d} = \frac{dy_i(t_i)}{dt_i}\cdot p_i \quad (8)$$

where $p_i = \dfrac{d\rho_i(t_d)}{dt_d}$. We also have $\mathbf{z}_i(t_i) = \mathbf{z}_d(t_d)$ from $\mathbf{y}_i(t_i) = \mathbf{y}_d(t_d)$. Using (8), we can rewrite (7) as follows:

$$u_i(\rho_i(t_d)) = \left(\bar{\mathbf{c}}^T\bar{\mathbf{b}}\right)^{-1}\left[\frac{dy_d(t_d)}{dt_d}\cdot p_i^{-1} - \bar{\mathbf{c}}^T(\bar{\mathbf{A}} + \bar{\mathbf{d}}\bar{\mathbf{c}}^T)\mathbf{z}_d(t_d)\right] \quad (9)$$

Let $d_1(\mathbf{z}_d(t_d)) = \left(\bar{\mathbf{c}}^T\bar{\mathbf{b}}\right)^{-1}\dfrac{dy_d(t_d)}{dt_d}$ and $d_2(\mathbf{z}_d(t_d)) = -\left(\bar{\mathbf{c}}^T\bar{\mathbf{b}}\right)^{-1}\left[\bar{\mathbf{c}}^T(\bar{\mathbf{A}} + \bar{\mathbf{d}}\bar{\mathbf{c}}^T)\mathbf{z}_d(t_d)\right]$, then we have

$$\begin{bmatrix} p_1^{-1} & 1 \\ p_2^{-1} & 1 \\ \vdots & \vdots \\ p_l^{-1} & 1 \end{bmatrix}\begin{bmatrix} \mathbf{d}_1(\mathbf{x}_d(t_d)) \\ \mathbf{d}_2(\mathbf{x}_d(t_d)) \end{bmatrix} = \begin{bmatrix} \mathbf{u}_1(\rho_1(t_d)) \\ \mathbf{u}_2(\rho_2(t_d)) \\ \vdots \\ \mathbf{u}_l(\rho_l(t_d)) \end{bmatrix} \quad (10)$$

or $\mathbf{W}\mathbf{d} = \bar{\mathbf{u}}_l$ where $\mathbf{d} = [\mathbf{d}_1^T(\mathbf{z}_d(t_d)),\ \mathbf{d}_2^T(\mathbf{z}_d(t_d))]^T$. Since $\mathbf{W}^T\mathbf{W}$ is invertible from the *Assumption* 2, we can solve $\mathbf{d}$ in (10). Recall that, from (6),

$$\mathbf{d}_1(\mathbf{x}_d(t_d)) + \mathbf{d}_2(\mathbf{x}_d(t_d))$$
$$= \left(\bar{\mathbf{c}}^T\bar{\mathbf{b}}\right)^{-1}\left[\frac{dy_d(t_d)}{dt_d} - \bar{\mathbf{c}}^T(\bar{\mathbf{A}} + \bar{\mathbf{d}}\bar{\mathbf{c}}^T)\mathbf{z}_d(t_d)\right]$$
$$= \mathbf{u}_d(t_d) \quad (11)$$

Combining (10) and (11), we obtain $\mathbf{u}_d(t_d)$ as shown in (4). ∇∇∇

From the above Theorem, we have known that DLC can be effectively utilized in the linear feedback system to get a perfect tracking even if the existing feedback controller cannot guarantee a precise tracking performance. Actually, DLC can generate the proper modification quantity required for the output of the feedback controller when the new desired output is given as a different pattern to the previous outputs only if the proportionality between previous outputs and new desired output is satisfied.

4. SIMULATION RESULTS

To show the validity and the performance of the proposed scheme in this paper, simulations are performed for the trajectory tracking control. Consider the open-loop system in (12) with PI controller where $K_P = K_I = 6$.

$$\dot{x}_1 = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}x_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u_1 \quad (12)$$
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix}x_1$$

The task of the system is assumed to be repetitive and let the desired output trajectory be given as (13) for one period as the follows:

$$y_d = [1 - \cos(\frac{\pi}{2}t_d)], \quad t_d \in [0, 2] \quad (13)$$

where the sampling period is 0.004[s].

Assume that we have obtained control inputs(u_{ILC}) corresponding to previously given outputs y_1 and y_2 for the system in Fig. 1. The corresponding control inputs are stored in the memory through learning for following outputs

$$y_{d,1} = [1 - \cos(\pi t_1)], \quad t_1 \in [0, 1]$$
$$y_{d,2} = [1 - \cos(\frac{\pi}{4}t_2)], \quad t_2 \in [0, 4]\ . \quad (14)$$

It is assumed that ILC has been used *a priori* to obtain additional control input profiles which make the tolerance error bound on the output error 0.02. Since the number of data for the desired control input in one period should be the same as the numbers of control input data for both y_1 and y_2, we select the sampling time as 0.002[s] for y_1 and 0.008[s] for y_2.

We can know that the proportionality holds since $t_1 = 0.5t_d$ and $t_2 = 2t_d$ from (14) and the Definition 1. Hence, the additional control input(u_{DLC}) in Fig. 2 is calculated using (4) as follows:

$$u_{DLC} = 1/3(u_1(t_1) + 2u_2(t_2)) \tag{15}$$

where u_1 and u_2 are the inputs corresponding to y_1 and y_2, respectively. In Fig. 3 and Fig. 4, the actual output trajectories through the iterative learning process and the corresponding desired output trajectories $y_{d,1}$ and $y_{d,2}$ are shown, respectively. The corresponding control inputs $u_1(t_1)$ and $u_2(t_2)$ from ILC are stored which may be used later for other proportional outputs.

When we find the additional control using DLC and apply it to the new desired output trajectory, the actual output is shown to converge well to the given desired output trajectory without any iterative learning process as shown in Fig. 5.
The control inputs previously learned (u_1, u_2) for previously given outputs (y_1, y_2) and u_{DLC} generated by DLC are shown in Fig. 6. Note that the time duration for u_1, u_2 and u_{DLC} are different each other. If we increase the number of data on u_1 and u_2, we can obtain more precise tracking performance for y_d.

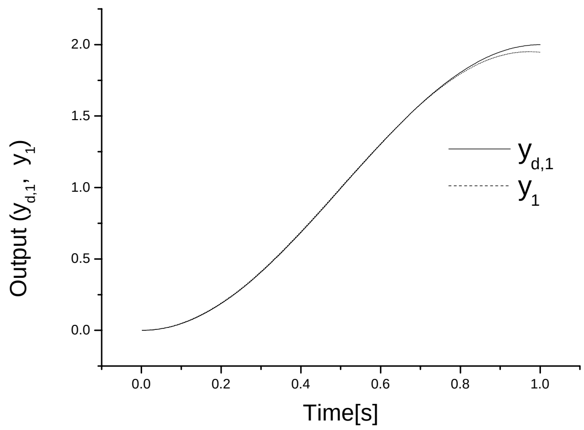

Fig. 3. Convergence of y_1 to $y_{d,1}$ by ILC.

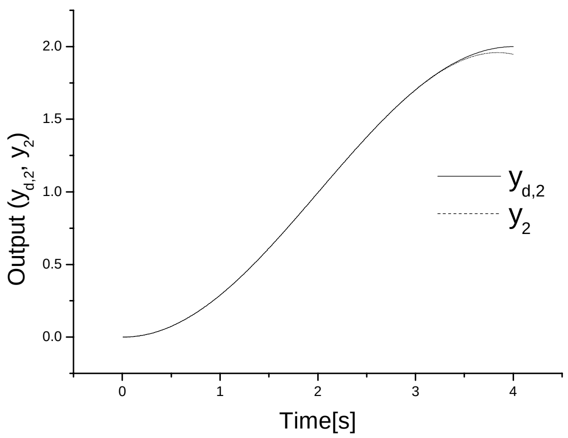

Fig. 4. Convergence of y_2 to $y_{d,2}$ by ILC.

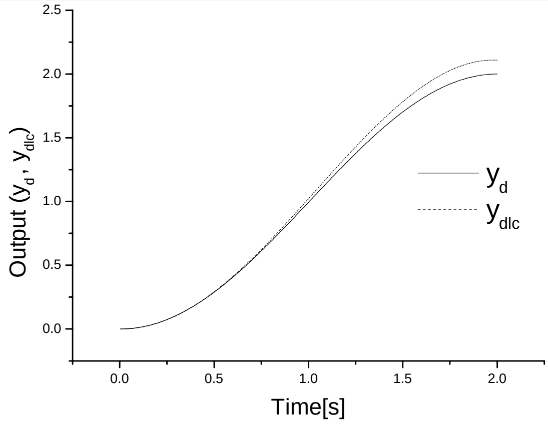

Fig. 5. Desired output and the actual output by DLC.

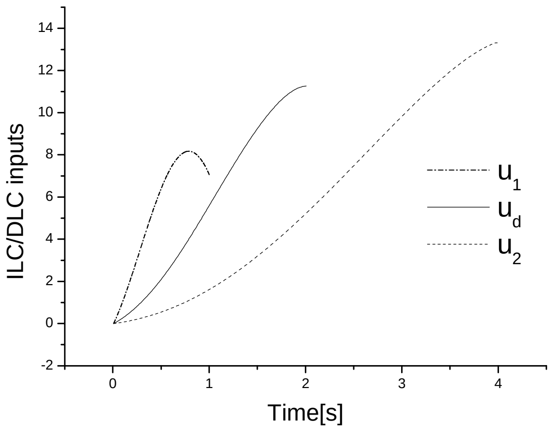

Fig. 6. Generation of new desired control input by DLC.

If we select the tolerance error bound in ILC smaller, the output error is much reduced. Moreover, only a few iterations are required if the output of DLC is utilized as the initial control input in next ILC process.

5. CONCLUSION

DLC-based generation of additional control efforts has been considered for a precise tracking in linear feedback systems. It was assumed the proportionality between the previous outputs and a new desired output is satisfied and the task of the system is repetitive. It was proved that DLC can generate the additional control input which can guarantee the perfect tracking performance even when the system modeling errors exist.

It was illustrated by simulation results that the proposed DLC method for feedback systems can find the corresponding additional control efforts for the new desired output and the output error can be reduced only if we set the tolerance error bound smaller in ILC process.

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