Robust Adaptive Output Feedback Control for Nonlinear Systems with Higher Order Relative Degree

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Abstract: In this paper, it is dealt with a controller design problem for nonlinear systems with higher order relative degree. A robust adaptive control for uncertain nonlinear systems with stable zero dynamics will be proposed based on the high-gain adaptive output feedback and backstepping strategies. The proposed method is useful in the case where only the output signal is available.

Keywords: Adaptive control, Output feedback control, Nonlinear systems, Backstepping

1. INTRODUCTION

It is well known that one can stabilize uncertain nonlinear systems with OFEP(output feedback exponentially passive) property by a high-gain output feedback based control with simple structure [1-3]. A nonlinear system is said to be OFEP if there exists an output feedback such that the resulting closed-loop system is exponentially passive [1]. The sufficient conditions for the nonlinear system to be OFEP are that (1) the system has relative degree of 1, (2) the system be globally exponential minimum-phase and (3) the nonlinearities of the system satisfy the Lipschitz condition. Under these conditions, there exists a static output feedback such that the resulting closed-loop system is exponentially passive [1,3]. As shown in [2,3], using the OFEP property of the controlled system, one can design an output feedback base controller. Since the adaptive control methods based on the OFEP property utilize only output signal in order to design the controller and it is not required to design an observer for control, the OFEP based control methods have very simple structure. Further since the methods have strong robustness with respect to bounded disturbances in spite of its simple structure, the methods are considered powerful control tools for uncertain nonlinear systems. Unfortunately the OFEP conditions give very severe restrictions to practical applications of the above-mentioned adaptive schemes because most practical systems do not satisfy the OFEP condition.

With this problem in mind, some alleviation methods to the OFEP condition have been proposed [3-5]. The method by [3] and [4] alleviated the OFEP condition by introducing a parallel feedforward compensator(PFC) in parallel to the controlled system. Although this method can solve the restriction for relative degree, since the controller is designed for an augmented controlled system with PFC, the bias error from the PFC output remain. The method by [5] is a robust control scheme for non-OFEP systems with nonlinear uncertainties but the method was for systems with relative degree of 1.

In this paper, we will propose a robust adaptive tracking control, which is based on high-gain output feedback based adaptive control, for a class of uncertain minimum-phase nonlinear systems with higher order relative degree. We extend the robust adaptive control method in [5] to uncertain nonlinear systems with higher order relative degree by utilizing 'backstepping' strategy. It is shown that if the upper bound of uncertain nonlinearities can be evaluated by a function of the output signal then one can design a stable adaptive control system by using only the output signal without a state observer. It is also shown that a suitable choice of design parameters guarantees the convergence of the tracking error to any design bound.

2. PROBLEM STATEMENT

Consider the following nth order nonlinear systems with relative degree of k ($1 \le i \le k$, $2 \le k \le n$):

$$\begin{aligned} \dot{x}_i &= f_i(\boldsymbol{x}, t) + \theta_i x_{i+1} \\ \dot{x}_k &= f_k(\boldsymbol{x}, t) + \theta_k u(t) + \boldsymbol{b}^T \boldsymbol{\eta} \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{f}_\eta(\boldsymbol{x}, t) + \boldsymbol{q}(y, \boldsymbol{\eta}) \\ y &= x_1 \end{aligned} \tag{1}$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^T \in R^n$, $\boldsymbol{\eta} = [x_{k+1}, \cdots, x_n]^T \in R^{n-k}$ are state vectors, $u(t) \in R$, $y(t) \in R$ are an input and an output, $f_1(\boldsymbol{x}, t), \cdots, f_k(\boldsymbol{x}, t)$, $\boldsymbol{f}_\eta(\boldsymbol{x}, t) = [f_{k+1}(\boldsymbol{x}, t), \cdots, f_n(\boldsymbol{x}, t)]^T$ are uncertain nonlinearties and $\boldsymbol{b}^T = [b_{k+1}, \cdots, b_n], \theta_1, \cdots, \theta_k$ are unknown vector and constants respectively.

Here we impose the following assumptions on the system (1):

Assumption 1. *The uncertain nonlinearities $f_i(\boldsymbol{x}, t)$, $\boldsymbol{f}_\eta(\boldsymbol{x}, t)$ can be evaluated by*

$$\begin{aligned} |f_i(\boldsymbol{x}, t)| &\le d_{1i}|\psi_i(y)| + d_{0i} \ (1 \le i \le k) \\ ||\boldsymbol{f}_\eta(\boldsymbol{x}, t)|| &\le d_{1\eta}|\psi_\eta(y)| + d_{0\eta} \end{aligned} \tag{2}$$

with unknown constants $d_{1i}, d_{1\eta}, d_{0i}, d_{0\eta}$ and known functions $\psi_i(y), \psi_\eta(y)$ which have the following property for any variables y_1 and y_2:

$$\begin{aligned} |\psi_i(y_1 + y_2)| &\le |\psi_{1i}(y_1, y_2)||y_1| + |\psi_{2i}(y_2)| \\ |\psi_\eta(y_1 + y_2)| &\le |\psi_{1\eta}(y_1, y_2)||y_1| + |\psi_{2\eta}(y_2)| \end{aligned} \tag{3}$$

with known functions $\psi_{1i}, \psi_{1\eta}$ and unknown smooth functions $\psi_{2i}, \psi_{2\eta}$.

Assumption 2. *There is an unknown positive constant θ_0 such that*

$$\bar{\theta}_{1k} := \prod_{i=1}^{k} \theta_i \ge \theta_0 > 0. \tag{4}$$

Assumption 3. *The function $\boldsymbol{q}(y, \boldsymbol{\eta})$ is globally Lipschitz, i.e., there exists a positive constant L_1 such that*

$$\|\boldsymbol{q}(y_1, \boldsymbol{\eta}_1) - \boldsymbol{q}(y_2, \boldsymbol{\eta}_2)\| \le L_1(|y_1 - y_2| + \|\boldsymbol{\eta}_1 - \boldsymbol{\eta}_2\|). \tag{5}$$

Assumption 4. *Nominal part of the system* (1) *is exponentially minimum-phase. That is, the zero dynamics of the nominal system:*

$$\dot{\boldsymbol{\eta}}(t) = \boldsymbol{q}(0, \boldsymbol{\eta}) \tag{6}$$

is exponentially stable.

Under these assumptions the control objective is to achieve the goal:

$$\lim_{t\to\infty} |y(t) - y^*(t)| \le \delta \tag{7}$$

for a given positive constant δ and a smooth reference signal $y^*(t)$ such as

$$|y^*(t)| \le \beta_0,\ |\dot{y}^*(t)| \le \beta_1. \tag{8}$$

3. CONTROLLER DESIGN

3.1 Virtual system

For the controlled system (1) we introduce the following $(k-1)$th order virtual filter:

$$\begin{aligned} &\dot{u}_{f_i} = -\lambda_i u_{f_i} + u_{f_{i+1}}\ (1 \le i \le k-2)\\ &\dot{u}_{f_{k-1}} = -\lambda_{k-1} u_{f_{k-1}} + u\\ &\lambda_i > 0,\ (1 \le i \le k-1). \end{aligned} \tag{9}$$

Considering the following variable transformation with filter signal u_{f_i} $(2 \le l \le k-1)$

$$\begin{aligned} \xi_l = &\prod_{i=1}^{l-1}\theta_i x_l - \prod_{i=1}^{k}\theta_i u_{fl-1} - \sum_{i=l-1}^{k-1}\lambda_i \prod_{i=1}^{l-2}\theta_i x_{l-1} \\ &+ \sum_{\substack{i_1 \le i_2 \\ i_1, i_2 = l-1, \cdots, k-1}} \lambda_{i_1}\lambda_{i_2}\prod_{i=1}^{l-3}\theta_i x_{l-2} + (-1)^{l-1}\sum_{\substack{i_1 \le \cdots \le i_{l-1} \\ i_1, \cdots, i_l = l-1, \cdots, k-1}} \lambda_{i_1}\cdots\lambda_{i_{l-1}} x_1 \end{aligned} \tag{10}$$

where

$$\sum_{\substack{i_1 \le \cdots \le i_l \\ i_1, \cdots, i_l = k_1, \cdots, k_m}}$$

denotes the sum of all combinations of indices $i_1, i_2, \cdots, i_l \subset k_1, k_2, \cdots, k_m$ and $i_1 < i_2 < \cdots < i_l$, we can obtain the following virtual system with u_{f_1} as the control input:

$$\begin{aligned} &\dot{y} = a(y, \boldsymbol{\xi}) + \bar{\theta}_{1k} u_{f_1} + f_1(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t)\\ &\dot{\boldsymbol{\xi}} = A_\xi \boldsymbol{\xi} + \boldsymbol{a}_\xi y + B_\xi \boldsymbol{\eta} + \boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t)\\ &\dot{\boldsymbol{\eta}} = \boldsymbol{q}(y, \boldsymbol{\eta}) + \boldsymbol{f}_\eta(\boldsymbol{x}, t)\\ &\dot{u}_{f_i} = -\lambda_i u_{f_i} + u_{f_{i+1}} (1 \le i \le k-2)\\ &\dot{u}_{fk-1} = -\lambda_{k-1} u_{fk-1} + u \end{aligned} \tag{11}$$

where $\boldsymbol{\xi} = [\xi_2, \cdots, \xi_l, \cdots \xi_{k-1}]^T$,

$$a(y, \boldsymbol{\xi}) = \sum_{i=1}^{k-1}\lambda_i y + \xi_2 \tag{12}$$

$$A_\xi = \begin{bmatrix} -\lambda_1 & 1 & 0 & \cdots & 0 \\ 0 & -\lambda_2 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & \cdots & 0 & -\lambda_{k-1} \end{bmatrix}, B_\xi = \begin{bmatrix} \boldsymbol{o} \\ \prod_{i=1}^{k-1}\theta_i \boldsymbol{b}^T \end{bmatrix} \tag{13}$$

and $\boldsymbol{a}_\xi = [a_{\xi_2}, \cdots, a_{\xi_l}, \cdots, a_{\xi_{k-1}}]^T$, $\boldsymbol{F} = [F_2, \cdots, F_l, \cdots, F_{k-1}]^T$

$$a_{\xi_l} = (-1)^{l-1}\sum_{\substack{i_1 \le \cdots \le i_{l-1} \\ i_1, \cdots, i_l = l-1, \cdots, k-1}} \lambda_{i1}\cdots\lambda_{il} \tag{14}$$

$$\begin{aligned} F_l(\boldsymbol{x}, t) = &\prod_{i=1}^{l-1}\theta_i f_l(\boldsymbol{x}, t) - \sum_{i=l-1}^{k-1}\lambda_i \prod_{i=1}^{l-2}\theta_i f_{l-1}(\boldsymbol{x}, t) \\ &- \sum_{\substack{i_1 \le \cdots \le i_{l-1} \\ i_1, \cdots, i_l = l-1, \cdots, k-1}} \lambda_{i_1}\cdots\lambda_{i_{l-1}} f_1(\boldsymbol{x}, t). \end{aligned} \tag{15}$$

From (12), $a(y, \boldsymbol{\xi})$ is globally Lipschitz with respect to $(y, \boldsymbol{\xi})$, so that there exists a positive constant L_2 such that

$$|a(y_1, \boldsymbol{\xi}_1) - a(y_2, \boldsymbol{\xi}_2)| \le L_2(|y_1 - y_2| + ||\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2||) \tag{16}$$

and A_ξ is a stable matrix from (13) so that for any positive definite matrix Q_ξ, there exists a positive matrix P_ξ such that

$$P_\xi A_\xi + A_\xi^T P_\xi = -Q_\xi. \tag{17}$$

Moreover, from assumption 1 and (15), the vector function $\boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t)$ can be evaluated by

$$||\boldsymbol{F}(y, \boldsymbol{\xi}, \boldsymbol{\eta}, t)|| \le p_1|\phi(y)| + p_0 \tag{18}$$

with unknown positive constants p_1, p_0 and known function $\phi(y)$ which have the following property for any variables y_1 and y_2:

$$|\phi(y_1 + y_2)| \le |\phi_1(y_1, y_2)||y_1| + |\phi_2(y_2)| \tag{19}$$

with known function $\phi_1(y_1, y_2)$ and unknown function $\phi_2(y_2)$ which is smooth for all $y_2 \in R$.

3.2 Controller design through Backstepping

[*Step.1*]Consider the tracking error $\nu = y - y^*$. The error system can be represented by

$$\begin{aligned} &\dot{\nu} = a(\nu + y^*, \xi) + \bar{\theta}_{1k} u_{f_1} + f_1(\nu + y^*) - \dot{y}^*\\ &\dot{\boldsymbol{\xi}} = A_\xi \boldsymbol{\xi} + \boldsymbol{a}_\xi[\nu + y^*] + B_\xi \boldsymbol{\eta} + \boldsymbol{F}(\nu + y^*, \boldsymbol{\xi}, \boldsymbol{\eta})\\ &\dot{\boldsymbol{\eta}} = \boldsymbol{q}(\nu + y^*, \boldsymbol{\eta}) + \boldsymbol{f}_\eta(\nu + y^*, \boldsymbol{\xi}, \boldsymbol{\eta}). \end{aligned} \tag{20}$$

For this system, we introduce a virtual input α_1 for u_{f_1}, which is designed through a robust adaptive high-gain feedback given by

$$\alpha_1(t) = -[k(t)\nu(t) + u_R(t)] \tag{21}$$

$$k(t) = k_I(t) + k_P(t) \tag{22}$$

$$\dot{k}_I(t) = \gamma_I \nu(t)^2 - \sigma_I k_I(t),\ k_I(0) \ge 0 \tag{23}$$

$$k_P(t) = \gamma_p[\phi_1(\nu, y^*)^4 + \psi_{1\eta}(\nu, y^*)^4]\nu(t)^2 \tag{24}$$

$$u_R(t) = \gamma_R \psi_1(y)^2 \nu(t), \tag{25}$$

where $\gamma_I, \gamma_{pi}, \gamma_R$ and σ_I are any positive constants.

Now, consider the following positive definite function:

$$\begin{aligned} V_0(\nu, \boldsymbol{\xi}, \boldsymbol{\eta}, k) = &\frac{1}{2}\nu^2 + \mu_0 \xi^T P_\xi \boldsymbol{\xi} + \mu_1 W(\boldsymbol{\eta}) \\ &+ \frac{\theta_0}{2\gamma_I}[k_I - k^*]^2, \end{aligned} \tag{26}$$

where μ_0, μ_1 are any positive constants and k^* is an ideal feedback gain to be determined later. $W(\boldsymbol{\eta})$ is a positive definite function which has the following properties:

$$\frac{\partial W(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}}\boldsymbol{q}(0,\boldsymbol{\eta}) \leq -\kappa_1\|\boldsymbol{\eta}(t)\|^2, \ \left\|\frac{\partial W(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}}\right\| \leq \kappa_2\|\boldsymbol{\eta}(t)\|$$
$$\kappa_4\|\boldsymbol{\eta}(t)\|^2 \leq \|W(\boldsymbol{\eta})\| \leq \kappa_3\|\boldsymbol{\eta}(t)\|^2. \quad (27)$$

Since the system (1) is exponentially minimum-phase from assumption 4, there exists such a positive definite function: $W(\boldsymbol{\eta})$[6]. The time derivative of (26) along the trajectories of (20) and (23) yields

$$\begin{aligned}\dot{V}_0 = \ &\nu\big[a(\nu+y^*,\xi) - \bar{\theta}_{1k}[k\nu+u_R] + \bar{\theta}_{1k}[u_{f_1}-\alpha_1] \\ &+ f_1(\nu+y^*) - \dot{y}^*\big] + \mu_0\boldsymbol{\xi}^T(A_\xi^T P_\xi + P_\xi A_\xi)\boldsymbol{\xi} \\ &+ 2\mu_0\big[\|\boldsymbol{a}_\xi\|[|\nu|+|y^*|] + \|B_\xi\|\|\boldsymbol{\eta}\| + \|\boldsymbol{F}\|\big]\|P_\xi\|\|\boldsymbol{\xi}\| \\ &+ \mu_1\frac{\partial W}{\partial \boldsymbol{\eta}}\big[\boldsymbol{q}(\nu+y^*,\boldsymbol{\eta}) + \boldsymbol{f}_\eta(\nu+y^*,\boldsymbol{\xi},\boldsymbol{\eta})\big] \\ &+ \frac{\theta_0}{\gamma_I}[k_I-k^*][\gamma_I\nu^2 - \sigma_I k_I]. \end{aligned} \quad (28)$$

It follows from assumptions 1∼4 and from (16)∼(19) that (28) can be evaluated as

$$\begin{aligned}\dot{V}_0 \leq \ &-(\theta_0 k^* - L_2)\nu^2 - \mu_0\boldsymbol{\xi}^T Q_\xi\boldsymbol{\xi} - \mu_1\kappa_1\|\boldsymbol{\eta}\|^2 + L_2|\nu|\|\boldsymbol{\xi}\| \\ &+ 2\mu_0\|B_\xi\|\|P_\xi\|\|\boldsymbol{\xi}\|\|\boldsymbol{\eta}\| + \mu_1\kappa_2 L_1\beta_0\|\boldsymbol{\eta}\| \\ &+ 2\mu_0\|\boldsymbol{a}_\xi\|\|P_\xi\|(|\nu|+|y^*|)\|\boldsymbol{\xi}\| + (L_2\beta_0 + d_{01} + \beta_1)|\nu| \\ &+ 2\mu_0\|P_\xi\|\|\boldsymbol{\xi}\|\big[p_1(|\phi_1(\nu,y^*)||\nu| + |\phi_2(y^*)|) + p_0\big] \\ &+ \mu_1\kappa_2\|\boldsymbol{\eta}\|\big[d_{1\eta}(|\psi_{1\eta}(\nu,y^*)||\nu| + |\psi_{2\eta}(y^*)|) + d_{0\eta}\big] \\ &- \bar{\theta}_{1k}k\nu^2 + \theta_0 k_I\nu^2 + \mu_1\kappa_2 L_1|\nu|\|\boldsymbol{\eta}\| \\ &- \frac{\theta_0}{\gamma_I}\sigma_I[k_I-k^*]k^* - \frac{\theta_0}{\gamma_I}\sigma_I[k_I-k^*]^2 \\ &- \bar{\theta}_{1k}u_R\nu + d_{11}|\psi_1(\nu+y^*)||\nu| + \bar{\theta}_{1k}\omega_1\nu, \end{aligned} \quad (29)$$

where $\omega_1 = u_{f_1} - \alpha_1$. Here, from assumption 2 and since $k(t) \geq 0$ from (23) and (24), we have

$$\begin{aligned} &-\bar{\theta}_{1k}k\nu^2 + \theta_0 k_I\nu^2 - \bar{\theta}_{1k}u_R\nu \\ &\leq -\theta_0 k\nu^2 + \theta_0 k_I\nu^2 - \theta_0\gamma_R\psi_1^2\nu^2 \\ &\leq -\theta_0 k_p\nu^2 - \theta_0\gamma_R\psi^2\nu^2. \end{aligned} \quad (30)$$

Thus the time derivative of V_0 can be evaluated from (29),(30) as follows:

$$\begin{aligned}\dot{V}_0 \leq \ &-[\theta_0 k^* - v_0]\nu^2 - [\mu_0\lambda_{min}[Q_\xi] - v_1]\|\boldsymbol{\xi}\|^2 \\ &-[\mu_1\kappa_1 - v_2]\|\boldsymbol{\eta}\|^2 - \theta_0\frac{\sigma_I}{\gamma_I}(1-\rho_{11})[k_I-k^*]^2 \\ &+\bar{\theta}_{1k}\nu\omega_1 + R_0 \end{aligned} \quad (31)$$

with any positive constants ρ_0 to ρ_{11}, where

$$v_0 = L_2 + \rho_{10} + \frac{(\mu_0 L_2)^2}{4\rho_9} + \frac{(\mu_1\kappa_2 L_1)^2}{4\rho_2} + \frac{(\mu_0\|P_\xi\|\|\boldsymbol{a}_\xi\|)^2}{\rho_4} \quad (32)$$

$$v_1 = \frac{(\mu_0\|B_\xi\|\|P_\xi\|)^2}{\rho_0} + \rho_3 + \rho_4 + \rho_5 + \rho_6 + \rho_9 \quad (33)$$

$$v_2 = \rho_0 + \rho_1 + \rho_2 + \rho_7 + \rho_8 \quad (34)$$

$$\begin{aligned} R_0 = \ &\frac{(\mu_1\kappa_2 L_1\beta_0)^2}{4\rho_1} + \frac{(\mu_0\|P_\xi\|\|\boldsymbol{a}_\xi\|\beta_0)^2}{\rho_3} + \frac{\theta_0\sigma_I}{4\rho_{11}\gamma_I}k^{*2} \\ &+ \frac{(\mu_0\|P_\xi\|)^2}{\rho_6}(p_1\phi_{2M} + p_0)^2 \\ &+ \frac{(\mu_1\kappa_2)^2}{4\rho_8}(d_{1\eta}\psi_{2\eta M} + d_{0\eta})^2 \\ &+ \frac{1}{4\rho_{10}}(L_2\beta_0 + d_{01} + \beta_1)^2 + \frac{1}{4\theta_0\gamma_R}d_1^2 \\ &+ \frac{1}{4\theta_0\gamma_p}\left(\frac{(\mu_0 p_1\|P_\xi\|)^2}{\rho_5} + \frac{(\mu_1\kappa_2 d_{1\eta})^2}{4\rho_7}\right)^2 \end{aligned} \quad (35)$$

and ϕ_{2M} and $\psi_{2\eta M}$ are constants such that $\phi_{2M} \geq \phi_2(y^*)$ and $\psi_{2\eta M} \geq \psi_{2\eta}(y^*)$ respectively. $\lambda_{min}[Q_\xi]$ denotes the minimum value of the eigenvalue of the matrix Q_ξ.

[*Step.2*]The control objective can be achieved if the filter signal u_{f_1} is identical with the virtual input α_1 which is constructed for the virtual system with relative degree of 1. In this step, the error system between u_{f_1} and α_1, ω_1-system, will be considered and a virtual input α_2 for the filter signal u_{f_2} is designed to make u_{f_1} identify α_1. The ω_1-system is obtained from (9) as follows:

$$\dot{\omega}_1 = -\lambda_1 u_{f_1} + u_{f_2} - \dot{\alpha}_1. \quad (36)$$

Since α_1 is constructed by y, y^* and k_I as in (21) to (25), the time derivative of α_1 is given by

$$\begin{aligned}\dot{\alpha}_1 &= \frac{\partial\alpha_1}{\partial y}\dot{y} + \frac{\partial\alpha_1}{\partial y^*}\dot{y}^* + \frac{\partial\alpha_1}{\partial k_I}\dot{k}_I \\ &= \frac{\partial\alpha_1}{\partial y}\left[\sum_{i=1}^{k-1}\lambda_i y + \xi_2 + f_1 + \bar{\theta}_{1k}u_{f1}\right] \\ &\quad + \frac{\partial\alpha_1}{\partial y^*}\dot{y}^* + \frac{\partial\alpha_1}{\partial k_I}[\gamma_I\nu^2 - \sigma_I k_I]. \end{aligned} \quad (37)$$

Then we design the virtual input α_2 for the filter signal u_{f_2} as follows:

$$\begin{aligned}\alpha_2 &= -c_1\omega_1 + \lambda_1 u_{f1} - \epsilon_1\Psi_1\omega_1 \\ &\quad + \frac{\partial\alpha_1}{\partial y}\sum_{i=1}^{k-1}\lambda_i y + \frac{\partial\alpha_1}{\partial k_I}(\gamma_I\nu^2 - \sigma_I k_I) \end{aligned} \quad (38)$$

$$\Psi_1 = (\psi_1^2 + u_{f1}^2 + 2)\left(\frac{\partial\alpha_1}{\partial y}\right)^2 + \left(\frac{\partial\alpha_1}{\partial y^*}\right)^2 \quad (39)$$

where c_1 and ϵ_1 are any positive constants.

Now we consider the following positive definite function:

$$V_1 = \frac{1}{2}\omega_1^2 + V_0. \quad (40)$$

The time derivative of V_1 yields that

$$\begin{aligned}\dot{V}_1 &= \omega_1(\dot{u}_{f_1} - \dot{\alpha}_1) + \dot{V}_0 \\ &= \omega_1\Big[-\lambda_1 u_{f_1} + (\omega_2 + \alpha_2) - \frac{\partial\alpha_1}{\partial y}(\sum_{i=1}^{k-1}\lambda_i y + \xi_2 \\ &\quad + f_1 + \bar{\theta}_{1k}u_{f1}) - \frac{\partial\alpha_1}{\partial y^*}\dot{y}^* - \frac{\partial\alpha_1}{\partial k_I}[\gamma_I\nu^2 - \sigma_I k_I]\Big] \\ &\quad + \dot{V}_0 \end{aligned} \quad (41)$$

where $\omega_2 = u_{f_2} - \alpha_2$. Substituting (38) and (39) to (41), the time derivative of V_1 can be evaluated by

$$\begin{aligned}\dot{V}_1 \leq \ &-(c_1 - \rho_{12})\omega_1^2 + \omega_1\omega_2 + \frac{1}{4\epsilon_1}|\xi_2|^2 + R_1 + R_0 \\ &-[\theta_0 k^* - v_0 - \frac{\bar{\theta}_{1k}^2}{4\rho_{12}}]\nu^2 - [\mu_0\lambda_{min}[Q_\xi] - v_1]\|\boldsymbol{\xi}\|^2 \\ &-[\mu_1\kappa_1 - v_2]\|\boldsymbol{\eta}\|^2 - \theta_0\frac{\sigma_I}{\gamma_I}(1-\rho_{11})[k_I - k^*]^2 \end{aligned} \quad (42)$$

$$R_1 = \frac{1}{4\epsilon_1}\left(\bar{\theta}_{1k}^2 + d_{01}^2 + d_{11}^2 + \beta_1^2\right) \quad (43)$$

where ρ_{12} is any positive constant.

[*Step.l*]$(3 \le l \le k-1)$If the filter signal $u_{f_{l-1}}$ is equivalent to α_{l-1}, then u_{f_1} converges to α_1 and the control objective will be attained. Therefore, in this step we consider the error system between $u_{f_{l-1}}$ and α_{l-1} and design a virtual input α_l so as to stabilize the error system. Define $\omega_{l-1} = u_{f_{l-1}} - \alpha_{l-1}$. The error system, ω_{l-1}-system, is given by

$$\dot{\omega}_{l-1} = -\lambda_{l-1} u_{f_{l-1}} + u_{f_l} - \dot{\alpha}_{l-1}. \tag{44}$$

Considering that the virtual input α_{l-1} is constructed by $y, y^*, k_I, u_{f1}, \cdots, u_{f_{l-2}}$, the time derivative of α_{l-1} can be expressed by

$$\begin{aligned}\dot{\alpha}_{l-1} &= \frac{\partial \alpha_{l-1}}{\partial y}\dot{y} + \frac{\partial \alpha_{l-1}}{\partial y^*}\dot{y}^* + \frac{\partial \alpha_{l-1}}{\partial k_I}\dot{k}_I + \sum_{i=1}^{l-2}\frac{\partial \alpha_{l-1}}{\partial u_{f_i}}\dot{u}_{f_i} \\ &= \frac{\partial \alpha_{l-1}}{\partial y}\left[\sum_{i=1}^{k-1}\lambda_i y + \xi_2 + f_1 + \bar{\theta}_{1k}u_{f1}\right] \\ &\quad + \frac{\partial \alpha_{l-1}}{\partial y^*}\dot{y}^* + \frac{\partial \alpha_{l-1}}{\partial k_I}(\gamma_I \nu^2 - \sigma_I k_I) \\ &\quad + \sum_{i=1}^{l-2}\frac{\partial \alpha_{l-1}}{\partial u_{f_i}}(-\lambda_i u_{f_i} + u_{f_{i+1}}).\end{aligned} \tag{45}$$

Then the virtual input α_l for filter signal u_{f_l} is designed as follows:

$$\begin{aligned}\alpha_l &= -c_{l-1}\omega_{l-1} - \omega_{l-2} + \lambda_{l-1}u_{f_{l-1}} - \epsilon_{l-1}\Psi_{l-1}\omega_{l-1} \\ &\quad + \frac{\partial \alpha_{l-1}}{\partial y}\sum_{i=1}^{k-1}\lambda_{l-1}y + \frac{\partial \alpha_{l-1}}{\partial k_I}(\gamma_I \nu^2 - \sigma_I k_I) \\ &\quad + \sum_{i=1}^{l-2}\frac{\partial \alpha_{l-1}}{\partial u_{f_i}}(-\lambda_i u_{f_i} + u_{f_{i+1}})\end{aligned} \tag{46}$$

$$\Psi_{l-1} = (\psi_1^2 + u_{f_1}^2 + 2)\left(\frac{\partial \alpha_{l-1}}{\partial y}\right)^2 + \left(\frac{\partial \alpha_{l-1}}{\partial y^*}\right)^2 \tag{47}$$

where c_{l-1} and ϵ_{l-1} are any positive constants.

Here we consider the following positive definite function:

$$V_{l-1} = \frac{1}{2}\omega_{l-1}^2 + V_{l-2}. \tag{48}$$

The time derivative of V_{l-1} yields that

$$\begin{aligned}\dot{V}_{l-1} &= \omega_{l-1}(\dot{u}_{f_{l-1}} - \dot{\alpha}_{l-1}) + \dot{V}_{l-2} \\ &= \omega_{l-1}\Bigg[-\lambda_{l-1}u_{f_{l-1}} + (\omega_l + \alpha_l) \\ &\quad - \frac{\partial \alpha_{l-1}}{\partial y}\Big(\sum_{i=1}^{k-1}\lambda_i y + \xi_2 + f_1 + \bar{\theta}_{1k}u_{f1}\Big) \\ &\quad - \frac{\partial \alpha_{l-1}}{\partial y^*}\dot{y}^* - \frac{\partial \alpha_{l-1}}{\partial k_I}[\gamma_I \nu^2 - \sigma_I k_I] \\ &\quad - \sum_{i=1}^{l-2}\frac{\partial \alpha_{l-1}}{\partial u_{f_i}}(-\lambda_i u_{f_i} + u_{f_{i+1}})\Bigg] + \dot{V}_{l-2}\end{aligned} \tag{49}$$

where $\omega_l = u_{f_l} - \alpha_l$. From (46) and (47), the time derivative of V_1 can be evaluated by

$$\begin{aligned}\dot{V}_{l-1} &\le -(c_1 - \rho_{12})\omega_1^2 - \sum_{i=2}^{l-1}c_i\omega_i^2 + \omega_{l-1}\omega_l \\ &\quad + \sum_{i=1}^{l-1}\frac{1}{4\epsilon_i}|\xi_2|^2 + \sum_{i=0}^{l-1}R_i \\ &\quad - [\theta_0 k^* - v_0 - \frac{\bar{\theta}_{1k}^2}{4\rho_{12}}]\nu^2 - [\mu_0\lambda_{min}[Q_\xi] - v_1]\|\boldsymbol{\xi}\|^2 \\ &\quad - [\mu_1\kappa_1 - v_2]\|\boldsymbol{\eta}\|^2 - \theta_0\frac{\sigma_I}{\gamma_I}(1-\rho_{11})[k_I - k^*]^2\end{aligned} \tag{50}$$

where

$$R_{l-1} = \frac{1}{4\epsilon_{l-1}}(\bar{\theta}_{1k}^2 + d_{01}^2 + d_{11}^2 + \beta_1^2). \tag{51}$$

[*Step.k*]This is the last step, an actual control input u is obtained. The input u is designed by the same way of *step.l* as follows:

$$\begin{aligned}&k = 2: \\ &\quad u = \alpha_2 \end{aligned} \tag{52}$$

$$\begin{aligned}&k \ge 3: \\ &\quad u = -c_{k-1}\omega_{k-1} - \omega_{k-2} + \lambda_{k-1}u_{f_{k-1}} - \epsilon_{k-1}\Psi_{k-1}\omega_{k-1} \\ &\qquad + \frac{\partial \alpha_{k-1}}{\partial y}\sum_{i=1}^{k-1}\lambda_{k-1}y + \frac{\partial \alpha_{k-1}}{\partial k_I}(\gamma_I \nu^2 - \sigma_I k_I) \\ &\qquad + \sum_{i=1}^{k-2}\frac{\partial \alpha_{k-1}}{\partial u_{f_i}}(-\lambda_i u_{f_i} + u_{f_{i+1}})\end{aligned} \tag{53}$$

$$\Psi_{k-1} = (\psi_1^2 + u_{f_1}^2 + 2)\left(\frac{\partial \alpha_{k-1}}{\partial y}\right)^2 + \left(\frac{\partial \alpha_{k-1}}{\partial y^*}\right)^2 \tag{54}$$

where c_{k-1} and ϵ_{k-1} are any positive constants.

In this step we again consider the following positive definite function:

$$V_{k-1} = \frac{1}{2}\omega_{k-1}^2 + V_{k-2}. \tag{55}$$

The time derivative of V_{k-1} can be evaluated as

$$\begin{aligned}\dot{V}_{k-1} &\le -(c_1 - \rho_{12})\omega_1^2 - \sum_{i=2}^{k-1}c_i\omega_i^2 + \sum_{i=0}^{k-1}R_i \\ &\quad - [\theta_0 k^* - v_0 - \frac{\bar{\theta}_{1k}^2}{4\rho_{12}}]\nu^2 \\ &\quad - \left[\mu_0\lambda_{min}[Q_\xi] - v_1 - \sum_{i=1}^{k-1}\frac{1}{4\epsilon_i}\right]\|\boldsymbol{\xi}\|^2 \\ &\quad - [\mu_1\kappa_1 - v_2]\|\boldsymbol{\eta}\|^2 - \theta_0\frac{\sigma_I}{\gamma_I}(1-\rho_{11})[k_I - k^*]^2\end{aligned} \tag{56}$$

where

$$R_{k-1} = \frac{1}{4\epsilon_{k-1}}(\bar{\theta}_{1k}^2 + d_{01}^2 + d_{11}^2 + \beta_1^2) \tag{57}$$

by using the same manner in *step.l.* Finally, setting $\rho_0 = \frac{12\mu_0|B_\xi P_\xi|^2}{\lambda_{min}[Q_\xi]}, \rho_1 = \rho_2 = \rho_7 = \rho_8 = \frac{\mu_1\kappa_1}{12}, \rho_3 = \rho_4 = \rho_5 = \rho_6 = \rho_9 = \frac{\mu_0\lambda_{min}[Q_\xi]}{12}, \rho_{11} = \frac{1}{2}, \rho_{12} = \frac{c_1}{2}, \mu_0 = \sum_{i=1}^{k-1}\frac{1}{\lambda_{min}[Q_\xi]\epsilon_i}$ and $\mu_1 = \frac{48\mu_0|B_\xi P_\xi|^2}{\lambda_{min}[Q_\xi]\kappa_1}$, we obtain

$$\begin{aligned}\dot{V}_{k-1} &\le -\frac{1}{2}c_1\omega_1^2 - \sum_{i=2}^{k-1}c_i\omega_i^2 - [\theta_0 k^* - v_0 - \frac{\bar{\theta}_{1k}^2}{2}]\nu^2 \\ &\quad - \frac{\mu_0\lambda_{min}[Q_\xi]}{4}\|\boldsymbol{\xi}\|^2 - \frac{\mu_1\kappa_1}{2}\|\boldsymbol{\eta}\|^2 \\ &\quad - \theta_0\frac{\sigma_I}{2\gamma_I}[k_I - k^*]^2 + R_T\end{aligned} \tag{58}$$

where

$$
\begin{aligned}
R_T :=& \sum_{i=0}^{k-1} R_i \\
=& \sum_{i=1}^{k-1} \frac{144\|B_\xi\|\|P_\xi\|^2\kappa_2^2}{\lambda_{min}^2[Q_\xi]\kappa_1^2\epsilon_i}\left[(L_1\beta_0)^2+(d_{1\eta}\psi_{2\eta M}+d_{0\eta})^2\right] \\
&+\sum_{i=1}^{k-1} \frac{12\|P_\xi\|^2}{\lambda_{min}^2[Q_\xi]\epsilon_i}\left[(\|\boldsymbol{a}_\xi\|\beta_0)^2+(p_1\phi_{2M}+p_0)^2\right] \\
&+\frac{1}{4\rho_{10}}(L_2\beta_0+d_{01}+\beta_1)^2+\frac{\theta_0\sigma_I}{2\gamma_I}k^{*2}+\frac{1}{4\theta_0\gamma_R}d_1^2 \\
&+\frac{1}{4\theta_0\gamma_p}\left(\frac{12\mu_0 p_1^2\|P_\xi\|^2}{\lambda_{min}[Q_\xi]}+\frac{3\mu_1\kappa_2^2 d_{1\eta}^2}{\kappa_1}\right)^2 \\
&+\sum_{i=1}^{k-1}\frac{1}{4\epsilon_i}(\bar{\theta}_{1k}^2+d_{01}^2+d_{11}^2+\beta_1^2). \qquad (59)
\end{aligned}
$$

Since it follows from (27) that $\|\boldsymbol{\eta}\|^2 \le \frac{1}{\kappa_3}W(\boldsymbol{\eta})$, we have

$$
\begin{aligned}
\dot{V}_{k-1} \le& -\frac{1}{2}c_1\omega_1^2-\sum_{i=2}^{k-1}c_i\omega_i^2-\frac{\kappa_1}{2\kappa_3}\left[\mu_1 W(\boldsymbol{\eta})+\frac{1}{2}\nu^2\right] \\
&-\frac{\mu_0}{4}\lambda_{min}[Q]\|\boldsymbol{\xi}\|^2-\frac{1}{2}\sigma_I\frac{\theta_0}{\gamma_I}[k_I-k^*]^2+R_T \qquad (60)
\end{aligned}
$$

by setting the ideal feedback gain k^* as

$$
\begin{aligned}
k^* \ge& \frac{1}{\theta_0}\left[\frac{\kappa_1}{4\kappa_3}+L_2+\rho_{10}+\frac{3\mu_0 L_2^2}{\lambda_{min}[Q_\xi]}\right. \\
&\left.+\frac{3\mu_1\kappa_2^2L_1^2}{\kappa_1}+\frac{12\mu_0\|P_\xi\|\|\boldsymbol{a}_\xi\|^2}{\lambda_{min}[Q_\xi]}+\frac{\bar{\theta}_{1k}^2}{2}\right]. \qquad (61)
\end{aligned}
$$

Consequently the time derivative of the positive definite function V_{k-1} can be evaluated by

$$
\dot{V}_{k-1} \le -\alpha_v V_{k-1}+R_T \qquad (62)
$$

$$
\alpha_v = \min\left[c_1, 2c_2\cdots, 2c_{k-1}, \frac{\kappa_1}{2\kappa_3}, \frac{\lambda_{min}[Q_\xi]}{4\lambda_{max}[P_\xi]}, \sigma_I\right] \qquad (63)
$$

where $\lambda_{max}[P_\xi]$ denotes the maximum value of the eigenvalue of the matrix P_ξ. It is apparent from (62),(63) that all the signals in the closed-loop system with the controller (52) or (53) are bounded. We also obtain that

$$
\lim_{t\to\infty} V_{k-1} \le R_T/\alpha_v. \qquad (64)
$$

From the fact that $\nu^2 \le 2V_{k-1}$, it follows that

$$
\lim_{t\to\infty} \nu^2 \le 2R_T/\alpha_v. \qquad (65)
$$

Thus, the goal (7) is achieved for $\delta^2 \le 2R_T/\alpha_v$. It can also be confirmed that the appropriate choice of μ_0, μ_1 and ρ_{10} and design parameters $\gamma_I, \gamma_p, \gamma_R$ and $\epsilon_1 \sim \epsilon_{k-1}$ ensures the goal (7) for any δ.

Now, we have the following theorem concerning the stability of the resulting control system.

Theorem 1. Under assumptions 1∼4, all the signals in the closed-loop system with the controller (52) or (53) are bounded and the goal (7) is achieved by appropriate choice of design parameters $\gamma_I, \gamma_p, \gamma_R$ and $\epsilon_1, \cdots, \epsilon_{k-1}$.

4. NUMERICAL SIMULATION

Here the effectiveness of the proposed control scheme will be confirmed through a numerical simulation.

Consider the following SISO affine nonlinear system:

$$
\begin{aligned}
\dot{x}_1 &= f_1(x_1,x_2)+\theta_1 x_2 \\
\dot{x}_2 &= f_2(x_1,x_2,\eta_3)+\theta_2 u+b\eta_3 \\
\dot{\eta}_3 &= f_\eta(x_1)+q(x_1,\eta_3) \\
y &= x_1 \qquad (66)
\end{aligned}
$$

where

$$
\begin{aligned}
&f_1(x_1,x_2) = x_1^2\cos x_2 \\
&f_2(x_1,x_2,\eta_3) = x_1^2\exp(-x_2^2)\mathrm{sgn}(\eta_3),\ f_\eta(x_1) = x_1^3 \\
&q(x_1,\eta_3) = x_1-\eta_3,\ \theta_1 = 2,\ \theta_2 = 3,\ b = -1.
\end{aligned}
$$

The controlled system given in (66) has a relative degree of 2 and is exponentially minimum-phase. In this simulation, it is supposed that we have *a priori* information about the controlled system such that the nonlinearity $q(y,\eta_3)$ is Lipshitz in (y,η_3) and nonlinear functions f_1, f_2 and f_η are not Lipshitz but can be evaluated by

$$
\begin{aligned}
&|f_1| \le d_{11}|\psi_1|+d_{01} \qquad (67) \\
&|f_2| \le d_{12}|\psi_2|+d_{02} \qquad (68) \\
&|f_\eta| \le d_{1\eta}|\psi_\eta|+d_{0\eta} \qquad (69) \\
&\psi_1 = \psi_2 = y^2,\ \psi_\eta = y^3.
\end{aligned}
$$

Here we introduce a virtual filter:

$$
u_{f_1} = -\lambda_1 u_{f_1}+u,\ \lambda_1 > 0. \qquad (70)
$$

Applying the following variable transformation:

$$
\xi_2 = \theta_1 x_2-\theta_1\theta_2 u_{f_1}-\lambda_1 x_1, \qquad (71)
$$

the controlled system (66) can be represented by

$$
\begin{aligned}
&\dot{y} = a(y,\xi)+\theta_1\theta_2 u_{f_1}+f_1(y,\xi_2) \\
&\dot{\xi}_2 = A_\xi\xi_2+a_\xi y+B_\xi\eta_3+F(y,\xi_2,\eta_3) \\
&\dot{\eta}_3 = q(y,\eta_3)+f_\eta(y,\eta_3) \qquad (72) \\
&A_\xi = -\lambda_1,\ a_\xi = -\lambda_1,\ B_\xi = \theta_1 b \\
&F(y,\xi_2,\eta_3) = \theta_1 f_2(y,\xi_2,\eta_3)-\lambda_1 f_1(y,\xi_2).
\end{aligned}
$$

From (67) and (68) the nonlinear function F can be evaluated by

$$
\begin{aligned}
|F(y,\xi_2,\eta_3)| \le& |\theta_1|(d_{12}|\psi_2|+d_{02})+\lambda_1(d_{11}|\psi_1|+d_{01}) \\
\le& p_1|\phi(y)|+p_0 \qquad (73)
\end{aligned}
$$

where $\phi(y) = y^2$ and p_0, p_1 are unknown positive constants. Furthermore, from (69) and (73) the nonlinear functions $\psi_\eta(y)$ and $\phi(y)$ can be decomposed as follows:

$$
\begin{aligned}
&|\psi_\eta(y)| \le |\psi_{1\eta}(y)||y| \qquad (74) \\
&|\phi(y)| \le |\phi_1(y)||y| \qquad (75) \\
&\psi_{1\eta}(y) = y^2,\ \phi_1(y) = y
\end{aligned}
$$

so as to satisfy (3) and (19).

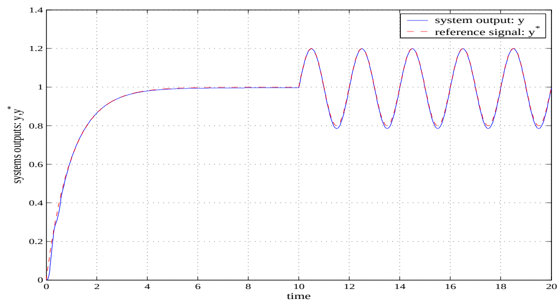

Fig. 1 Systems outputs

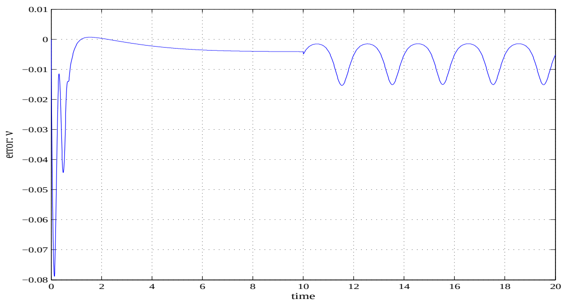

Fig. 2 Output error

By utilizing functions $\psi_1(y), \psi_{1\eta}(y), \phi_1(y)$, the control input is obtained as

$$u = -c_1\omega_1 + \lambda_1 u_{f_1} - \epsilon_1 \Psi_1 \omega_1 + \frac{\partial \alpha_1}{\partial y}\lambda_1 y + \frac{\partial \alpha_1}{\partial k_I}(\gamma_I \nu^2 - \sigma_I k_I) \tag{76}$$

$$\Psi_1 = (\psi_1^2 + u_{f_1}^2 + 2)\left(\frac{\partial \alpha_1}{\partial y}\right)^2 + \left(\frac{\partial \alpha_1}{\partial y^*}\right)^2$$

$$\alpha_1 = -[k\nu + u_R], \ k = k_I + k_p$$

$$\dot{k}_I = \gamma_I \nu^2 - \sigma_I k_I, \ k_p = \gamma_p[\phi_1^4 + \psi_{1\eta}^4]\nu^2$$

$$u_R = \gamma_R \psi_1^2 \nu, \ \nu = y - y^*, \ \omega_1 = u_{f_1} - \alpha_1$$

$$\frac{\partial \alpha_1}{\partial y} = -\big(4\gamma_p[\phi_1^3 + 2y\psi_{1\eta}^3]\nu^3 + 2k_p + k + 4\gamma_R \psi_1 y \nu + \gamma_R \psi_1^2\big)$$

$$\frac{\partial \alpha_1}{\partial y^*} = -2k_p + k - \gamma_R \psi_1^2, \ \frac{\partial \alpha_1}{\partial k_I} = -\nu.$$

In this simulation the reference signal $y^*(t)$ is given by

$$y^* = y_1^* + y_2^* \tag{77}$$

$$y_1^* = (1 - \exp\{-t\}), \ y_2^* = \begin{cases} 0 & t < 10 \\ 0.2 \sin \pi(t-10) & t \geq 10 \end{cases}$$

and we set the design parameters as follows:

$$c_1 = 1, \ \lambda_1 = 1, \ \epsilon_1 = 0.1, \ k_I(0) = 4, \ \gamma_I = 1000 \tag{78}$$

$$\gamma_p = \gamma_R = 100, \ \sigma_I = 0.1, \ \psi_1 = \psi_{1\eta} = y^2, \ \phi_1 = y.$$

Fig.1∼Fig.4 show the simulation results of the proposed method. It is confirmed that the proposed controller gives a good tracking performance even though there exist unknown nonlinearities in controlled system with relative degree of greater than 1.

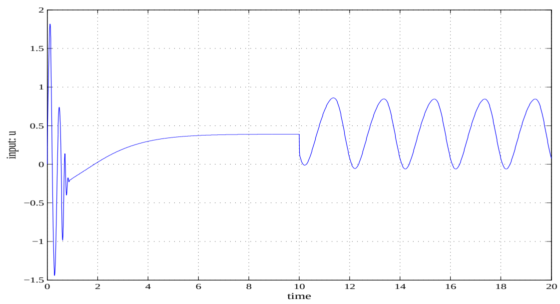

Fig. 3 Input

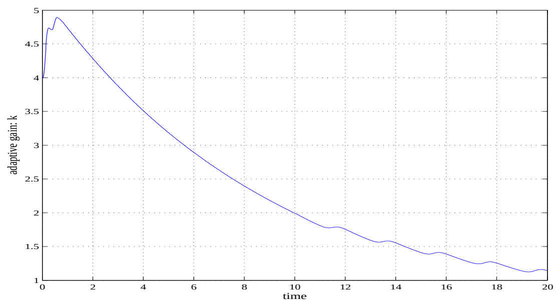

Fig. 4 Adaptive gain

5. CONCLUSION

In this paper, we proposed a robust high-gain adaptive output feedback control for a class of minimum-phase nonlinear systems with higher order relative degree. It was shown that we can design the robust high-gain adaptive output feedback controller via backstepping strategy by introducing a virtual filter in the case where only output signal is available. It was also confirmed that the appropriate choice of design parameters ensures the tracking error be small, *i.e*, the tracking error converges into any given bound.

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