Search Vector Method for Solution Domain Renewal

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Abstract: A band function model paired comparison method (BMPC method) is a kind of a paired comparison methods. Considering the human ambiguities, the BMPC method expressing the human judgment characteristics as a monotonous increase function with some width. Since function types are not specified in a BMPC method, the solution is obtained from inequalities, and the solution is given as a domain. To solve the simultaneous inequalities, the sequential renew method is used in the previous BMPC method. However, the sequential renew method requires much computational effort and memories. Generally, in BMPC method, it is able to solve only a paired comparison table which has less 12-13 samples. For that purpose, a new fast solution algorithm is required. In this paper, we proposed a new "search vector method" which renews the solution domain without creating new edge vectors. By using the method, it is able to decrease the necessary memory spaces and time to solve. The proposed method makes it able to solve more than 15 samples paired comparison inspections which are impossible to solve by previous method.

Keywords: Sensory evaluation, Simultaneous inequality, sequential renew method

1. Introduction

Measuring human senses clearly is very important for a quality control or marketing. To measure the unclear quantity such as human senses, a paired comparison method is one of the effective techniques. A band function model paired comparison method (BMPC method)[1] is a kind of paired comparison methods. Considering the human ambiguities, the BMPC method expressing the human judgment characteristics as a monotonous increase function with some width. Since function types are not specified in a BMPC method, the solution is obtained from inequalities, and the solution is given as a domain.

Solving simultaneous inequalities can be characterized as vertex enumeration problem of n-dimensional convex polyhedron. Due to the human ambiguity, the simultaneous inequalities which appears in BMPC method has contradicted inequalities. Then, the solution of the simultaneous inequalities becomes null. To escape the problem, sequential renew method is used to solve simultaneous inequalities in previous method[2]. However, the sequential renew method requires much computational effort and memories. Generally, in BMPC method, it is able to solve only a paired comparison table which has less 12-13 samples. For that purpose, a new fast solution algorithm is required.

In this paper, we proposed a new "search vector method" which renews the solution domain without creating new edge vectors. By using the method, it is able to decrease the necessary memory spaces and time to solve.

2. Algorithm of BMPC method

2.1. Generation of simultaneous inequality

The BMPC method supposes that a bigger reaction occurs when the stimulation difference between samples is bigger. The scale value for sample X_i (i=1,2,$\cdots$,n) is defined as m_i(i=1,2,$\cdots$,n). The relationship between relative evaluation φ_{ij} for X_i, X_j and the relative evaluation φ_{kl} for X_k, X_l can be shown by the monotonous increase function f_c,

$$\varphi_{ij} = f_c(m_i - m_j) \quad (1)$$

$$\varphi_{kl} = f_c(m_k - m_l) \quad (2)$$

Therefore, it is possible to deduce the inequality of relationship among m_i, m_j, m_k, m_l,

$$m_i - m_j > m_k - m_l \quad (3)$$

$$\Rightarrow m_i - m_j - m_k + m_l \quad > \quad 0 \quad (4)$$

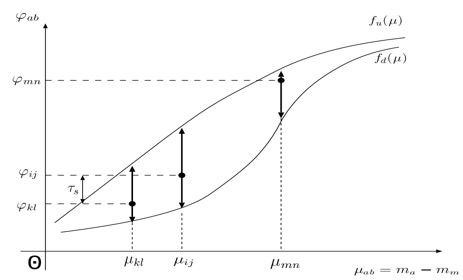

Fig. 1. Band function model

Because the human sense has an ambiguity, examinees' judgments are not always unique. Then the judgment characteristic function is accurately expressed using two monotonous increase functions f_d, f_u as shown in Fig.1.

In this function, φ_{ij} must have the following relationship:

$$f_d(m_i - m_j) \leq \varphi_{ij} \leq f_u(m_i - m_j) \quad (5)$$

Considering that there is arbitrariness in the units and the origin for m_i, the conditions of equations (7) and (8) are added, but no generality is lost by this operation.

Solving the BMPC method is deduced by solving the following simultaneous inequalities

$$m_i - m_j - m_k + m_l > 0 \quad (6)$$

$$\sum_{i=1}^{n} m_i = 0 \quad (7)$$

$$\sum_{i=1}^{n} m_i^2 = 1 \quad (8)$$

2.2. The solution method of simultaneous inequality

The simultaneous inequality (6) made from a band function is now expressed by the matrix form as

$$\mathbf{Am} = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_k \\ \vdots \\ \mathbf{a}_K \end{bmatrix} \begin{bmatrix} m_1 \\ \vdots \\ m_n \end{bmatrix} > 0 \quad (9)$$

$\mathbf{A}$ is the $K \times n$ matrix, and this is called the coefficient matrix. The n-dimensional line vector $\mathbf{a}_k$, which is the component, is called the coefficient vector, and $\mathbf{m}$ is an n-dimensional column vector. Equation(7) and (8) are, then, expressed as follows.

$$\mathbf{a}_0 \cdot \mathbf{m} = [1, 1, \ldots, 1] \cdot \mathbf{m} = 0 \quad (10)$$

$$|\mathbf{m}| = 1 \quad (11)$$

The solution of one inequality in the equation (9)

$$\mathbf{a}_k \cdot \mathbf{m} > 0 \quad (12)$$

is one open half-space which is determined according to the hyperplane in the n-dimensional space which passes through the origin and orthogonalized with $\mathbf{a}_k$. The solution of equation (9) becomes the solution domain shown by the common part of a half-open space of the K pieces, i.e., the solution obtained using equations (9) and (10) becomes the entire inner point of some convex multi-dimensional cone which is surrounded by hyperplanes and has an origin at a point, as shown in Fig.2. Equation (11) shows that the solution is the hypersphere which is a distance from the origin of 1.

The edge vector is a vector which took the end point on each edge of this convex multi-dimensional cone and takes the start point at the origin, and is expressed as $\mathbf{g}_i; i \in 1, \cdots, r$, where r indicates the number of edge vectors. The solution domain is shown by the set of edge vectors.

3. Search vector method

Solving simultaneous inequalities can be characterized as vertex enumeration problem of n-dimensional convex polyhedron. Due to the human ambiguity, the simultaneous inequalities which appears in BMPC method has contradicted inequalities. Then, the solution of the simultaneous inequalities becomes null. Therefore, it is necessary to choose a contradicted inequality from the simultaneous inequalities.

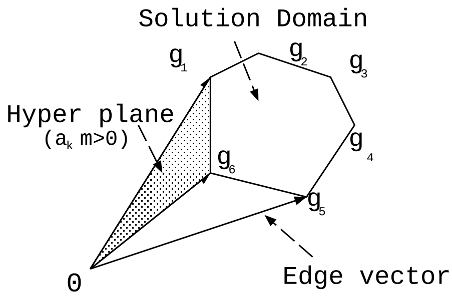

Fig. 2. Concept of solution domain

In the BMPC method, the contradicted inequalities are determined by following rules. "If an inequality with a certain reliability is contradicted to the solution domain which constituted only by inequalities which has higher reliability, the inequality is rejected from simultaneous inequality as contradicted inequality." To realize the rule, a sequential renew method is used to solve simultaneous inequalities as following.

An inequality is added to the solution domain sequentially from higher reliability. In the case that the inequality is not contradicted to the solution domain, renew the solution domain by the inequality. On the other hand, when the inequality is contradicted to the solution domain, inequalities which constituted the solution domain have higher reliability, so the ineuqlity is rejected as a contradicted inequality.

In the process of using sequential renew method, many edge vectors, which shows the vertex of the convex polyhedron, are appears. It is well known that it requires much computational effort and memories to create edge vectors. However, the most of edge vectors are not used for the solution domain. Thus, we proposed the new "Search vector method" to renew solution domain without creating edge vectors.

3.1. Search vector method

The purpose of the search vector method is to find a point which is in an inside of a final solution domain which satisfies all inequalities ecsept contradicted inequalities.

Let consider a given solution domain S_n which has a point p^n (search point) in an inside. If it is able to search a point in solution domain S_{n+1} which renewed by a new hyper-plane generated by an inequality $A_{n+1} > 0$, it is able to find a point in the final solution domain by the mathematical induction. The point p^{n+1} which is in a new solution domain S_{n+1}, satisfies new inequality and inequalities which constitute the solution domain S_n. To find the search point p^{n+1}, the search point p^n is moved by a search vector sequentially as Figure 3. After all inequalities are added, the final search point p^z is in an inside of the final solution domain. From the final search point p^z, it is able to create the final solution domain.

3.1.1 Search point movement by the search vector

The search point p^{n+1} is required to satisfy a half-space which generated by the new inequality $A_{n+1} > 0$. Therefore, let the search vector as a normal of the hyper-plane which generated by the inequality $A_{n+1} > 0$. However, as shown in Fig.4, sometimes other hyper-plane A_k may be exist before

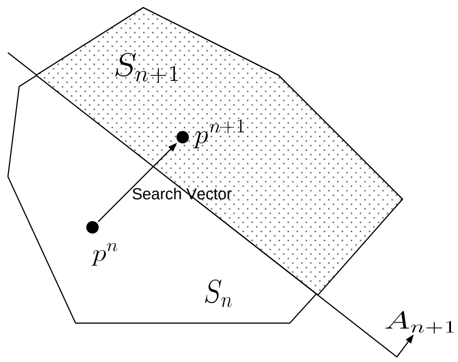

Fig. 3. Solution domain renewal

hyper-plane A_{n+1}. In such a case, the search vector changes its direction to follow the hyper-plane A_k. On this bases, movement of the search point is determined as follows.

1. Let the first search vector v_0 as the nomal of the hyper-plane $A_{n+1} > 0$.
2. Move search point p_i^n to
$$p_{i+1}^n = p_i^n + (d(A_{n+1}, p_i^n) + \epsilon) \cdot v_i \tag{13}$$
when $d(x, y)$ is distance of x and y.
3. If p_{i+1}^n does not satify the inequality A_k which constitute the solution domain S_n,
$$p_{i+1}^n = p_i^n + (d(A_k, p_i^n) - \epsilon) \cdot v_i \tag{14}$$
and make a new search vector as
$$v_{i+1} = v_i + d \cdot A_k \tag{15}$$
$$d = -\frac{v_i \cdot A_k}{||A_k||} \tag{16}$$
$i := i + 1$ and back to 2
4. If p_{i+1}^n is in an inside of solution domain S_n and it satisfies the new inequality $A_{n+1} > 0$, p_{i+1}^n is in an inside of S_{n+1}.
5. $n := n + 1$ and back to 1

The final solution domain is obtained by operating the algorithm to all inequalities.

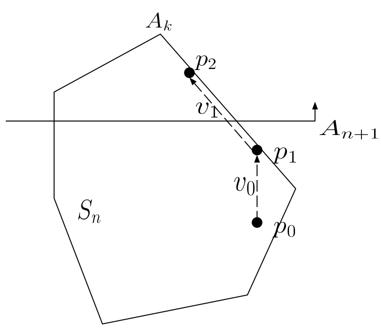

Fig. 4. Change Direction of Search Vector

3.1.2 Contradicted inequalities

Due to the human ambiguity, the simultaneous inequalities which appears in BMPC method has contradicted inequalities. When the contradicted inequality added to a solution domain, the solution domain become null. In such a case, the physical relationship of contradicted hyper-plane and the solution domain becomes as it shown in Fig.5. In this case, there is no solution domain S_{n+1} which satifty A_{n+1} and inequalities which constituted the solution domain S_n, thus, the search point continues movement infinitely. Then, the following end conditions are added to the algorithm of the search vector method.

- The case when the search vector shows the opposite direction to the normal of hyper-plane.(Fig.6)
- The case when $d(A_k, p_i^n) < \epsilon$ in algorithm 3, i.e. the case when the search point cannot move to search vectcor direction.(Fig.7)

When the end conditions are satisfied, we regard the added inequality A_{n+1} as a contradicted inequalit, and rejected it. A flow chart of the algorithm with end conditions is shown in Fig.8.

After finding the point in the final solution domain, it is able to make final solution domain.

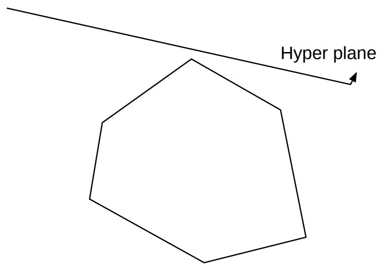

Fig. 5. Contradicted Inequality

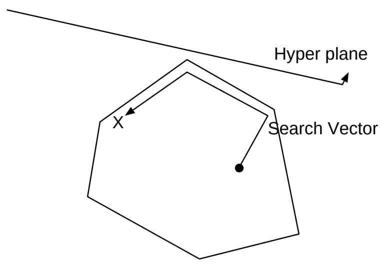

Fig. 6. Search vector shows opposite direction of hyper plane

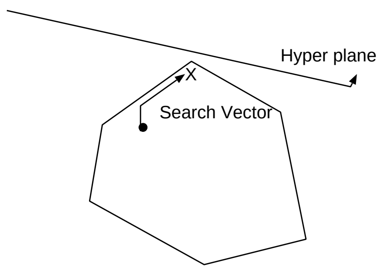

Fig. 7. Can not move search point to search vector direction

4. Simulation

4.1. Simulation condition

To check the validity, the proposed method is applied to the simulation. We use 100 paired comparison tables which have

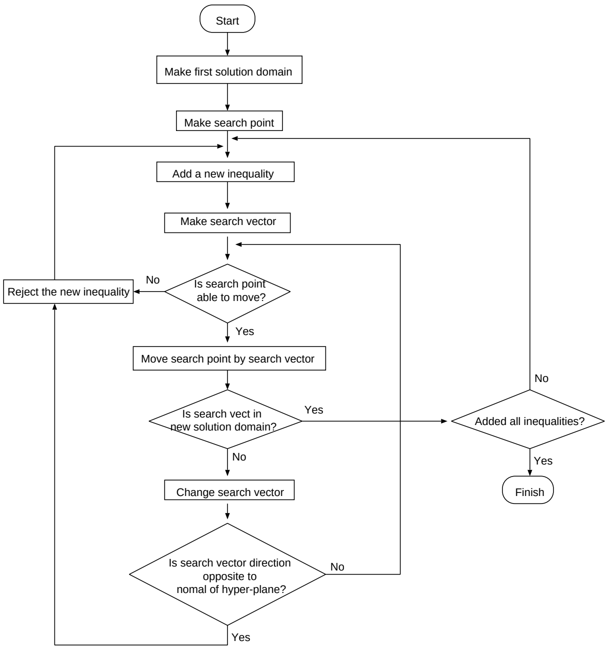

Fig. 8. Flow chart of the search vector algorithm

4-13 samples each. We solved each paired comparisons by the proposed algorithm and the recent algorithm.

4.2. Simulation result

4.2.1 Error of solution domain

Table 1 shows the average of angle between the solution domain which obtained by the proposed method and previous method. This result makes it clear that it is able to obtain almost same solution domains by proposed method and the previous method.

Table 1. Average angle between solution domains which obtained by proposed method and previous method

Num	no contradiction[deg]	with contradiction[deg]
4	0.00	0.00
5	0.0404	0.524
6	0.0386	2.66
7	0.213	3.44
8	0.147	6.81
9	0.00015	7.65
10	0.00041	7.71
11	0.0241	8.09

4.2.2 Effect of decreasing computational effort and memories

Table 2 shows the average time to solve BMPC method by proposed method and previous method.(Using Intel Xeon 2.8GHz 2G-Byte memories, Linux) From this result, when the sample numbers becomes large, the effort of proposed method becomes large. As to memories, the proposed method requires only memories of simultaneous inequalities, search point and search vector. Comparing to the previous method which required many memories for the edge vectors, the proposed method reduced the required memory sharply.

Table 2. Average time to solve BMPC method

num	proposed method	previous method
7	0.279[sec]	0.075[sec]
8	1.34	0.19
9	3.63	0.60
10	11.2	3.6
11	27.5	17.0
12	67.3	1851
13	762	3704

5. Conclusion

In this paper, we proposed the new search vector algorithm for BMPC method. The proposed method makes it able to solve more than 13 samples paired comparison inspections which are impossible to solve by previous method.

The sequential paired comparison method [2] makes it possible to inspect with paired comparison which has large number of samples without burden to panels. However, it have been impossible to solve them by the previous method. By using the sequential paired comparison method and the search vector algorithm together, it may be easy to inspect the paired comparison which has large number of samples.

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