On the Undershoot Compensation in MIMO Systems with Nonminimum Phase Zeros

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Abstract: In the control system analysis and synthesis, the nonminimum phase system has some difficulties due to the undershoot behaviour and the constrained sensitivity function. SISO problems has been widely investigated in the literatures, and it is well known that the undershoot cannot be eliminated by any linear feedback control. However, the undershoot compensation in MIMO system is less studied, and this paper is to deal with the zero property and the nonminimum phase behaviour of the MIMO system. Firstly, some definitions of the zeros will be introduced. Second, some systems including nonminimum phase transmission zeros are exemplified to show that the undershoot behaviour could be eliminated by a linear feedback in MIMO systems.

Keywords: Blocking zero, transmission zero, nonminimum phase system, MIMO system

1. INTRODUCTION

Control system analysis and design problems in LTI systems are mainly focused on system poles rather than zeros since poles are related to most of the system performances as well as the stability. Thus properties of zeros have little attention in the literature. In SISO systems, it is well known that the left half plane zero is concerned with the overshoot [5]. Moreover, if the number of right half plane zero is odd, then the system must have the undershoot[7]. If a single pole is placed on the imaginary axis, then the system is marginally stable. On the other hand, the zero on the imaginary axis yields some difficulties. Specially, if the system has least one zero at the origin, it must have a diverse input for the tracking performance[3]. Therefore, the origin has a role as the critical point.

It is also known in SISO systems that the undershoot problem due to the unstable zero cannot be eliminated by any linear feedback control since the zero position cannot be changed by the linear feedback. The undershoot might be eliminated by using a nonlinear control. For example, Tsai and Li[6] have proposed a fuzzy control to eliminate the undershoot, where most part of the undershoot in the transient response looks like time delay. But this control law cannot guarantee the stability of the closed-loop system.

The system design problem for the frequency response is very interested in the MIMO system. Because, we can get the robustness and the worst case performance from frequency characteristics. However, it has been less investigated about the zero properties in MIMO system. Therefore, this paper deals with the system's zeros and the nonminimum phase behaviour of the MIMO system.

The typical example of the nonminimum phase MIMO system with the undershoot phenomenon is the human body. When we want to go forward, if our foots only go forward, then our body must drop to backward direction, which means the nonminimum phase characteristic. But we can walk forward without backward steps, because our body is a multivariable system with a number of inputs for canceling the undershoot characteristics.

This paper is organized as follows: First, the undershoot phenomenon is exemplified using an inverted pendulum in the next section. Section 3 deals with some definitions of zeros in the MIMO systems. And Section 4 shows two examples of MIMO systems with nonminimum phase zeros for the undershoot compensation. One case is shown to be able to eliminate the undershoot by a feedback, but another case is not. The last section gives some conclusions and further works.

2. PRELIMINARIES AND NONMINIMUM PHASE CASE EXAMPLE

It is well known that if an LTI SISO system has odd number of nonminimum phase zeros then it must have undershoot regardless of any linear feedback. As an popular example, the inverted pendulum can not go forward without undershoot.

Example 1 (Inverted Pendulum):

Fig. 1 is description of the inverted pendulum [1]. Intuitionally, as shown in this figure, if the cart go to the forward direction, the rod must be falling backward.

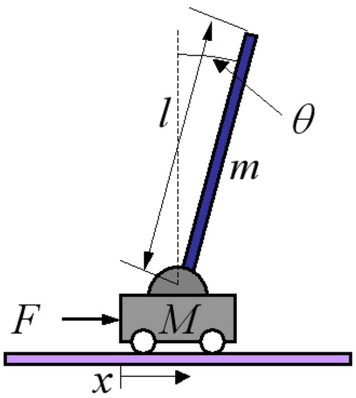


Fig. 1 Inverted Pendulum

In the above figure, let $M=10$, $m=1$, $l=2$ and acceleration of gravity $g=9.8$, then our linearized model is as follows:

$$\dot{x}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & -9.33 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 5.13 & 0\end{bmatrix}x+\begin{bmatrix}0\\ 0.095\\ 0\\ -0.024\end{bmatrix}u \quad (1)$$

$$y=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix} \quad (2)$$

This system has a transmission zero at 2.213 in complex plane. In order to improve the tracking performance, an integral action of the error signal is added to a feedback control system as follow[2]:

$$\dot{e}=r-y \quad (3)$$

Then the structure of the control system can be described by Fig. 2.

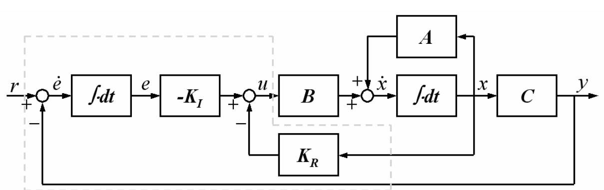


Fig. 2 LQ Tracking Control Scheme with Integration

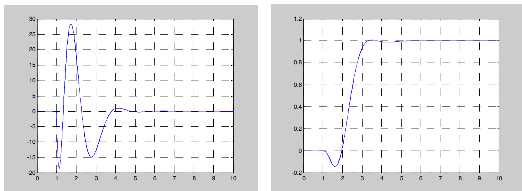
If we take a quadratic cost function of these control scheme for the tracking performance

$$J=\int_0^{t_f}\begin{bmatrix}x^T & e^T\end{bmatrix}\begin{bmatrix}Q_x & 0\\ 0 & Q_e\end{bmatrix}\begin{bmatrix}x\\ e\end{bmatrix}+u^T Ru\, dt \quad (4)$$

then it is a typical LQR problem, and we can easily compute the control gain. The detailed structure can be described as follows:

$$u=-K\begin{bmatrix}x\\ e\end{bmatrix}=-K_R x-K_I e. \quad (5)$$

In the case of $Q_e=I$, $R=10^{-5}$ and Q_x must to be the zero matrix for the tracking performance, the control result is shown in Fig. 3.



(a) Input (b) Output

Fig. 3 The Result of the Control

Where the unit is normalized. This system shows an undershoot, and it cannot be eliminated by the feedback control, which is widely known in SISO system. However, in MIMO systems some nonminimum phase effects can be eliminated and thus can avoid the undershoot phenomenon. Next section is to deal with the property of the nonminimum phase multivariable system using definition of the row and column blocking zero.

3. BLOCKING AND TRANSMISSION ZEROS

In linear multivariable systems, the eigenvalues of the system matrix are to be poles, which is exactly same as in SISO systems. But the definition of zeros is not unique, but there are several definitions of zeros, for example, transmission zero, invariant zero, decoupling zero, system zero, blocking zero, fixed zero, and so on. Below definition is of the blocking zero by Ferreira and Bhattacharyya[10].

Definition 1 (Blocking zero)[10]:

The unique monic polynomial $\beta(s)$ which is the greatest common divisor(gcd) of the numerators of the elements of $G(s)$ is called the *blocking polynomial* of $G(s)$. The roots of

$\beta(s) = 0$, counting multiplicities, are called the *blocking zeros* of $G(s)$.

In other words, it is said to be the blocking zero that the complex value makes the transfer function matrix a zero matrix. Therefore, all outputs are to be blocked at the frequency of the blocking zero. In many cases of the control problems, all outputs do not require the perfect tracking performance in the multivariable system. For example, one output requires the tracking performance and another requires only the stability. Sometimes the next definition is effective.

Definition 2 (Row and Column Blocking Zero):

Let a transfer function matrix $G(s) \in R^{m \times n}(s)$ be given. It is said to be the *i*th *row blocking zero* that makes the *i*th row vector of $G(s)$ the zero row vector, where $1 \le i \le m$. And similarly, *j*th *column blocking zero* makes the jth column vector of $G(s)$ the zero column vector, where $1 \le j \le n$.

Assume that a transfer function matrix $G(s) \in R^{m \times n}(s)$ has not any pole-zero cancelation in the right half plane. If the system has an odd number of row blocking zeros in the right half plane, then any controller cannot be designed to eliminate the undershoot phenomenon. It must have a type A undershoot[11]. However, if the ith row of the transfer function matrix has no row blocking zero in the right half plane, then the undershoot of the ith output can be avoided even though there are other unstable blocking zeros in the system.

This property can be explained by the following block diagrams. If the subsystems $G_{1j}(s)$, $1 \le j \le n$ have any common elements, then the block diagram can be redrawn as Fig. 6 by using $G_1^{'}(s)$, where $G_1^{'}(s)$ is the common element of $G_{1j}(s)$, $1 \le j \le n$. Note that the zeros of $G_1^{'}(s)$ is the blocking zeros.

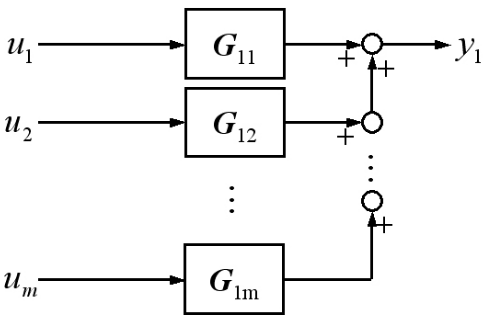


Fig. 4 One output of a MIMO system

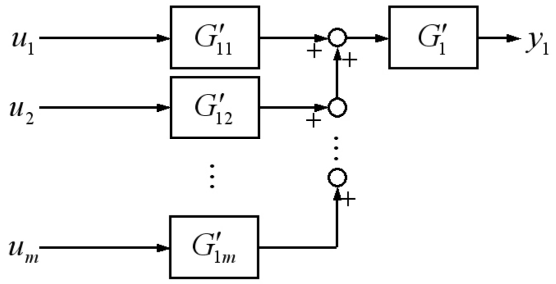


Fig. 5 Tied block of the common elements

If $G_1^{'}(s)$ has nonminimum phase zeros in $G_1^{'}(s)$, it is impossible to avoid the undershoot phenomenon, which can be easily checked by the initial value theorem in the odd number case of the nonminimum phase zeros.

Definition 3 (Transmission zero)[8]:

Let $G(s) \in R^{m \times n}(s)$ be a rational transfer-function matrix with Smith-McMillan form $M(s)$, and define the zero polynomial $z(s) = \varepsilon_1(s) \cdots \varepsilon_r(s)$. Then roots of $z(s)$ are called the *transmission zeros* of $G(s)$.

It is noted that the blocking zero is defined as zeros of the first factor $\varepsilon_1(s)$ of the zero polynomial $G(s)$. If an MIMO system has no blocking zero among transmission zeros in the right half plane, then it is possible to avoid the undershoot problem by a linear feedback, which will be shown via some examples in the next section.

4. UNDERSHOOT PHENOMENA IN MIMO SYSTEMS WITH UNSTABLE ZEROS

Following examples is an case of the tracking control problem having nonminimum phase transmission zero.

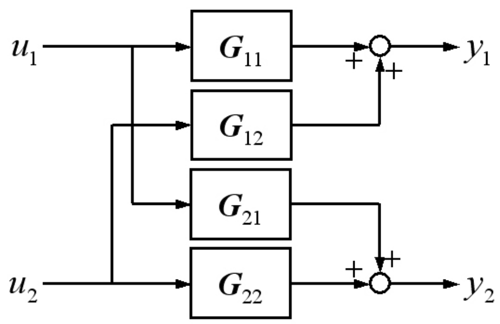


Fig. 6 2×2 multivariable system

Example 2 (Transmission zero case):

Let us consider a 2×2 MIMO system as shown in Fig. 6. The constructed LQ outputs and states feedback control system is shown Fig. 2, where sizes of A, B, C, K_I, K_R, x, r, y, u and e are each 8×8, 8×2, 2×8, 2×2, 2×8, 2, 2, 2 and 2, respectively. n Fig. 6, it is assumed that G_{11} is the linearized model of the inverted pendulum in Section 2 and that other subsystems are defined as follows:

$$G_{12} = \frac{1}{s+1}$$

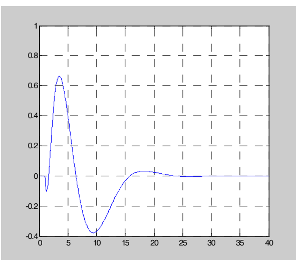
$$G_{21} = \frac{0.1}{s+1}$$

$$G_{22} = \frac{-0.1s+1}{s^2+2s+1}$$

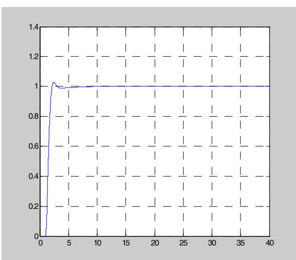
Unstable transmission zeros of the system are each 2.274 and 0.905. The control result of the system using LQ control in Section 3 is as shown in Fig. 7. Design parameters are used as follows:

$$Q_e = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = 1$$

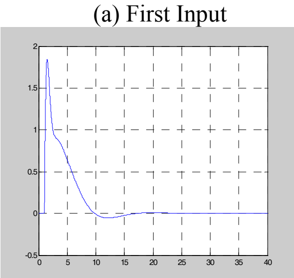
Where Q_x is a 8×8 zero matrix and Q is a block diagonal matrix of the Q_x and Q_e.



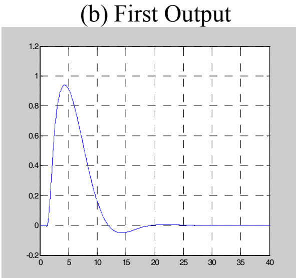
(a) First Input



(b) First Output



(c) Second Input



(d) Second Output

Fig. 7 Control Result including Nonminimum Phase Transmission Zeros

The overall system containing controller has still nonminimum phase transmission zeros. Nevertheless, first output graph has not undershoot phenomenon, that is, its unstable row blocking zeros in the plant are avoided. Where each elements of the overall transfer function $F(s)$ and the control gains are as follows:

$$F_{11}(s) = \frac{-1.066\times10^{-14}s^9 + 9.964s^8 + 84.3s^7 + 290.6s^6 + 533.7s^5 + 577.4s^4 + 385.1s^3 + 160.7s^2 + 40.21s + 4.667}{s^{10} + 13.05s^9 + 78.04s^8 + 272.4s^7 + 596.9s^6 + 843.1s^5 + 776.3s^4 + 466.7s^3 + 181s^2 + 42.56s + 4.667}$$

$$F_{12}(s) = \frac{-1.421\times10^{-14}s^9 + 0.08518s^8 + 0.777s^7 + 2.844s^6 + 5.465s^5 + 6.067s^4 + 3.982s^3 + 1.47s^2 + 0.2421s - 3.819\times10^{-14}}{s^{10} + 13.05s^9 + 78.04s^8 + 272.4s^7 + 596.9s^6 + 843.1s^5 + 776.3s^4 + 466.7s^3 + 181s^2 + 42.56s + 4.667}$$

$$F_{21}(s) = \frac{-1.421\times10^{-14}s^9 - 1.082s^8 + 1.922s^7 + 51.23s^6 + 185.6s^5 + 302s^4 + 360.2s^3 + 119.8s^2 + 24.21s - 1.776\times10^{-14}}{s^{10} + 13.05s^9 + 78.04s^8 + 272.4s^7 + 596.9s^6 + 843.1s^5 + 776.3s^4 + 466.7s^3 + 181s^2 + 42.56s + 4.667}$$

$$F_{22}(s) = \frac{-1.421\times10^{-14}s^9 + 0.09112s^8 + 0.6814s^7 + 1.942s^6 + 0.4275s^5 - 7.8s^4 - 10.08s^3 + 3.352s^2 + 11.22s + 4.667}{s^{10} + 13.05s^9 + 78.04s^8 + 272.4s^7 + 596.9s^6 + 843.1s^5 + 776.3s^4 + 466.7s^3 + 181s^2 + 42.56s + 4.667}$$

$$K_R = \begin{bmatrix} -5.5991 & -15.9760 & -649.7580 & -289.2693 & -0.9459 & 0.0997 & 0.9849 & 2.0821 \\ 4.5930 & 1.2661 & 8.1201 & 3.6599 & 3.5624 & 0.0056 & 0.0238 & 0.0858 \end{bmatrix}$$

$$K_I = \begin{bmatrix} 0.8518 & -0.9964 \\ -9.9637 & -0.0852 \end{bmatrix}$$

Example 3 (Row blocking zero case):

Assume that the transfer function matrix of the MIMO system of Fig. 6 is given as follows:

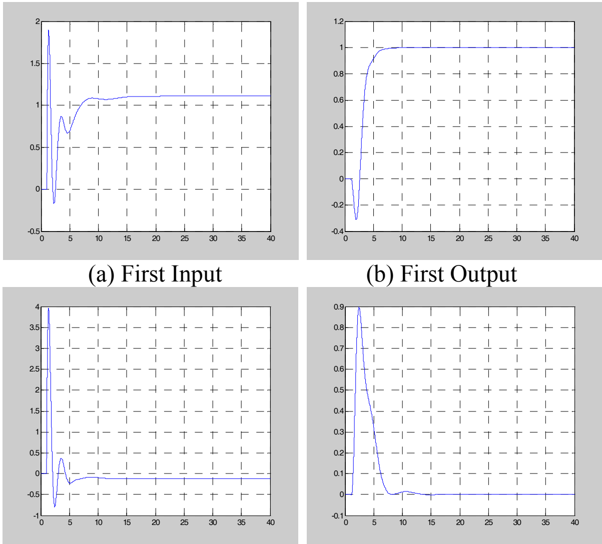
$$G_{11}(s) = \frac{-s+1}{4s^3+3s^2+2s+1}$$

$$G_{12}(s) = \frac{-s+1}{2s^3+3s^2+4s+1}$$

$$G_{21}(s) = \frac{0.1}{s+1}$$

$$G_{22}(s) = \frac{-0.1s+1}{s^2+2s+1}$$

This system has two unstable transmission zeros at 3.31 and 1. Moreover, there is an unstable blocking zero at 1. Following figure is the control result.



(a) First Input (b) First Output

(c) Second Input (d) Second Output

Fig. 8 Simulation result with the blocking zero

We can check the undershoot in this result. This system may have always undershoot using any design parameters, if tracking performance is kept. Where each elements of the overall transfer function $F(s)$ and the control gains are follows:

$$F_{11}(s)=\frac{\begin{array}{c}1.776\times10^{-15}s^{10}+4.263\times10^{-14}s^{9}-17.67s^{8}-66.29s^{7}-95.2s^{6}\\-49.45s^{5}+39.08s^{4}+89.29s^{3}+70.24s^{2}+26.45s+3.558\end{array}}{\begin{array}{c}s^{11}+10.67s^{10}+56.49s^{9}+190.6s^{8}+437s^{7}+703.1s^{6}\\+810.7s^{5}+674.5s^{4}+397.4s^{3}+156.5s^{2}+36.24s+3.558\end{array}}$$

$$F_{12}(s)=\frac{\begin{array}{c}3.553\times10^{-14}s^{9}-0.01367s^{8}-0.06637s^{7}-0.1084s^{6}\\-0.04262s^{5}+0.07037s^{4}+0.1004s^{3}+0.05166s^{2}+0.00862s+3.695\times10^{-13}\end{array}}{\begin{array}{c}s^{11}+10.67s^{10}+56.49s^{9}+190.6s^{8}+437s^{7}+703.1s^{6}\\+810.7s^{5}+674.5s^{4}+397.4s^{3}+156.5s^{2}+36.24s+3.558\end{array}}$$

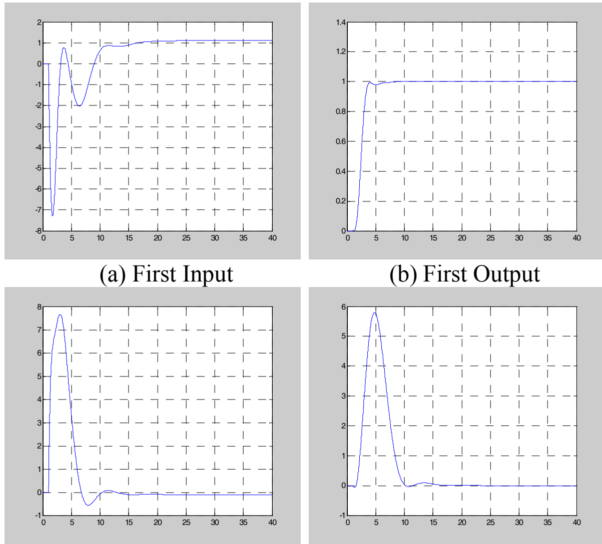
$$F_{21}(s)=\frac{\begin{array}{c}3.553\times10^{-15}s^{10}-1.31s^{9}+23.05s^{8}+108.7s^{7}+252.7s^{6}\\+361.8s^{5}+332.4s^{4}+196.1s^{3}+65.81s^{2}+8.62s-2.58\times10^{-13}\end{array}}{\begin{array}{c}s^{11}+10.67s^{10}+56.49s^{9}+190.6s^{8}+437s^{7}+703.1s^{6}\\+810.7s^{5}+674.5s^{4}+397.4s^{3}+156.5s^{2}+36.24s+3.558\end{array}}$$

$$F_{22}(s)=\frac{\begin{array}{c}-0.1352s^{9}-0.6878s^{8}-1.12s^{7}+3.112s^{6}\\+21.96s^{5}+52.78s^{4}+71.42s^{3}+57.4s^{2}+24.02s+3.558\end{array}}{\begin{array}{c}s^{11}+10.67s^{10}+56.49s^{9}+190.6s^{8}+437s^{7}+703.1s^{6}\\+810.7s^{5}+674.5s^{4}+397.4s^{3}+156.5s^{2}+36.24s+3.558\end{array}}$$

$$K_R=\begin{bmatrix}1.9392 & 4.5964 & 10.4334 & 2.3730 & 7.4502 & 20.7459 & -0.0830 & -0.6913 & -1.6043\\1.6390 & 6.5778 & 18.9629 & 3.2142 & 12.5006 & 37.7334 & 0.0427 & 0.3469 & 0.8164\end{bmatrix}$$

$$K_I=\begin{bmatrix}-14.8295 & 0.8832\\-27.9300 & -0.4689\end{bmatrix}$$

Now, let us assume the nonminimum phase zero of the $G_{12}(s)$ is changed as 0.5. Then the result is as following figure.



(a) First Input (b) First Output

(c) Second Input (d) Second Output

Fig. 9 Simulation result excluding the undershoot

Where each elements of the overall transfer function $F(s)$ and the control gains are as follows:

$$F_{11}(s)=\frac{\begin{array}{c}-1.421\times10^{-14}s^{10}-1.492\times10^{-13}s^{9}+0.03267s^{8}+17.05s^{7}+83.08s^{6}\\+182.1s^{5}+236.4s^{4}+196.9s^{3}+103.5s^{2}+30.48s+3.558\end{array}}{\begin{array}{c}s^{11}+10.33s^{10}+52.94s^{9}+174s^{8}+395.5s^{7}+640.8s^{6}\\+750.9s^{5}+636.4s^{4}+381.8s^{3}+152.9s^{2}+35.88s+3.558\end{array}}$$

$$F_{12}(s)=\frac{\begin{array}{c}-1.776\times10^{-15}s^{10}-4.263\times10^{-14}s^{9}+0.3536s^{8}1.703s^{7}3.726s^{6}\\+4.914s^{5}+4.213s^{4}+2.299s^{3}+0.7095s^{2}+0.08669s+4.516\times10^{-13}\end{array}}{\begin{array}{c}s^{11}+10.33s^{10}+52.94s^{9}+174s^{8}+395.5s^{7}+640.8s^{6}\\+750.9s^{5}+636.4s^{4}+381.8s^{3}+152.9s^{2}+35.88s+3.558\end{array}}$$

$$F_{21}(s)=\frac{\begin{array}{c}-1.776\times10^{-15}s^{10}-4.472s^{9}-7.382s^{8}+19.7s^{7}+172.4s^{6}\\+557.1s^{5}+922.4s^{4}+930.3s^{3}+501.9s^{2}+86.69s+5.911\times10^{-13}\end{array}}{\begin{array}{c}s^{11}+10.33s^{10}+52.94s^{9}+174s^{8}+395.5s^{7}+640.8s^{6}\\+750.9s^{5}+636.4s^{4}+381.8s^{3}+152.9s^{2}+35.88s+3.558\end{array}}$$

$$F_{22}(s)=\frac{\begin{array}{c}-5.329\times10^{-15}s^{10}+0.0004133s^{9}-0.867s^{8}-4.606s^{7}-11.72s^{6}\\-15.35s^{5}-2.854s^{4}+19.06s^{3}+30.44s^{2}+19.44s+3.558\end{array}}{\begin{array}{c}s^{11}+10.33s^{10}+52.94s^{9}+174s^{8}+395.5s^{7}+640.8s^{6}\\+750.9s^{5}+636.4s^{4}+381.8s^{3}+152.9s^{2}+35.88s+3.558\end{array}}$$

$$K_R=\begin{bmatrix}1.9712 & 3.3525 & -4.5263 & 0.9232 & 0.5678 & -9.1458 & -0.0675 & -0.5799 & -1.3223\\0.4155 & 2.9624 & 12.713 & 3.8737 & 10.9362 & 23.9172 & -0.0723 & -0.6985 & -1.4937\end{bmatrix}$$

$$K_I=\begin{bmatrix}22.4259 & 0.7050\\-22.2952 & 0.7092\end{bmatrix}$$

Even though the control effort looks very high, the result shows that the undershoot phenomenon could be avoided.

5. CONCLUSIONS AND FURTHER WORKS

Sometimes, the control system designers want to have neither undershoot nor overshoot in the control system designed. The undershoot problems are considerably investigated in SISO systems, and it is known to be unavoidable by any linear feedback control. This paper deals with the undershoot compensation problem in MIMO systems and proposes some examples to avoid the undershoot problem by a linear feedback, which says that it is possible to solve the problem in MIMO systems. However, it requires further works to get a rigorous result via some complete analyses.

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