Critical Control Systems Design via LTR Technique

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Abstract: A new method for designing critical control systems is proposed in this paper. The controller structure is chosen as a Davison type integral controller with an observer. The proposed method consists of two steps. First, the state feedback critical control system is designed using a quadratic performance index with tunable parameters. Second, the observer gain matrix is determined by the formal LTR procedure using a Riccati equation. Consequently, the search space can be reduced considerably compared with the conventional approach. Although the proposed method sacrifices a large freedom for the choice of controller structure provided by the principle of matching, the controller structure used in this paper is not excessively complex and can be used for most applications. An illustrative design example is presented.

Keywords: Critical control systems, The principle of matching, Davison type integral controllers, Loop transfer recovery

1. INTRODUCTION

A control problem is called *critical* if the controlled variables are required to be less than prescribed values for all time. As a systematic method for designing critical control systems, Zakian [13, 14] has proposed the principle of matching which requires modeling of exogenous inputs. For several classes of exogenous inputs, the matching conditions which ensure that a control system satisfies a critical condition have been derived. The conditions are expressed in terms of inequality conditions on the norm of closed loop responses. The design of a critical control system is reduced to find a controller satisfying the matching condition: First, the designer chooses an appropriate controller structure with tuning parameters. Then parameters satisfying the matching conditions are found by a numerical search algorithm such as the moving boundary process [6, 11], the genetic algorithm [4] and the simulated annealing [10].

Although a large freedom exists in the choice of controller structure, it is not clear *a priori* whether controller parameters satisfying the matching conditions exist for a given controller structure. In addition, the search often requires much time to find controller parameters satisfying the matching conditions because the convexity of the objective functions can not be guaranteed. Although an application of LMI to a class of critical control problems has recently been reported [8], it seems difficult to deal with more general class of the problems.

To alleviate the above difficulties of the conventional approach, a new systematic design method is proposed in this paper. Since the principle of matching [14] suggests that a controller with integral action is preferable for most applications, the Davison type integral controller [2] with the observer is adopted as a basic controller structure. The controller parameters are determined by the classical LQ method with the loop transfer recovery (LTR) technique [1, 7]. First, the state feedback controller is determined to satisfy the matching conditions using a weighting matrix of the quadratic performance index as tuning parameters. The observer gain matrix is determined by the formal LTR procedure recovering the closed loop properties of the state feedback design in the output feedback with the observer. In the proposed method, the parameter search required in each design step is simpler than that required in the conventional design.

This paper is organized as follows: In Section 2, the problem is formulated. The proposed design method is described in detail in Section 3. A design example is presented in Section 4. Concluding remarks are given in Section 5.

2. PROBLEM FORMULATION

2.1 Davison type integral controller

Consider a plant described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B\left[u(t) + d(t)\right], \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ is a state vector, $u(t) \in R^m$ is a control input, $y(t) \in R^m$ is an output vector and $d(t) \in R^m$ is a disturbance vector. It is assumed that the matrix pair (A,B) is controllable and that (C, A) is observable. In addition, the transfer function matrix $G(s) \triangleq C(sI - A)^{-1}B$ is minimum phase.

To construct the Davison type integral controller [2] shown in Fig. 1 for the plant given (1), m integrators are augmented to the plant output. Then the extended system is described by

$$\begin{aligned} \dot{\xi}(t) &= \Phi\xi(t) + \Gamma\left[u(t) + d(t)\right], \\ \eta(t) &= H\xi(t), \end{aligned} \tag{2}$$

where $\eta(t) \in R^m$ is a state vector (the output) of the integrators, and

$$\begin{aligned} &\xi'(t) \triangleq \begin{bmatrix} x'(t) & \eta'(t) \end{bmatrix}', \\ &\Phi \triangleq \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \ \Gamma \triangleq \begin{bmatrix} B \\ 0 \end{bmatrix}, \ H \triangleq \begin{bmatrix} 0 & I \end{bmatrix}. \end{aligned} \tag{3}$$

Define the quadratic performance index as

$$J \triangleq \int_0^\infty \left[\eta'(t)Q\eta(t) + u'(t)Ru(t)\right]dt \ . \tag{4}$$

The optimal state feedback regulator minimizing (4) is given by

$$u(t) = -F\xi(t) \ , \tag{5}$$

where F is the optimal feedback gain matrix given by

$$F \triangleq R^{-1}\Gamma'\Pi, \tag{6}$$

with Π being a positive definite solution of the Riccati equation

$$\Phi'\Pi + \Pi\Phi - \Pi\Gamma R^{-1}\Gamma'\Pi + H'QH = 0 \ . \tag{7}$$

The Davison type integral controller is obtained from the optimal regulator (5) for the extended system (3) by using a simple trick: The state vector $x(t)$ in $\xi(t)$ is replaced by the estimate $\hat{x}(t)$ generated by the observer and the reference

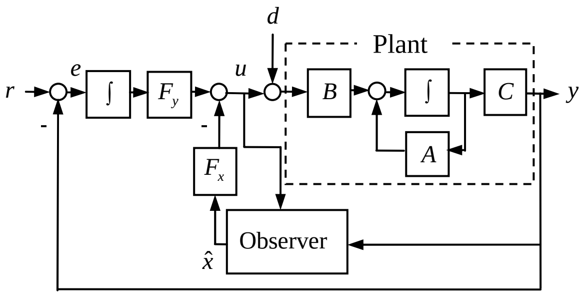


Fig. 1 Structure of Davison type integral controller.

input is inserted to the output side of the plant. Then the controller dynamics can be expressed as

$$u(t) = -F_x\hat{x}(t) + F_y\int_0^t\left[r(\sigma) - y(\sigma)\right]d\sigma,$$
$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K\left[y(t) - C\hat{x}(t)\right], \tag{8}$$

where $r(t)\in R^m$ is a reference input, K is an observer gain matrix and the matrices F_x and F_y are the partition matrices of F:

$$F = \begin{bmatrix} F_x & F_y \end{bmatrix}. \tag{9}$$

2.2 Critical control problem

A critical control problem for the plant (1) with the controller (8) is considered. To define the problem, it is necessary to model the exogenous inputs and to give the design specifications regarding the responses of interest. Although the problem can be formulated for the multivariable case ($m\geq 2$) without fundamental difficulty, it requires complicated notations. For convenience of presentation, the following discussions assume that $m=1$

The reference input $r(t)$ and the disturbance $d(t)$ are considered as the exogenous inputs of interest. For simplicity, the reference input $r(t)$ is assumed to be a rate limited signal with the known rate, *i.e.*,

$$r\in\tilde{F}_\infty(D) \triangleq \left\{ f : \|\dot{f}\|_\infty \leq D,\ f(0)=0 \right\}, \tag{10}$$

while the disturbance $d(t)$ is assumed to be a magnitude limited signal with the known bound, *i.e.*,

$$d\in F_\infty(M) \triangleq \left\{ f : \|f\|_\infty \leq M,\ f(0)=0 \right\}. \tag{11}$$

The responses of interest are assumed to be the tracking error

$$e(t) \triangleq r(t) - y(t), \tag{12}$$

and the control input $u(t)$. The design specifications are given by

$$\hat{\varepsilon}_1(F,K) \triangleq \sup_{r,d}\|e\|_\infty \leq \varepsilon_1,\quad \hat{\varepsilon}_2(F,K) \triangleq \sup_{r,d}\|u\|_\infty \leq \varepsilon_2, \tag{13}$$

where ε_1 and ε_2 are admissible bounds specified by the designer. The inequalities in (13) should be satisfied for all possible exogenous inputs defined in (10) and (11).

It is known that the design specification (13) can be replaced by the practical matching conditions [13, 14]

$$\hat{\varepsilon}_1(F,K) = D\|g_{er}(h,F)\|_1 + M\|g_{ed}(\delta,F,K)\|_1 \leq \varepsilon_1, \tag{14}$$

$$\hat{\varepsilon}_2(F,K) = D\|g_{ur}(h,F)\|_1 + M\|g_{ud}(\delta,F,K)\|_1 \leq \varepsilon_2, \tag{15}$$

where $g_{er}(h,F)$ denotes the unit step response of the transfer function from r to e, $g_{ed}(\delta,F,K)$ denote the unit impulse response of the transfer function from d to e ; the meanings of the notations $g_{ur}(h,F)$ and $g_{ud}(\delta,F,K)$ are obvious. It should be noted that the expressions related to the responses to the reference input r do not include the observer gain matrix K since it can easily be shown that they are really independent of K.

The design of the critical control system is reduced to find controller parameters F and K which satisfy the set of the inequalities (14) and (15). It might be possible to find all the parameters at once by an appropriate search method such as the moving boundary process [6, 11]. However, this approach requires $2n+1$ dimensional search space for which it is difficult to assume simple topography for the set of solutions for the inequalities (14) and (15).

To alleviate the search problem in the standard design of critical control systems, a new design method is proposed. The proposed method is a two-step design procedure. The fist step is to find a state feedback controller satisfying the design specifications (14) and (15). For the state feedback design, it is proposed to use a weighting coefficient of the quadratic performance index as a tuning parameter. The second step is to determine the observer gain matrix K by using the well known asymptotic property of the optimal state estimator (Kalman filter) [1, 3].

3. NEW DESIGN METHOD

3.1 State feedback design

For the state feedback case, the integral controller is given by

$$u(t) = -F_x x(t) - F_y\eta(t),$$
$$\dot{\eta}(t) = y(t) - r(t). \tag{16}$$

The structure of the controller is shown in Fig. 2. It follows from (1) and (16) that

$$\dot{\xi}(t) = \Phi_F\xi(t) + \Gamma_r r(t) + \Gamma_d d(t),$$
$$y(t) = \Theta\xi(t), \tag{17}$$

where

$$\Phi_F \triangleq \begin{bmatrix} A-BF_x & -BF_y \\ C & 0 \end{bmatrix},$$
$$\Gamma_r \triangleq \begin{bmatrix} 0 \\ -1 \end{bmatrix},\quad \Gamma_d \triangleq \begin{bmatrix} B \\ 0 \end{bmatrix},\quad \Theta \triangleq \begin{bmatrix} C & 0 \end{bmatrix}. \tag{18}$$

From (17), the explicit representations of the transfer functions related to the design specification (14) and (15) can easily be obtained as

$$G^*_{er}(s) = 1 - \Theta(sI-\Phi_F)^{-1}\Gamma_r$$
$$= 1 - s^{-1}G(s)\left[I + (F_x + s^{-1}F_yC)(sI-A)^{-1}B\right]^{-1}F_y, \tag{19}$$

$$G^*_{ed}(s) = -\Theta(sI-\Phi_F)^{-1}\Gamma_d$$
$$= -G(s)\left[1 + (F_x + s^{-1}F_yC)(sI-A)^{-1}B\right]^{-1}, \tag{20}$$

$$G^*_{ur}(s) = -F(sI-\Phi_F)^{-1}\Gamma_r$$
$$= s^{-1}\left[1 + (F_x + s^{-1}F_yC)(sI-A)^{-1}B\right]^{-1}F_y, \tag{21}$$

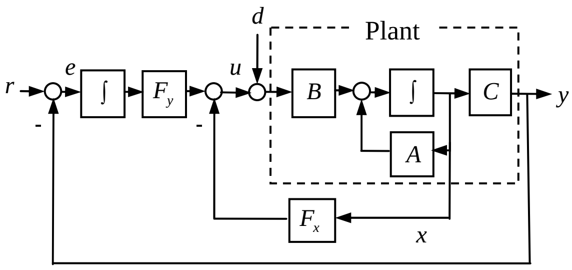


Fig. 2 Structure of the state feedback controller.

$$G_{ud}^{*}(s) = -F(sI-\Phi_F)^{-1}\Gamma_d$$
$$= -\left[1+(F_x+s^{-1}F_yC)(sI-A)^{-1}B\right]^{-1} \quad (22)$$
$$(F_x+s^{-1}F_yC)(sI-A)^{-1}B,$$

where the meaning of the suffixes are obvious and the asterisks are used to indicate that they are transfer functions related to the state feedback design. Note that $G(s)$ in (19) and (20) is the plant transfer function $G(s)=C(sI-A)^{-1}B$.

For the transfer functions (19)-(22), the following result is essential for the state feedback design of the critical control system.

Property 1: Consider the four transfer functions defined in (19)-(22). The transfer functions $G_{er}^{*}(s)$ and $G_{ed}^{*}(s)$ have a zero at $s=0$. The transfer function $G_{ur}^{*}(s)$ has a zero at $s=0$ if the plant transfer function $G(s)=C(sI-A)^{-1}B$ has a pole at $s=0$. The transfer function $G_{ud}^{*}(s)$ has no zero at $s=0$.
Proof: Omitted. □

Choose $Q=q$ and $R=1$ in the quadratic performance index (4). A state feedback matrix F satisfying the state feedback versions of the practical matching conditions (14) and (15) are found by a numerical search using the weight q as a tuning parameter. This can reduce the number of tuning parameters compared with the direct search of the feedback gain matrix F. Another important advantage is that the stability and sufficient stability margins are always guaranteed for any choice of $q>0$.

The state feedback version of the practical matching conditions (14) and (15) can be written as

$$\hat{\varepsilon}_1^{*}(q) = D\left\| g_{er}^{*}(h,q)\right\|_1 + M\left\| g_{ed}^{*}(\delta,q)\right\|_1 \le \varepsilon_1, \quad (23)$$

$$\hat{\varepsilon}_2^{*}(q) = D\left\| g_{ur}^{*}(h,q)\right\|_1 + M\left\| g_{ud}^{*}(\delta,q)\right\|_1 \le \varepsilon_2, \quad (24)$$

where the tuning parameter q is explicitly included to emphasize the dependence.

3.2 Asymptotic properties of state feedback responses

It is useful to clarify the asymptotic behavior of the responses of interest when the tuning parameter q tends to infinity. From the well-know result for the standard optimal regulators, [1, 3] the asymptotic behavior of the optimal feedback gain matrix F can easily be identified as follows.

Property 2: Consider the quadratic performance index (4) with $Q=q$ and $R=1$. Denote the corresponding optimal feedback gain matrix as

$$F(q) = \left[\, F_x(q) \quad F_y(q)\,\right] \quad (25)$$

to emphasize the dependence on q. Then the sub-matrices have the asymptotic properties

$$\lim_{q\to\infty} q^{-1/2}F_x(q)=0, \quad \lim_{q\to\infty} q^{-1/2}F_y(q)=I. \quad (26)$$

Proof: Omitted. □

The asymptotic behavior of the transfer functions related to the responses of interest can be obtained by applying the above results.

Property 3: Consider the transfer functions defined in (19)-(22) with the additional inclusion of the tuning parameter q to denote the dependence. These transfer functions have the following asymptotic properties:

$$\lim_{q\to\infty} G_{er}^{*}(s,q)=0 \quad (27)$$

$$\lim_{q\to\infty} G_{ed}^{*}(s,q)=0 \quad (28)$$

$$\lim_{q\to\infty} G_{ur}^{*}(s,q)=G^{-1}(s) \quad (29)$$

$$\lim_{q\to\infty} G_{ud}^{*}(s,q)=-1 \quad (30)$$

Proof: The results follow by using the asymptotic properties (26) in the expressions (19)-(22). □

From the above results, it turns out that the L_1 norms of the time responses in the practical matching conditions (23) and (24) have the following asymptotic properties:

$$\lim_{q\to\infty} \left\| g_{er}^{*}(h,q)\right\|_1 = 0 \quad (31)$$

$$\lim_{q\to\infty} \left\| g_{ed}^{*}(\delta,q)\right\|_1 = 0 \quad (32)$$

$$\lim_{q\to\infty} \left\| g_{ur}^{*}(h,q)\right\|_1 = \infty \quad (33)$$

$$\lim_{q\to\infty} \left\| g_{ud}^{*}(\delta,q)\right\|_1 = \infty \quad (34)$$

These results readily provide the following result.

Property 4: The left sides of the practical matching conditions (23) and (24) satisfy the asymptotic properties

$$\lim_{q\to\infty} \hat{\varepsilon}_1^{*}(q)=0, \quad \lim_{q\to\infty} \hat{\varepsilon}_2^{*}(q)=\infty, \quad (35)$$

respectively. △

The above property implies that there is a fundamental limitation on the choice of the admissible bounds ε_1 and ε_2.

3.3 Computation of L_1 norms

Unlike H_2 and H_∞ norms, a simple method for computing L_1 norms has not been discovered. Rutland and Lane [5] has proposed a method for computing L_1 norms of the impulse response using a state space description. Their method seems to be best currently available.

The expressions in (20) and (22) suggest that the L_1 norms of the impulse responses $g_{ed}^{*}(\delta,q)$ and $g_{ud}^{*}(\delta,q)$ in the practical matching conditions (23) and (24) can be computed by the method of Rutland and Lane using the state space representations $(\Phi_F, \Gamma_d, -\Theta)$ and $(\Phi_F, \Gamma_d, -F)$, respectively.

To compute the L_1 norms of the step responses $g_{er}^{*}(h,q)$ and $g_{ur}^{*}(h,q)$ required in (23) and (24), the state space representations given by (19) and (21) can not directly be used with the method of Rutland and Lane. The following results provide simple modifications to overcome this difficulty.

Property 5: The step response $g_{er}^*(h,q)$ can be computed as the impulse response of the realization $(\Phi_F, \Gamma_{er}, \Theta)$ where Γ_{er} is a vector satisfying the relation

$$\begin{bmatrix} \Phi_F & \Gamma_r \\ -\Theta & 1 \end{bmatrix} \begin{bmatrix} \Gamma_{er} \\ 1 \end{bmatrix} = 0 . \tag{36}$$

In addition, if the plant transfer function $G(s) = C(sI - A)^{-1}B$ has a pole at $s = 0$, the step response $g_{ur}^*(h,q)$ can be computed as the impulse response of the realization $(\Phi_F, \Gamma_{ur}, \Theta)$ where Γ_{ur} is a vector satisfying the relation

$$\begin{bmatrix} \Phi_F & \Gamma_r \\ -F & 0 \end{bmatrix} \begin{bmatrix} \Gamma_{ur} \\ 1 \end{bmatrix} = 0 . \tag{37}$$

Proof: As shown in Property 1 the transfer function $G_{er}^*(s)$ defined in (19) has a zero at $s = 0$. Then, by the definition of the zero [1], there exists a non-zero vector Γ_{er} satisfying the relation (36) which can be rewritten as

$$\begin{aligned} &\Phi_F \Gamma_{er} + \Gamma_r = 0, \\ &\Theta \Gamma_{er} = 1. \end{aligned} \tag{38}$$

It follows from (19) and (38) that

$$\begin{aligned} G_{er}^*(s) &= 1 - \Theta(sI - \Phi_F)^{-1}\Gamma_r \\ &= \Theta\left[I + (sI - \Phi_F)^{-1}\Phi_F \right]\Gamma_{er} \\ &= s\Theta(sI - \Phi_F)^{-1}\Gamma_{er} . \end{aligned} \tag{39}$$

Consequently, the step response $g_{er}^*(h,q)$ can be obtained as the impulse response of $\Theta(sI - \Phi_F)^{-1}\Gamma_{er}$. The result for the step response $g_{ur}^*(h,q)$ can be proved in a similar way. □

The above results make it possible to compute the step responses by the method for computing L_1 norm of the impulse responses proposed by Rutland and Lane [5].

3.4 Application of LTR procedure

Once the parameter q satisfying the practical matching conditions (23) and (24) for the state feedback design is found, the next step is to determine the observer gain matrix K which satisfies the practical matching conditions (14) and (15) for the output feedback case. The well known LTR procedure using the asymptotic property of the Kalman filter [1, 3, 7] is used for this purpose.

Consider the Kalman filter as an observer for the plant (1). For a disturbance covariance matrix W and an observation noise covariance matrix V, the Kalman filter gain matrix is given by

$$K \triangleq P_f C' V^{-1} , \tag{40}$$

where P_f is a non-negative definite solution of the Riccati equation

$$AP_f + P_f A' - P_f C' V^{-1} C P_f + W = 0 . \tag{41}$$

The special choice of the covariance matrices

$$V = I, \quad W = \sigma BB' \tag{42}$$

where σ is a positive scalar parameter, is usually used. Let $K(\sigma)$ denote the optimal filter gain matrix (40) corresponding to the covariance matrices (42). It is well-known that, for sufficiently large σ, $K(\sigma)$ can be expressed simply as

$$K(\sigma) \approx \sigma^{1/2} B . \tag{43}$$

The parameter σ is chosen as the tuning parameter to determine the observer gain matrix K which is used in the output feedback controller. Rewrite the practical matching conditions (14) and (15) for the output case as

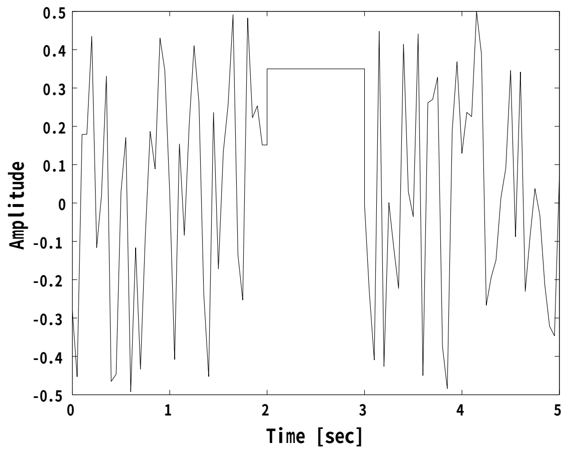


Fig. 3 Test disturbance signal

$$\hat{\varepsilon}_1(q,\sigma) = D \left\| g_{er}^*(h,q) \right\|_1 + M \left\| g_{ed}(\delta,q,\sigma) \right\|_1 \le \varepsilon_1 , \tag{44}$$

$$\hat{\varepsilon}_2(q,\sigma) = D \left\| g_{ur}^*(h,q) \right\|_1 + M \left\| g_{ud}(\delta,q,\sigma) \right\|_1 \le \varepsilon_2 . \tag{45}$$

Note that that the responses related to the reference input $r(t)$ are independent of the parameter σ and already fixed by q found in the state feedback design step.

The asymptotic behavior of the responses related to the disturbance $d(t)$ is given as follows.

Property 6: Assume that the observer gain matrix K is determined as the Kalman filter gain matrix for the covariance matrices (42). Let $G_{ed}(s,\sigma)$ and $G_{ud}(s,\sigma)$ denote the transfer function from the disturbance to the tracking error and that to the control input, respectively. Then, the transfer function matrices satisfy

$$\lim_{\sigma\to\infty} G_{ed}(s,\sigma) = G_{ed}^*(s), \quad \lim_{\sigma\to\infty} G_{ud}(s,\sigma) = G_{ud}^*(s), \tag{46}$$

where $G_{ed}^*(s)$ and $G_{ud}^*(s)$ are defined in (20) and (22), respectively.

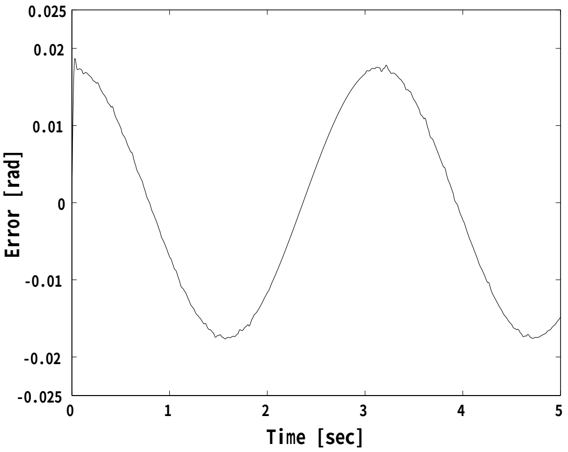
Proof: It can easily be shown that

$$\begin{aligned} G_{ed}(s,\sigma) = -G(s)\Big\{ I + \Big(I + F_x[sI - A + K(\sigma)C]^{-1}B \Big) \\ \Big(F_x[sI - A + K(\sigma)C]^{-1}K(\sigma) + s^{-1}F_y \Big) G(s) \Big\}^{-1} , \end{aligned} \tag{47}$$

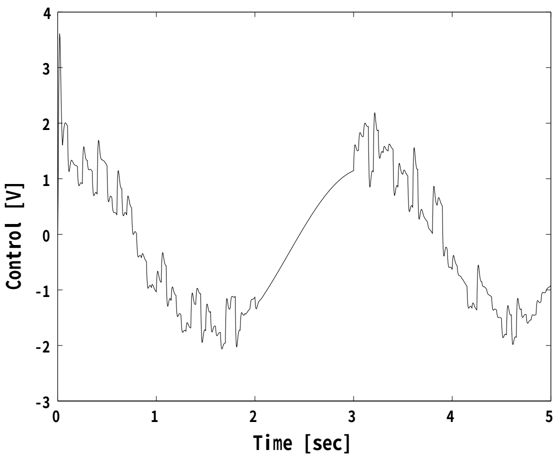
$$\begin{aligned} G_{ud}(s,\sigma) = -\Big\{ I + \Big(I + F_x[sI - A + K(\sigma)C]^{-1}B \Big)^{-1} \\ \Big(F_x[sI - A + K(\sigma)C]^{-1}K(\sigma) + s^{-1}F_y \Big) G(s) \Big\}^{-1} \\ \Big(I + F_x[sI - A + K(\sigma)C]^{-1}B \Big)^{-1} \\ \Big(F_x[sI - A + K(\sigma)C]^{-1}K(\sigma) + s^{-1}F_y \Big) G(s). \end{aligned} \tag{48}$$

Note that the following asymptotic properties can be obtained from (43):

$$\begin{aligned} &\left[sI - A + K(\sigma)C \right]^{-1} B \\ &\quad = (sI - A)^{-1}B\left[I + \sigma^{1/2}C(sI - A)^{-1}B \right]^{-1} \\ &\quad \to 0 \ \ (\sigma \to \infty) \end{aligned} \tag{49}$$

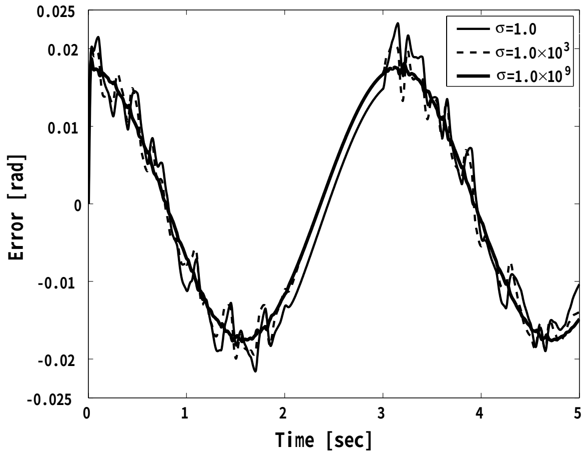


(a) Tracking error

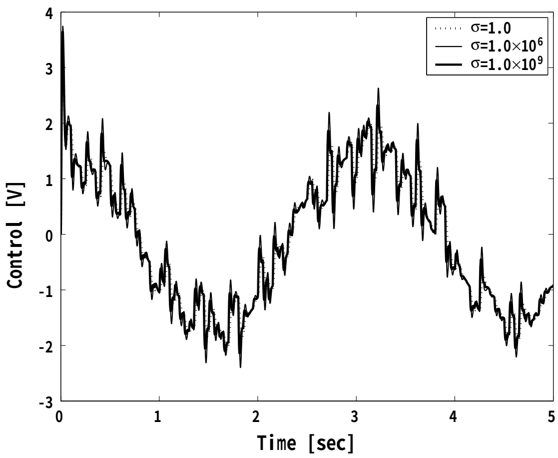


(b) Control input

Fig. 4 Responses of the state feedback case



(a) Tracking error



(b) Control input

Fig. 5 Responses of the output feedback case

$$\begin{aligned}&\left[sI - A + K(\sigma)C\right]^{-1} K(\sigma)\\ &\quad= \sigma^{1/2}(sI-A)^{-1}B\left[I + \sigma^{1/2}C(sI-A)^{-1}B\right]^{-1} \\ &\quad\to (sI-A)^{-1}BG^{-1}(s)\ \ (\sigma \to \infty)\end{aligned} \tag{50}$$

It can easily be checked that the asymptotic relations in (46) hold by substituting the expression (49) and (50) in (47) and (48). □

The above property suggests that, for sufficiently large σ, the practical matching conditions (44) and (45) for the output feedback controller are satisfied provided the parameter q is chosen such that the conditions (23) and (24) for the state feedback case are satisfied.

4. DESIGN EXAMPLE

A simple design example is presented to illustrate the effectiveness of the proposed design method. An earth scanning satellite antenna control problem discussed by Whidborne and Liu [9] is considered. The plant transfer function is given by

$$G(s) = \frac{27697}{s(s^2 + 1429s + 42653)}\ . \tag{51}$$

Note that the transfer function has a pole at $s = 0$. The reference input $r(t)$ and the disturbance $d(t)$ are assumed to belong to the class $\tilde{F}_\infty(D)$ with $D = 1$ and $F_\infty(M)$ with $M = 0.5$, respectively. The design specifications are given by

$$\hat{\varepsilon}_1(q, \sigma) \le \varepsilon_1 = 0.020944\ \text{[rad]}, \tag{52}$$

$$\hat{\varepsilon}_2(q, \sigma) \le \varepsilon_2 = 19.5\ \text{[V]}. \tag{53}$$

The first condition corresponds to the tracking error bound ± 1.2 degree which turns out to be more stringent than the second requirement.

First, the determination of the weighting parameter q for the quadratic performance index satisfying the practical matching conditions (23) and (24) for the state feedback design is considered. It is confirmed numerically that the index $\hat{\varepsilon}_1^*(q)$ is a monotonously decreasing function of q while $\hat{\varepsilon}_2^*(q)$ is monotonously increasing. In addition, it is found that the specification on $\hat{\varepsilon}_2^*(q)$ is satisfied for the wide range of q satisfying the specification on $\hat{\varepsilon}_1^*(q)$. Therefore it is possible to determine q satisfying the both specifications by a one dimensional search on the first index $\hat{\varepsilon}_2^*(q)$. It is found by the bisection search that q satisfying $\hat{\varepsilon}_1^*(q) = \varepsilon_1$ is given by $q = 7.4985 \times 10^9$ and then the second design specification is satisfied with $\hat{\varepsilon}_2^*(q) = 6.369$.

The sinusoidal signal $r(t) = 0.5\sin 2t$ is taken as a test reference input signal belonging to the class $\tilde{F}_\infty(D)$ with $D = 1$. A random signal shown in Fig. 3 is generated as a test disturbance signal belonging to the $F_\infty(M)$ with $M = 0.5$.

The responses to the test input signals for the state feedback design are shown in Fig. 4. It is seen that the two design specifications are satisfies. Figure 5 shows the responses of the output feedback control system to the test input. Note that the design specification on $\hat{\varepsilon}_1(q,\rho)$ can not be satisfied for relatively small σ but it can be satisfied by sufficiently large σ.

5. CONCLUSION

The new method for the design of critical control systems has been proposed. The proposed method utilizes the fruits of LQG/LTR technique to decompose the original design problem in the two steps. The parameter search required in each design step is much simpler than that required by the conventional approach. In addition, the tuning parameters have clear system-theoretic meaning, which provides the designer clear perspective.

The proposed method assumes the use of the Davison type integral controller as a basic controller. Although a large freedom for the choice of controller structure provided by the principle of matching is sacrificed, the controller structure used in this paper is not excessively complex and can be used for most applications.

Extensions of the proposed method to non-minimum phase plants are possible and will be reported elsewhere.

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