Fresnel computer-generated holograms for 3-D display of Real objects

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Abstract

Computer-generated holograms of real existing are synthesized by using a series of projection images of an incoherently illuminated object. The principle of computer tomography is applied to obtain the 3-D Fourier spectrum of the object. A Fresnel hologram is calculated directly from the 3-D Fourier spectrum. Experimental results with simulation are presented and some optical properties of reconstructed images are discussed.

1. Introduction

Computer-generated holograms(CGHs) can reconstruct non-existing 3-D objects[1,2]. Many methods for synthesizing CGHs have been discussed by many authors[3-6]. Since synthetic holograms enable us to make reconstruction of virtual 3-D objects, their applications to different fields have been discussed[7,8]. The difficulty of synthesizing CGHs of real existing 3-D objects, however, limits their application. Recently, Li et al. proposed a new method[9] in which projection images are recorded using an incoherent source and their 1-D Fourier transforms are calculated. Since projection images are recorded along one axis, however, out-of-focus images are reconstructed in the direction perpendicular to the recording axis. This hologram, consisting of 1-D Fourier holograms, needs two cylindrical lenses for reconstruction. Because of the peak corresponding to the bias component at the center of the synthesized hologram, the dynamic range of the CGH is limited.

It is well known that a 3-D Fourier spectrum of the 3-D object is calculated using a series of projection images from different points of view in the computer tomography(CT)[10]. Recording projection images with an angular range of 180° makes it possible to obtain the entire 3-D Fourier spectrum of an object, but not all Fourier spectra are required to synthesize a hologram with a limited viewing area. This means that it should be possible to calculate a Fresnel hologram from a 3-D Fourier spectrum by extracting the required spectral component.

In this paper, we discuss a method for synthesizing Fresnel holograms which does not need a lens for reconstruction and generates no defocused reconstructed images during 3-D reconstruction. The method proposed here consists of three steps. In the first step, projection images are recorded and the 3-D Fourier spectrum of a 3-D object is obtained based on the principle of CT. In the second step, the spectrum components required to generate a CGH are extracted from the 3-D Fourier spectrum, and finally a Fresnel hologram is calculated from the extracted components.

2. Principle and Theory

2.1 Recording projection images and their Fourier spectrum

Consider the actual recording setup as shown in Fig. 1 in the first step. We assume here that light is

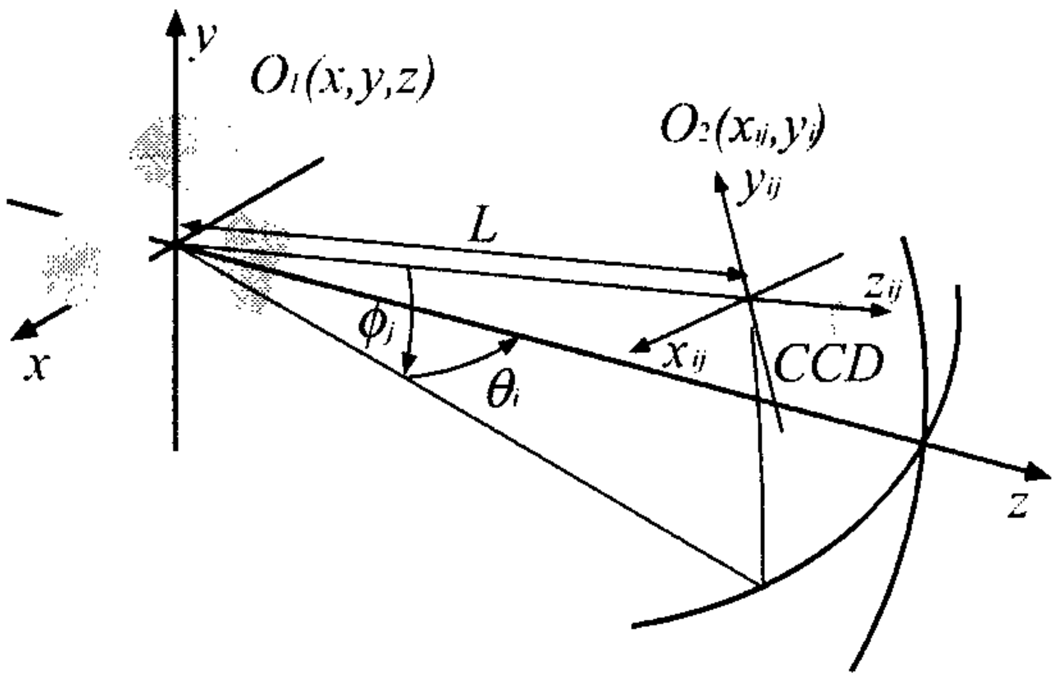

Figure 1: Schematic of a recording system for projection images from different points of view.

isotropicly scattered at the object surface. Specifically, $O_1(x,y,z)$ and $O_2(x_{ij},y_{ij})$ represent the reflection intensity of light at the surface of 3-D objects and the projection image recorded at a projection angle (θ_i,ϕ_j), respectively. If the depth of the objects is much smaller than the distance L from the origin of (x,y,z) to the CCD camera, it is possible to deal with projection images $O_2(x_{ij},y_{ij})$ as orthogonal projections. According to the principle of CT, we obtain

$$O_3(u,v) = \iint O_2(x_{ij},y_{ij}) \exp\left[-i2\pi\left(ux_{ij}+vy_{ij}\right)\right] dx_{ij}dy_{ij}. \tag{1}$$

The 2-D spectrum of Eq. (1) corresponds to one of the sectional planes in the 3-D spectrum of the object. The entire 3-D spectrum is calculated using a series of 2-D sections of Eq. (1) to gain different points of view. The next step is extraction of the 2-D Fourier component of the CGH from the 3-D spectrum.

2.2 Extraction of 2-D Fourier component

By the geometric consideration of Fig. 1 and converting the integral variables in Eq. (1) to (x,y,z), it is possible to describe Eq. (1) as follows:

$$O_3(u,v) = \iiint O_1(x,y,z) \, exp\big\{i2\pi\big[(v\theta_i\phi_j - u)x + vy + (u\theta_i + v\phi_j)z\big]\big\}dxdydz, \tag{2}$$

where we use following the small-angle approximation:$\cos\theta_i \cong 1, \sin\theta_i \cong \theta_i, \cos\phi \cong 1, \sin\phi \cong \phi$. We have the following relations in extracting one component form Eq. (1):

$$u = -\frac{2}{\lambda}\theta_i, v = -\frac{2}{\lambda}\phi_j. \tag{3}$$

Varing the proportional factors leads to change the magnification of z axis, but we do not describe this here. By substituting Eq. (3), Eq. (2) becomes

$$\begin{aligned} F_{\theta_i,\phi_j} =& O_3(u=-2\theta_i/\lambda, v=-2\phi_j/\lambda) \\ =& \iiint O_1(x,y,z)\exp\left\{\frac{i4\pi}{\lambda}\left[\theta_i(1-\phi_j^2)x + \phi_j y + (\theta_i^2+\phi_j^2)z\right]\right\}dxdydz. \end{aligned} \tag{4}$$

Using $1-\phi_j^2 \cong 1$, $u_0 = -2f\theta_i, and v_0 = -2f\phi_j$, we obtain

$$F_{\theta_i,\phi_j} = \iiint O_1(x,y,z)\exp\left\{-\frac{i2\pi}{\lambda}\left[\frac{u_0x+v_0y}{f} - \frac{(u_0^2+v_0^2)z}{2f^2}\right]\right\}dxdydz. \tag{5}$$

Eq. (5) completely coincides with the field at the back focus plane of a lens where the 3-D object is located at the vicinity of the forward focus plane. We note that we can obtain one Fourier component of the 3-D object from one projection image. Changing the projection angles allows the full distribution, $F(u_0,v_0)$, to be obtained at the Fourier plane. The extent of the acquired Fourier plane is proportional to the width of the projection angle during recording, but the width of the projection angle is limited by the area where small-angle approximation is effective.

2.3 Calculation of the Fresnel hologram

A Fresnel hologram pattern is calculated from $F(u_0, v_0)$. Calculating the Fresnel diffraction from the Fourier spectrum F gives the Fresnel diffraction pattern $h(x_0, y_0)$ described as follows:

$$\begin{aligned} h(x_0, y_0) &= C\mathcal{F}^{-1}[F] * \exp\left[\frac{i\pi}{\lambda R}(x_0^2 + y_0^2)\right] \\ &= C'\mathcal{F}^{-1}\left\{F(\nu_x, \nu_y)\exp\left[-i\lambda R\pi(\nu_x^2 + \nu_y^2)\right]\right\} \end{aligned} \quad (6)$$

where C and C' are proportional factors, the symbols $*$ and $\mathcal{F}$ denote the convolution integral and Fourier transform, and (ν_x, ν_y) is spatial frequency equivalent of (u_0, v_0), respectively. A call-oriented CGH is calculated.

3. Projections and optical reconstruction

For verifying the proposed method and simplification of the recording process, projection images are generated in the computer. Some examples of the projection images are shown in Fig. 2. The angular ranges of both θ_i and ϕ_j are ±16°, and the angular increment between two adjacent projection images is 1°. The number of projection images is $33 \times 33 = 1089$, and each image consists of 256×256 pixels. The objects are a club, a star, eyes and an exit mark.

First, we demonstrate the simulation of reconstruction images. The Fresnel diffraction reconstructed images from CGH at various positions are shown in Fig. 3. The objects, eyes in Fig. 3(a), club in (b), star in (c) and exit mark in (d) are finely focused without any blur. Since a object is focused only at the position, it means that the objects is reconstructed three dimensionally using this method.

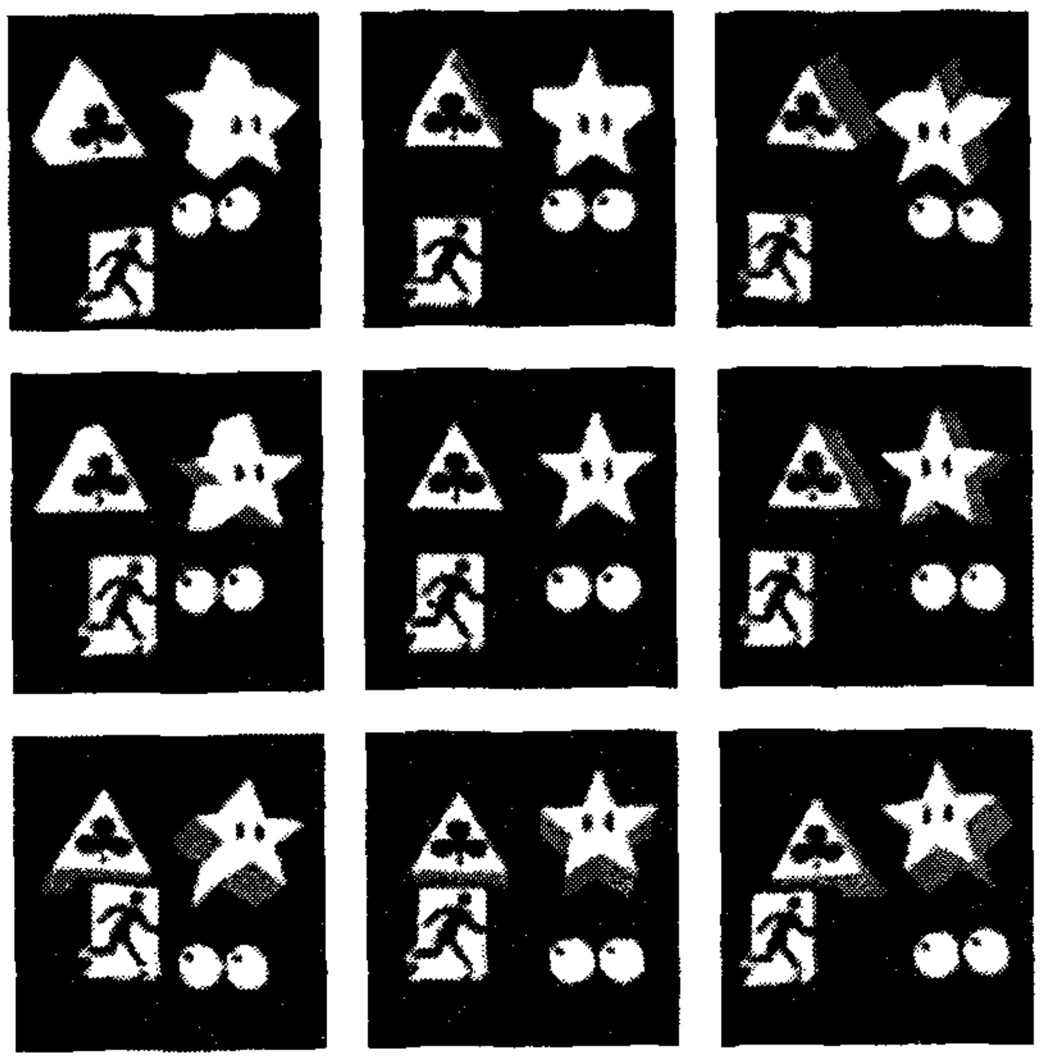

Figure 2: Projection images generated in the computer.

Optical reconstruction images from the Fresnel hologram are shown in Fig. 4. Comparison of Fig. 3 with Fig. 4 verifies the success of our method. The differences for the focused positions of the objects derive from the error of the photographic reduction.

4. Conclusion

In conclusion, we propose a method of synthesizing a Fresnel hologram from projection images recorded using an incoherent light source. This method both numerically and experimentally is verified in the case of a reflected object. The method proposed here enables us to display 3-D real objects in many applications such as CG display and so on.

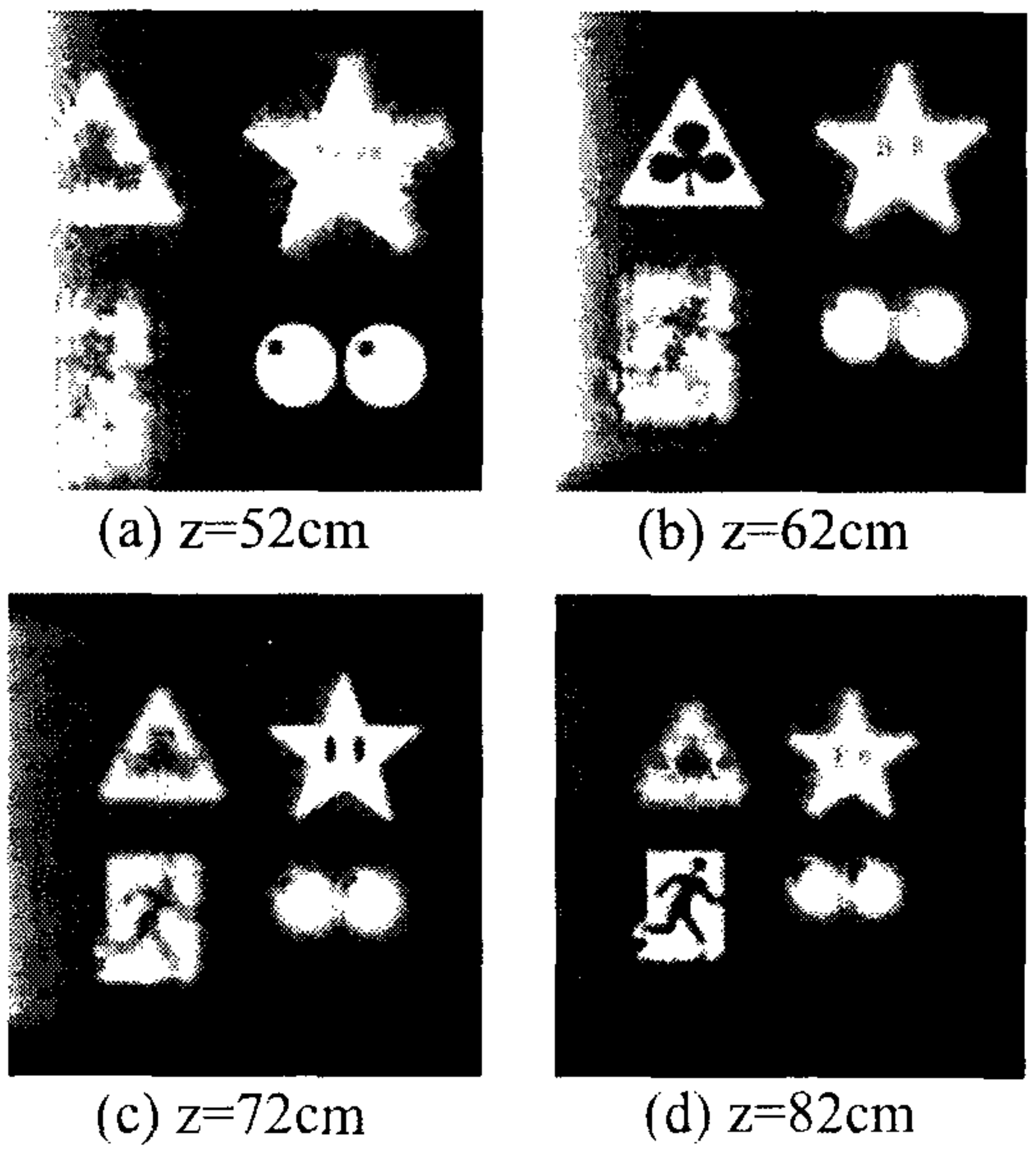

Figure 3: **Computer simulation: reconstructed images from the Fresnel CGH.**

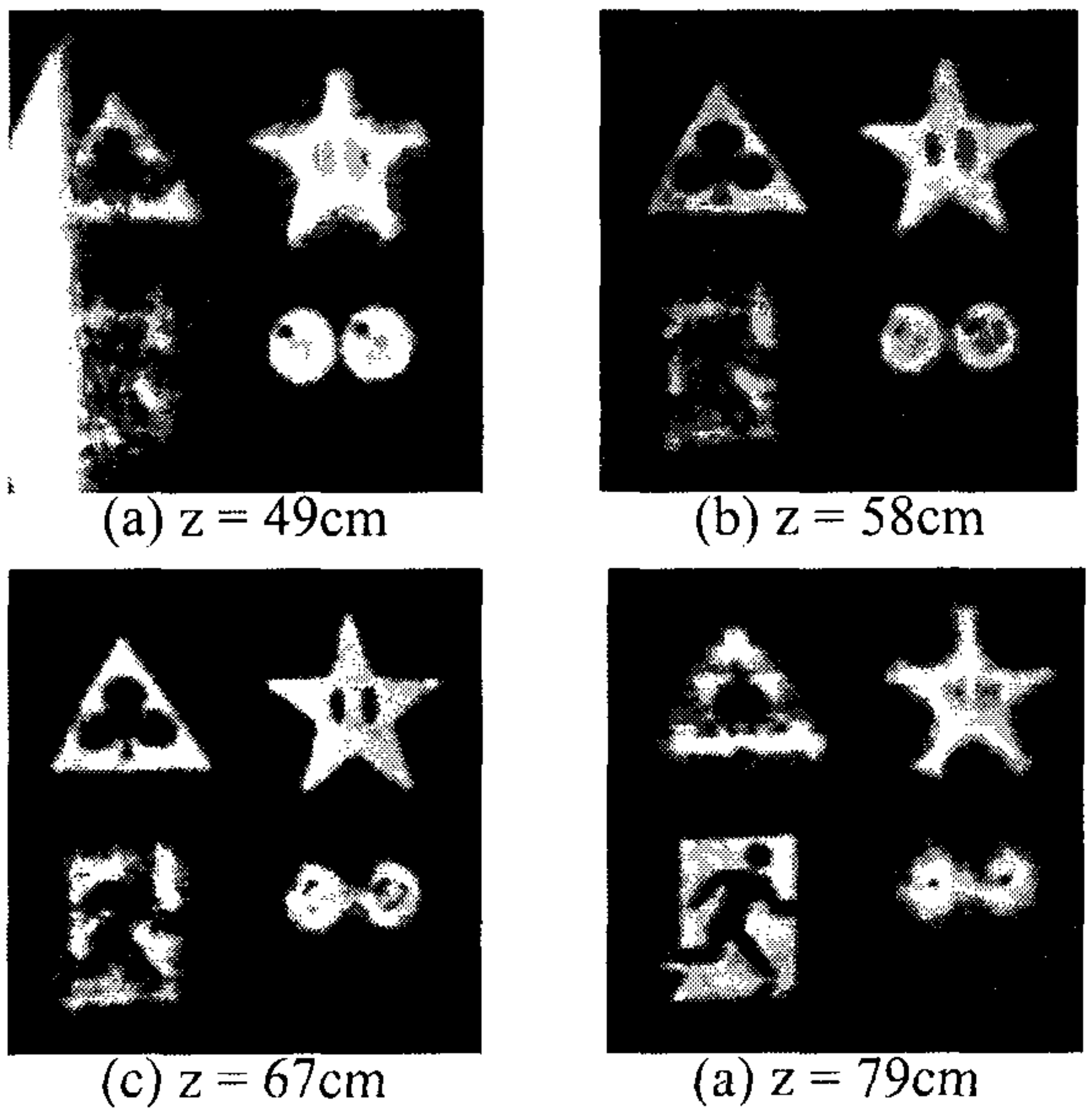

Figure 4: **Experimental results of optical reconstruction.**

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