Power Analysis for Tests Adjusted for Measurement Error

Sunyeong Heo[1] and John L. Eltinge[2]

Abstract

In man cases, the measurement error variances may be functions of the unknown true values or related covariate. In some cases, the measurement error variances increase in proportion to the value of predictor. This paper develops estimators of the parameters of a linear measurement error variance function under stratified multistage random sampling design and additional conditions. Also, this paper evaluates and compares the power of an asymptotically unbiased test with that of an asymptotically biased test. The proposed method are applied to blood sample measurements from the U.S. Third National Health and Nutrition Examination Survey(NHANES III)

Keywords : Complex sample survey; Errors-in-variables model; Linear regression model with unequal variances; Small-error approximation; Power of two-side test.

1. Introduction

A measurement error generally is defined as the difference between an observed value and an underlying true value. Some authors, e.g., Grove (1991), refer to measurement errors as observed errors. If measurement errors are nontrivial, then estimators from classical methods may have corresponding nontrivial biases.

Since the 1940s, people have been concerned about various problems associated with measurement errors. See, e.g., Dalenius (1981) for a review of some early literature, and Biemer *et al.* (1991) for a more recent review. For specific work with measurement error problems, see, e.g., Fuller (1987, sec.1.1.) and Carroll and Stefanski (1990).

Carroll and Stefanski (1990) gave some results of small measurement error approximation. One of their important results is that when measurement error is small, under additional conditions one can directly use observed values without accounting explicitly for the errors. Fuller (1991) gave a complex survey sample

1) Full-time Lecturer, Department of Statistics, Changwon National University, Changwon, 641-773, Korea.
E-mail : syheo@sarim.changwon.ac.kr

2) Senior Mathematical Statistician, U.S. Bureau of Labor Statistics, PSB 4915, 2 Massachusetts Aveue NE, Washington, DC 20212 U.S.A.

based errors-in-variables estimator of a vector of regression coefficients. under both non-small error and small error conditions.

In man cases, the measurement error variances may be functions of the unknown true values or related covariate. In some cases, the measurement error variances increase in proportion to the value of predictor. In this paper, we develop methods for estimation and inference for a linear measurement error variance function model under stratified multistage random sampling design and additional conditions.

We apply the ideas of Fuller (1991) to develop error-in-variables estimators of the parameters of the linear measurement error variances function. After that, we use the methods of Carroll and Stefanski(1990) to derive estimators of the parameters when measurements errors are small.

In section 2, we define a measurement error model and a measurement error variance function. In section 3, we define a sampling design and develop estimators of the parameters defined in section 2 under both non-small error and small error conditions. In section 4, we give theoretical basis for our simulation work with power curves. In section 5, we gives the details of simulation method, evaluate powers for a two-tailed test and interpret the power curves evaluated from simulation work. We apply the methods of this paper to data from the U.S. Third National Health and Nutrition Examination Survey (NHANES III).

2. Measurement Error Model and Measurement Error Variance Function

Let W be a proxy for X. Assume that W is unbiased for X and two replicate measurements are taken at each design point. Then following the notation of Carroll and Stefanski (1990), for a given x_t the model will be written as

$$W_{tr} = x_t + \delta U_{tr} \tag{2.1}$$

with

$$E(U_{tr}|x_t) = 0 \text{ and } Var(U_{tr}|x_t) = \Omega(x_t, \gamma)$$

for $t = 1, 2, \cdots, n$; $r = 1, 2$ where δ is a positive scale factor and $\Omega(x_t, \gamma)$ is an known function of parameter γ and x_t.

The model (2.1) may be rewritten by

$$W_{tr} = x_t + (\delta \Omega_t^{1/2}) d_{tr} \tag{2.2}$$

where $\Omega_t = \Omega(x_t, \gamma)$ and $d_{tr} = \Omega_t^{-1/2} U_{tr}$ are independent and identically distributed with mean 0 and variance 1 for all t and r. In the following work we will denote $\Omega(x_t, \gamma)$ as Ω_t if it is not necessary to emphasize Ω_t being a function of (x_t, γ).

In many cases, the measurement error variances increase proportionally as the values of predictors increase or decrease. To a first order approximation, it sometimes suffices to model the measurement error variance $\Omega(x_t, \gamma)$ as a linear function of x.

The linear measurement error variance function of x will be written as

$$\Omega_t = \gamma_0 + \gamma_1 x_t \qquad (2.3)$$

for a given x_t.

Davidian and Carroll (1987) and Davidian (1990) discussed several method of variance function estimation. Davidian (1990) showed through simulation results that sample variances from small numbers of replicates (2 or 3 replicates) may be more efficient than the log and the identity under normal distribution conditions.

Following Davidian (1900), we will consider the linear measurement error variance estimation based on the sample variances.

Under model (2.1), for a given value x_t and a known δ, an unbiased estimator of Ω_t is $\delta^{-2}S_t^2$ where $S_t^2 = (W_{t1} - W_{t2})^2/2 = \delta^2 S^2_{Ut}$ and standard deviation $S^2_{Ut} = (U_{t1} - U_{t2})^2/2$ is the sample variance within the tth unit. In addition, an unbiased estimator of x_t is $\overline{W}_{t\cdot} = \frac{1}{2}(W_{t1} + W_{t2})$.

3. Design Based Estimation of Measurement Error Variance

3.1 Sampling Design and conditions

We assume the following design condition which is quoted with minor modifications from the stratified multistage sampling design in Shao (1996, p.205-206).

(D.1) The population has been stratified into L strata with N_h clusters in the hth stratum. For the hth stratum, $n_h = 2$ clusters are selected independently across the strata. These first-stage clusters are selected with unequal selection probabilities and with replacement. Within the ith first-stage cluster in the hth stratum, $n_{hi} \geq 1$ ultimate units are sampled according to some sampling methods, $i = 1, \cdots, n_h$, $h = 1, \cdots, L$. The total number of ultimate units in the population is $N=\sum_{h=1}^{L}\sum_{i=1}^{N_h} N_{hi}$ and in the sample is $n=\sum_{h=1}^{L}\sum_{i=1}^{n_h} n_{hi}$. The total number of first-stage clusters in the sample is $n_F=\sum_{h=1}^{L} n_h$.

Based on the sampling design (D.1) and model (3.1), we will use the following conditions.

(C.1) The finite population expectations of x^p exist and are bounded for $0 \le p \le 4$ as $k \rightarrow \infty$.

(C.2) The sixth moment about the origin of d exists and is finite.

(C.3) $E_\xi(d^3)=0$ where $E_\xi(\ )$ denotes expectations taken based on only the model (3.1).

3.2 A Linear Measurement Error Variance Model

Under model (2.1) and (2.3) when we have n observed values, $(Y_{\delta hij}, X_{\delta hij}) = (\delta^{-2} S^2_{hij}, \overline{W}_{hij\cdot})$, for a given δ instead of (Ω_{hij}, x_{hij}), we can write a model,

$$y_{hij} = \gamma_0 + \gamma_1 x_{hij} + q_{hij}$$
$$(Y_{\delta hij}, X_{\delta hij}) = (y_{hij}, x_{hij}) + (\omega_{hij}, u_{hij}) \tag{3.1}$$

where q_{hij} are independent and identically distributed with mean 0 and a finite variance σ^2_q for all (hij), $y_{hij}=\Omega_{hij}$, $\omega_{hij}=S^2_{Uhij}-\Omega_{hij}$ and $u_{hij}=\delta\,\overline{U}_{hij\cdot}$. The $\overline{U}_{hij\cdot}=\frac{1}{2}(U_{hij1}+U_{hij2})$. The variable ω_{hij} is an independent $(0, \sigma_{\omega\omega hij})$ random variable with $\sigma_{\omega\omega hij}=\mathrm{Var}(S^2_{Uhij})$ and the variable u_{hij} is an independent $(0, \sigma_{uuhij})$ random variable with $\sigma_{uuhij}=\delta^2\Omega_{hij}/2$. Note that under model (2.2) $S^2_{Uhij}=\Omega_{hij}S^2_{dhij}$ where $S^2_{dhij}=2^{-1}(d_{hij1}-d_{hij2})^2$ and S^2_{dhij} is independent and identically distributed with mean 1 and a constant variance, say c. Therefore $\mathrm{Var}(S^2_{Uhij})=c\Omega^2_{hij}$. If we assume that the U_{hijr} follow a normal distribution, then $c=2$. We will assume here that the errors, q_{hij}, in the regression equation (3.1) are independent of $(x_{hij}, \omega_{hij}, u_{hij})$ for all (hij).

For convenience, we will replace the triple subscript (hij) with the single subscript t from the following expressions if it is not necessary to specify strata, clusters and ultimate units.

3.3 Estimation of moments of the finite population

Under the survey design (D.1) and model (3.2), when we have N true values, (y_t, x_t), define

$$\Sigma_{xx}=N^{-1}\sum_{t=1}^{N}(1\ x_t)'(1\ x_t)\ \text{and}\ \Sigma_{xy}=N^{-1}\sum_{t=1}^{N}(1\ x_t)'y_t$$

and when we have N observations of $(Y_{\delta t}, X_{\delta t})$ instead of (y_t, x_t), define

$$\Sigma_{X_\delta X_\delta}=N^{-1}\sum_{t=1}^{N}(1\ X_{\delta t})'(1\ X_{\delta t})\ \text{and}\ \Sigma_{X_\delta Y_\delta}=N^{-1}\sum_{t=1}^{N}(1\ X_{\delta t})'Y_{\delta t}$$

and also define

$$\Sigma_{uu}=\text{diag}(0,\ \sigma_{uu})$$

with $\sigma_{uu}=N^{-1}\sum_{t=1}^{N}\sigma_{uut}$ where $\sigma_{uut}=\delta^2\Omega_t/2$.

When we have n sampled true values, (y_t, x_t), from a finite population, Σ_{xx} and Σ_{xy} can be estimated by

$$M_{xx}=N^{-1}\sum_{t=1}^{n}w_t(1\ x_t)'(1\ x_t)\ \text{and}\ M_{xy}=N^{-1}\sum_{t=1}^{n}w_t(1\ x_t)'y_t$$

where w_t is a unit-level survey weight. In addition, when we have observations of $(Y_{\delta t}, X_{\delta t})$ in the place of (y_t, x_t), we may have

$$M_{X_\delta X_\delta}=N^{-1}\sum_{t=1}^{n}w_t(1\ X_{\delta t})'(1\ X_{\delta t})\ \text{and}\ M_{X_\delta Y_\delta}=N^{-1}\sum_{t=1}^{n}w_t(1\ X_{\delta t})'Y_{\delta t}$$

instead of M_{xx} and M_{xy}. In addition, Σ_{uu} can be estimated by $S_{\delta uu}=\text{diag}(0,\ \hat{\sigma}_{\delta uu})$ where $\hat{\sigma}_{\delta uu}=N^{-1}\sum_{t=1}^{n}w_t\,\hat{\sigma}_{uut}$ and $\hat{\sigma}_{uut}=S_t^2/2$.

3.4 Measurement error variance estimation

3.4.1 When δ is not small

Under the design (D.1) and model (3.1) , we have an estimator of $\gamma=\Sigma_{xx}^{-1}\Sigma_{xy}$ given by $\hat{\gamma}=M_{xx}^{-1}M_{xy}$. The M_{xx} and M_{xy} can be estimated by

$$\widehat{M}_{xx}=M_{X_\delta X_\delta}-S_{uu}\ \text{and}\ \widehat{M}_{xy}=M_{X_\delta Y_\delta}.$$

Thus, an estimator of γ from n pairs of $(Y_{\delta t}, X_{\delta t})$ is

$$\hat{\gamma}_{EIV,\,svy} = \hat{M}_{xx}^{-1}\hat{M}_{xy} \tag{3.2}$$

where $\hat{\gamma}_{EIV,\,svy} = (\hat{\gamma}_{EIV,\,svy,\,0}, \hat{\gamma}_{EIV,\,svy,\,1})'$. Under design (D.1), model (3.1), conditions (C.1)-(C.3) and additional regularity conditions, $\hat{\gamma}_{EIV,\,svy}$ is a consistent estimator of γ (Heo, 1999).

An estimator of the variance of the asymptotic distribution of $\hat{\gamma}_{EIV,\,svy}$ is

$$\widehat{Var}(\hat{\gamma}_{EIV,\,svy}) = (\sum_{t=1}^{n} w_t)^{-2}\, \hat{M}_{xx}^{-1}\, \hat{V}\, \hat{M}_{xx}^{-1} \tag{3.3}$$

where

$$\hat{V} = \sum_{h=1}^{L} \frac{n_h}{(n_h-1)} \sum_{i=1}^{n_h} (z_{hi} - \bar{z}_h)(z_{hi} - \bar{z}_h)'$$

$$z_{hi} = \sum_{j=1}^{n_{hi}} w_{hij} d_{hij}, \quad \bar{z}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} z_{hi}$$

$$d_{hij} = (\hat{v}_{\delta hij}, X_{\delta hij}\hat{v}_{\delta hij} + 2^{-1} S^2_{hij}\hat{\gamma}_{EIV,\,svy,\,1})'$$

and $\hat{v}_{\delta hij} = Y_{\delta hij} - (1, X_{\delta hij})\hat{\gamma}_{EIV,\,svy}$ (Fuller, 1991, p. 624).

3.4.2 Under small error approximation

When the measurement error is small, by the result of Carroll and Stefanski (1990, p. 654), we can rewrite the model (3.2) such as

$$Y_{\delta t} = \gamma_0 + \gamma_1 X_{\delta t} + \psi_t \tag{3.4}$$

where $\psi_t = (q_t + \omega_t) - \gamma_1 u_t$ with

$$E_{\xi}(\psi_t | x_t) = 0 \text{ and } V_{\xi}(\psi_t | x_t) = \sigma_q^2 + c\Omega_t^2 + \gamma_1^2(\delta^2 \Omega_t / 2).$$

The variance of ψ_t for given value x_t, $\mathrm{Var}(\psi_t | x_t)$, goes to $\sigma_q^2 + c\Omega_t^2$ as $\delta \to 0$. The model (3.4) has the form of a simple linear regression model with unequal variances. Therefore, under the sampling design (D.1) we have an ordinary least squares estimator of γ,

$$\hat{\gamma}_{REG,\,svy} = M^{-1}_{X_\delta X_\delta} M_{X_\delta Y_\delta} \tag{3.5}$$

where $\hat{\gamma}_{REG,\,svy} = (\hat{\gamma}_{REG,\,svy,\,0}, \hat{\gamma}_{REG,\,svy,\,1})'$. Under the design (D.1), model (3.4) and additional regularity conditions, $\hat{\gamma}_{REG,\,svy}$ is a consistent estimator of $\gamma = \Sigma^{-1}_{X_\delta X_\delta} \Sigma_{X_\delta Y_\delta}$.

4. The Power of the Test

In this section, we will examine the relative performance of an asymptotically unbiased test using $\hat{\gamma}_{EIV,svy}$ defined in expression (3.2) relative to an asymptotically biased test using $\hat{\gamma}_{REG,svy}$ defined in expression (3.5). This comparison centers on evaluation of the powers of tests based on these two estimators. This power evaluation uses simulation work based on blood sample measurements collected in NHANES III.

4.1 Confidence interval method

Assuming that the true γ_1 is $\gamma_{1,0}$ and that one tests

$$H_0 : \gamma_1 = \phi\gamma_{1,0} \qquad vs. \qquad H_1 : \gamma_1 \neq \phi\gamma_{1,0} \tag{4.1}$$

where $\phi \neq 0$ is a constant. Note that for $\phi \neq 1$ one is to test a wrong null hypothesis.

Let $\hat{\gamma}_1$ be an approximately unbiased point estimator of $\gamma_{1,0}$ and let $(\hat{\gamma}_{1,L}, \hat{\gamma}_{1,U})$ be a confidence interval based on $\hat{\gamma}_1$. Then, $H_0 : \gamma_1 = \phi\gamma_{1,0}$ will be rejected if a confidence interval $(\hat{\gamma}_{1,L}, \hat{\gamma}_{1,U})$ does not cover $\phi\gamma_{1,0}$.

Thus, note that

$$\alpha = \Pr\{\gamma_1 \notin (\hat{\gamma}_{1,L}, \hat{\gamma}_{1,U}) | \phi = 1\}.$$

and also note that the power of the test (4.1) is given by

$$1-\beta = \Pr\{\gamma_1 \notin (\hat{\gamma}_{1,L}, \hat{\gamma}_{1,U}) | \phi\}. \tag{4.2}$$

That is, in the confidence interval method the power is defined by the rate of confidence intervals not covering an incorrectly stated null parameter value when we take samples repeatedly from a given population through a given design, and evaluate confidence intervals based each sample.

4.2 Confidence intervals for the parameters of measurement error variance function

In section 3.4.1, we obtained an approximately unbiased point estimator of γ_1 expressed by $\hat{\gamma}_{EIV,svy,1}$ in expression (3.2). A confidence interval for γ_1 is given by

$$(\hat{\gamma}_{EIV,1,L}, \hat{\gamma}_{EIV,1,U}) = \hat{\gamma}_{EIV,svy,1} \pm t(\frac{\alpha}{2}, df) \cdot \hat{\sigma}(\hat{\gamma}_{EIV,svy,1}) \tag{4.3}$$

where $t(\frac{\alpha}{2}, df)$ is an upper $(\alpha/2)$th percentile point of t distribution on df

degrees of freedom and $\hat{\sigma}^2(\hat{\gamma}_{EIV, svy, 1})$ is the second diagonal element of $\widehat{Var}(\hat{\gamma}_{EIV, svy})$ obtained from expression (3.3).

In general, under a complex survey design, the degree of freedom of t distribution is determined by

$$df = \sum_{h=1}^{L}(n_h - 1) - p$$

where L is the number of strata, n_h the number of primary sampling units within each hth stratum and p the number of parameters to be estimated in a model. See, e.g., Korn and Graubard (1990)

When $\phi = 1$, the confidence interval expressed in (4.3) will approximately achieve the nominal level of significance a.

In section 3.4.2, we have an estimator from the regression method expressed by $\hat{\gamma}_{REG, svy}$ in expression (3.5). The $\hat{\gamma}_{REG, svy}$ gives an asymptotically biased estimator of γ for non-small δ (Fuller, 1987, sec. 1.1). The confidence interval for γ_1 obtained from $\hat{\gamma}_{REG, svy, 1}$ is

$$(\hat{\gamma}_{REG, 1, L}, \hat{\gamma}_{REG, 1, U}) = \hat{\gamma}_{REG, svy, 1} \pm t(\frac{a}{2}, df) \cdot \hat{\sigma}(\hat{\gamma}_{REGsvy, 1}) \tag{4.4}$$

where $\hat{\sigma}^2(\hat{\gamma}_{REG, svy, 1})$ is the design based estimator of the variance of $\hat{\gamma}_{REG, svy, 1}$. The confidence interval in (4.4) generally will give a biased test.

5. Application to the U.S. NHANES III Data

5.1 The U.S. NHANES III Data

The U.S. Third National Health and Nutrition Examination Survey (NHANES III) was conducted for the U.S. National Center for Health Statistics (NCHS) to assess the health and nutritional status of the non-institutionalized civilian population in the United States. NHANES III is a large-scale sample survey based on a stratified multistage design with 49 strata. Within each stratum, two primary sample units (PSUs, roughly equivalent to counties) are selected with unequal probabilities. Additional levels of sampling select secondary units (roughly equivalent to city blocks), households and individual persons. Each selected person is asked to complete a questionnaire and to participate in a very thorough medical examination.

As part of NHANES III, the NCHS considered using a formal two-phase sampling design to obtain replicate measurements from a relatively small subset of the group of original respondents.

5.2 Simulation methods to evaluate for power curve

For simulation work to evaluate power curves, this section was focused on that how well an asymptotically unbiased estimator $\hat{\gamma}_{EIV, svy}$ and a asymptotically biased estimator $\hat{\gamma}_{REG, svy}$ detect a wrong null hypothesis for different sizes of measurement error scale factors, δ, and different rates of deviation of null hypothesis value from the true parameter.

The detailed steps are described in the following subsections.

5.2.1 Construction of a large population

First, we selected 2,940 PSUs out of 98 original PSUs through simple random sampling with replacement (srswr). After that, we randomly divided the set of 2940 PSUs into 49 strata, with 60 PSUs in each stratum. Specifically, the first selected PSU through the 60th selected PSU were assigned to stratum 1, the next 61th through the 120th to stratum 2, and so on. The resulting stratified population of 2,940 primary units was then hold fixed through the remainder of the simulation work in this chapter.

5.2.2. Random sample

Within each stratum with 60 PSUs, we selected two PSUs through srswr. Thus, this new sample is composed of 49 strata with two PSUs each. We defined the first selected PSU as PSU ONE and the second selected PSU as PSU TWO within each stratum. All persons belonging to selected PSUs are included into the new sample.

5.2.3. Parameter values

For our simulation work, we used this $\hat{\gamma}_{EIV, svy}$ as the true γ. In the current discussion of our simulation work, we will focus on results related to the cholesterol measurements in blood samples of NHANES III.

In cholesterol measurements in blood samples of NHANES III, $\hat{\gamma}_{EIV, svy} = (-1425.728, 20.656)'$. Therefore, we will consider

$$\Omega_t = -1425.728 + 20.656 x_t \qquad (5.1)$$

as the true recalled measurement error variance for cholesterol measurements for the t-th person. In addition, we used the average of two original measurements, $\overline{W}_{t\cdot}$, for each t-th person as the true x_t.

5.2.4 Random errors

In model (2.2), the rth observation for t-th person is expressed by

$$W_{tr} = x_t + (\delta \Omega_t^{1/2}) d_{tr}$$

where the random variables d_{tr} are independent and identically distributed with mean 0 and variance one for all t and r. In our simulation work, we assumed that

$$d_{tr} \overset{iid}{\sim} N(0,1)$$

for $t = 1, \cdots, n$, $r = 1, 2$. Using the random number generator, ***uniform***( ), and the inverse cumulative normal function, ***invnorm***(), of the Stats package (StataCorp, 1997, section 20.3.2), we generated d_{tr}.

5.2.5 Simulated power curve

For each δ being equal to $0.125, 0.25, 0.375, 0.5, 0.625, 0.875, 1.0, 1.5$ and 2.0, we generated m samples of 98 primary units selected from the stratified population described in section 5.2.1. From the each sample persons included in a given sampled 98 selected PSUs, we computed errors-in-variables estimates, $\hat{\gamma}_{EIV,svy}$, from expression (3.2) and regression estimates, $\hat{\gamma}_{REG,svy}$, from expression (3.5). As the result, we had total of m $\hat{\gamma}_{EIV,svy}$ estimates and $\hat{\gamma}_{REG,svy}$ estimates, respectively.

As a result, we had m confidence intervals from each of the errors-in-variables and the regression methods for a two-tailed test.

For a given δ, we determined whether each confidence interval evaluated from each individual sample covers $\phi\gamma_{1,0}$ or not for different ϕ being equal to

$$0.1, 0.3, 0.5, 0.7, 0.9, 0.95, 1, 1.05, 1.1, 1.2, 1.3, 1.5, 1.7, 1.9.$$

Out of m intervals for a given ϕ and a given method, we counted the number c_m of confidence intervals, which were covering the null value $\phi\gamma_{1,0}$. Then, the estimate, $1-\hat{\beta}$, of $1-\beta$ in expression (4.2) is given by $1 - c_m/m$, for a given δ and ϕ. Using these estimated powers we compared the powers of the errors-in-variables method using $\hat{\gamma}_{EIV,svy}$ to the regression method using $\hat{\gamma}_{REG,svy}$.

5.2.6 Confidence bounds for power

For each possible combination of (δ, ϕ), we may consider the event in which a confidence interval that is evaluated from each simulated sample covers the null hypothesis value $\phi\gamma_{1,0}$ as a binomial trial.

Let X_{δ_0,ϕ_0} be the number of confidence intervals not covering $\phi_0\gamma_{1,0}$ out of m simulated samples given $\delta=\delta_0$. Then, X_{δ_0,ϕ_0} has a binomial distribution with parameters m and $1-\beta_{\delta_0,\phi_0}$ where β_{δ_0,ϕ_0} is the true coverage rate of $\phi_0\gamma_{1,0}$ when $\delta=\delta_0$.

Under the assumption that m is large enough for all relevant combinations of (δ, ϕ) to apply the central limit theorem for proportions, a 95% confidence interval for power $1-\beta$ point is evaluated by,

$$(1-\hat{\beta})\pm 1.96[m^{-1}\{\hat{\beta}(1-\hat{\beta})\}]^{1/2}. \qquad (5.2)$$

5.3 Power Curves and Interpretation

We now evaluated powers for all possible 150 combinations of (δ, ϕ), that we preassigned, according to the steps described in section 5.2. To draw three-dimensional power surfaces, we put $\phi\gamma_{1,0}=20.656\phi$ on the x-axis, δ on the y-axis and estimated power, $1-\hat{\beta}$, on the z-axis. Using 150 evaluated power points and linear spatial interpolation, we drew three-dimensional power surfaces for both methods with $m=5,000$ simulated samples.

Figure 5.1 shows that for all $\delta\in(0,2]$, the errors-in-variables method gives an approximately unbiased test. However as δ is increased, the power of the errors-in-variables method is very small.

Figure 5.2 shows that as δ increases the bias of the regression method is severe. The power at γ_1 that is positively deviated from true $\gamma_{1,0}$, that is $\phi>1$, is much larger than the errors-in-variables method, but at the true $\gamma_{1,0}=20.656$ the type I error is nontrivial for the regression method. For the negatively deviated γ_1, that is $\phi<1$, the errors-in-variables method has larger power than the regression method.

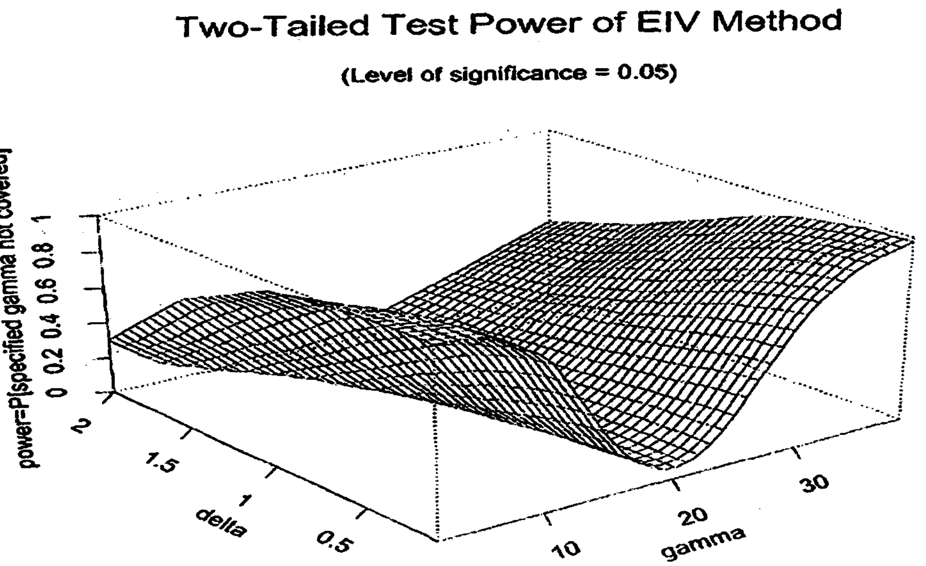

Figure 5.1. Three dimensional power surface for the errors-in-variables method for cholesterol measurements in blood samples of NHANES III. Powers were estimated with 5000 simulated samples using a stratified cluster design as described in sections 5.2.1. and 5.2.2. The nominal Type I error rate is $\alpha = 0.05$.

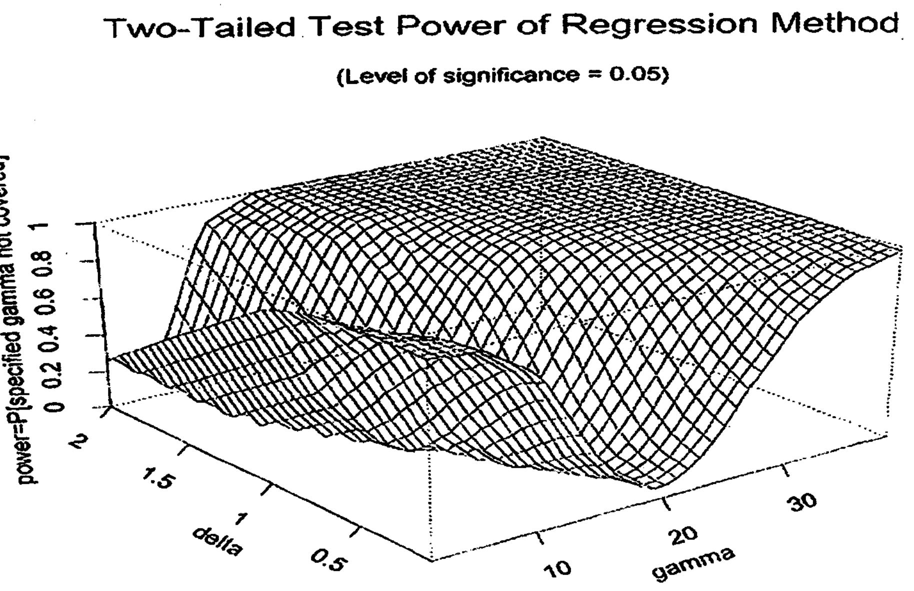

Figure 5.2. Three dimensional power surface for the regression method for cholesterol measurements in blood samples of NHANES III. Powers were estimated with 5000 simulated samples using a stratified cluster design as described in sections 5.2.1 and 5.2.2.. The nominal Type I error rate is $\alpha = 0.05$.

6. Conclusion

We have defined a superpopulation measurement error model and developed point estimators for the parameters of the associated linear measurement error variance function. Following Fuller (1991), we developed a consistent errors-in-variables estimator of the coefficients of the linear measurement error variance functions. Following Carroll and Stefanski (1990), we developed a consistent estimators under small measurement error approximation.

Based on those estimators, we evaluated power of two-sided tests and compared the power of an asymptotically unbiased test using the errors-in-variables point estimator, $\hat{\gamma}_{EIV,svy}$, with that of an asymptotically biased test using the regression point estimator, $\hat{\gamma}_{REG,svy}$.

The simulation work indicated the following: for two-tailed tests applied under the conditions considered here, the errors-in-variables method gives slightly biased test, but it may be considered as an approximately unbiased test because the amount of th bias was small. Even though the regression method has higher powers for the specific constellations of parameters γ and δ in the analysis, it has non-trivial bias for all predetermined δ.

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