Taylor's Power Law and Quasilikelihood

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ABSTRACT

In ecological studies, animal science, or entomology, the variance of count is considered to have the power of the mean relationship with the mean count as Taylor (1961) presented his famous "Taylor's Power Law". In this talk, we are going to review the development of TPL and its extension toward pest management sampling scheme. Different estimation methods are compared. Quasilikelihood approach is suggested to incorporate covariate information. Possible extensions will be discussed.

Key Words : Variance Modeling, Pseudo-Likelihood, Negative Binomial

1. Introduction

In addition to information related to population density, the variability in count response can provide important information on the spatial pattern of a animal/insect population. Traditionally spatial patterns have been classified into three categories: Uniform, Random, and Aggregated. A uniform spatial pattern describes some degree of repulsion between individuals and an aggregated spatial pattern indicates "live-together" phenomena in their behavior. Without any rigorous evidence, people used to think the spatial pattern as random if the variance equals the mean in sample counts. For most populations, however, the variance is larger than the mean, which we call "over-dispersed" in many statistics literatures (McCullagh and Nelder, 1989). This overdispersed pattern could occur when the oviposition (laying eggs) or food preference exists in insect populations.

For data sets containing sampling units, one method for evaluating the spatial pattern of a species is to fit discrete probability distributions such as the Poisson distribution and the Negative Binomial distributions. The probability distribution function of the count number (x) with given mean, μ , in Poisson distribution is

$$P(x) = \frac{e^{-\mu}\mu^{x}}{x!}$$

where $V(X) = \mu$ and the mean is estimated by sample mean, $\bar{x}$.

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In the case where the variance is larger than the mean such as in ecological studies, the negative binomial distribution is commonly used (Bliss, 1941) because the probability distribution function is

$$P(x)=\frac{(k+x-1)!}{x!(k-1)!}p^x q^{-(x+k)}$$

and $\mu=kp$, $V(X)=kp+kp^2$, which means $V(X)=\mu+\frac{1}{k}\mu^2$ This mean-variance relationship provides (Pedigo and Buntin, 2000) the proper estimate of k as

$$\hat{k}=\frac{\bar{x}^2}{s^2-\bar{x}}\ .$$

These probability distributions have been fitted to the count data obtained from field studies and evaluated by using chi-square goodness-of-fit test.

2. Indices for Dispersion Pattern

The prime concern may lies on the knowledge of spatial pattern not the whole distribution. Or, the sample size may not be big enough for the goodness of fit test. There have been practical indices to indicate the spatial pattern used in ecology studies.

The most simple index is the Variance-to-Mean ratio by calculating the corresponding sample moments,

$$\frac{s^2}{\bar{x}}>1 \quad \text{Aggregated,}$$

$$=1 \quad \text{Random}$$

$$>1 \quad \text{Regular (Uniform)}$$

Or, the negative binomial k (Waters, 1959) is used to indicate the degree of aggregation:

$$\frac{1}{k}=\frac{\sigma^2-\mu}{\mu^2}=0 \quad \text{Random}$$

$$>0 \quad \text{Aggregated}$$

$$<0 \quad \text{Regular}$$

However, these methods have been criticized because the estimated pattern is highly dependent on quadrat size and the mean (Pielou, 1977).

3. Quasilikelihood and TPL

For many insect and animal species, Taylor (1961) found that a power law function could be used to model the relationship between the mean and variance:

$$s^2=a\bar{x}^b$$

or equivalently in log scale,

$$\log(s^2) = \log(a) + b\log(\bar{x}).$$

Taylor (1961) considered the coefficient b to be an aggregation index as "aggregated" for $b > 1$ and b was a constant that depended on the species. However, several researches (Southwood, 1978, Davis and Pedigo, 1989 etc.) pointed out that the power law coefficients have been shown to vary for different sampling location and quadrat sizes.

Perry(1981) suggested other estimation method for parameter a, b in gamma distribution frame work or weighted least squares, others suggested orthogonal regression for the log-scaled linear equation guarding against the measurement error effect.

However, as Grout (1985) and Taylor et. al (1998) suggested, the parameter could change in different regions, crop variety, phenological stages, part of plants etc. Recently, Park and Cho (1999, 2003) suggested to use quasilikelihood estimation combined with pseudolikelihood variance estimation to utilize the covariate information within Taylor's Power Law framework.

4. Discussion

Although the covariate information is used in the quasilikelihood variance function modeling as Park and Cho (2003) suggested, they assumed the independence between sampling units due to the property of quasilikelihood. Similar approaches within the Taylor's Power Law structure may be investigated for the correlated error variances such as in nested sampling or longitudinal collected data.

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