# A NEW APPROACH OF CAMERA MODELING FOR LINEAR PUSHBROOM IMAGES 

Hyung-Sup Jung<br>Department of Earth System Sciences, Yonsei University<br>134 Sinchon-dong, Seodaemun-gu, Seoul 120-749, Korea<br>geohyung@yonsei.ac.kr<br>Myung-Ho Kang, Yong-Woong Lee<br>Agency for Defense Development<br>35 Yusung, Daejeon 305-600, Korea<br>Joong-Sun Won<br>Department of Earth System Sciences, Yonsei University


#### Abstract

The methods of the geometric reconstruction and sensor calibration of satellite linear pushbroom images are investigated. The model of the sensor used is based on the SPOT model that is developed by Kraiky. The satellite trajectory is a Keplerian trajectory in the approximation. Four orbit parameters, longitude of the ascending node $(\omega)$, inclination of the orbit plan(I), latitude argument of the satellite(W) and distance between earth center and satellite, are used for the camera modeling. Time-dependent orbit parameters are expressed by quadratic polynomials. SPOT- 5 images have been used for validation tests. The results are that the RMSE acquired from 20 GCPs is 1.763 m and the RMSE of 5 checking points 2.470 m . Because the ground resolution of SPOT- 5 is 2.5 m , the result obtained in this study has a good accuracy. It demonstrates that the sensor model developed by this study can be used to reconstruct the geometry of satellite image using pushbroom camera.


Keywords: Camera modeling, Pushbroom, SPOT-5

## 1. Introduction

The satellite images bring many benefits. One can find the changed region using two satellite images acquired in different times, generate the DTM (Digital Terrain Model) from the geometry of the stereo image, and acquire the military intelligence about the world.

In order to use this satellite image, it is important to study the geometry of satellite images. Because current commercial satellites such as IRS-1C/1D, SPOT, IKONOS, and QuickBird generally use linear arrays in pushbroom mode, most of researchers are interested in
the geometric reconstruction of linear pushbroom images. A variety of approach including affine transform, direct linear transformation (DLT), method based on extended collinear equations, and orbital parameter model, is used for geometric reconstruction [1-3].

In our study, the sensor model is based on a SPOT model developed by Kraiky [4]. The satellite trajectory is a Keplerian trajectory in the approximation. Four orbit parameters including longitude of the ascending node $(\omega)$, inclination of the orbit plan(I), latitude argument of the satellite( W ), and distance between earth center and satellite, were used for the camera modeling. These parameters are expressed by quadratic polynomials over time and updated using ground control points (GCPs).

Theory of sensor model is presented in section 2. In Section 3 and 4, we show the test data and results. Conclusions follow them in section 5 .

## 2. Theory

Generally, the satellite used to acquire the image is moving along a well defined close-to-circular elliptical orbit. Fig. 1 represents the relationships between satellite position in orbit and ground position on the earth to explain position vectors. The position vector $\vec{p}$ is the distance from earth center to a point P on earth, $\vec{u}$ is LOS (line-of-sight) vector, and satellite position vector $\vec{S}$ is the distance between satellite position S and earth center. They satisfy the following equation.

$$
\begin{equation*}
\vec{p}=\vec{s}+\mu \vec{u} \tag{1}
\end{equation*}
$$

where, $\mu$ is arbitrary factor.


Fig. 1. The relationship between the ground point and satellite position.

## 1) Coordinate Systems

In order to reconstruct the geometry of satellite image, several coordinate systems must be defined. We follow the convention below:

Earth Centered Interval (ECI) frame - ECI frame has its origin at the center of mass of the Earth but has a fixed inertial direction along the intersection of the Earth equatorial plane and the ecliptic plane.

Earth Centered Fixed (ECF) frame - ECF frame has its origin at the center of mass of the Earth but is fixed in the Earth with its axis through the Greenwich meridian (zero longitude).

Local Orbital Reference (LOR) frame - LOR frame has its origin at the center of mass of the satellite. Z is zenith direction, X is the direction of satellite velocity and $Y$ form a right handed reference frame.

Attitude Reference (AR) frame - AR frame is identical to the LOR frame when the satellite attitude angle are all zero. So, this frame is linked to the satellite.

Imaging Reference (IR) frame - IR frame is given by pixel (j) and scanline (i) position.

## 2) Geometric Reconstruction

In LOR frame, $\vec{s}$ and $\vec{u}$ can be expressed as

$$
\begin{align*}
& \vec{s}=\left[\begin{array}{lll}
0, & 0, & \rho
\end{array}\right]^{T}  \tag{2}\\
& \vec{u}=\left[\begin{array}{lll}
u_{x}, & u_{y}, & u_{z}
\end{array}\right]^{T} \tag{3}
\end{align*}
$$

where $\rho$ is the distance between earth center and satellite center.

In ECI frame, $\vec{p}$ can be expressed as

$$
\vec{p}=\left[\begin{array}{lll}
p_{x}, & p_{y}, & p_{z} \tag{4}
\end{array}\right]^{T}
$$

This vector is transformed into LOR frame using a rotation matrix $\mathbf{M}$. Equation (1) can be written as

$$
\left[\begin{array}{lll}
m_{11} & m_{12} & m_{13}  \tag{5}\\
m_{21} & m_{22} & m_{23} \\
m_{31} & m_{32} & m_{33}
\end{array}\right]\left[\begin{array}{c}
p_{x} \\
p_{y} \\
p_{z}
\end{array}\right]=\left[\begin{array}{l}
0 \\
0 \\
\rho
\end{array}\right]+\mu\left[\begin{array}{l}
u_{x} \\
u_{y} \\
u_{z}
\end{array}\right]
$$

Using $\mu$, we can represent as

$$
\begin{align*}
& F_{1}=\frac{m_{11} p_{x}+m_{12} p_{y}+m_{13} p_{z}}{m_{31} p_{x}+m_{32} p_{y}+m_{33} p_{z}-\rho}-\frac{u_{x}}{u_{z}}=0  \tag{6}\\
& F_{2}=\frac{m_{21} p_{x}+m_{22} p_{y}+m_{23} p_{z}}{m_{31} p_{x}+m_{32} p_{y}+m_{33} p_{z}-\rho}-\frac{u_{y}}{u_{z}}=0 \tag{7}
\end{align*}
$$

where rotation matrix M is represented by three orbital parameters of longitude of the ascending node $(\omega)$, inclination of the orbit plan (I), and latitude argument of the satellite(W) (Fig.2). And $u_{x}, u_{y}$ and $u_{z}$ can be expressed as

$$
\left[\begin{array}{l}
u_{x}  \tag{8}\\
u_{y} \\
u_{z}
\end{array}\right]=\left[\begin{array}{lll}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}\right]\left[\begin{array}{l}
u_{x}^{\prime} \\
u_{y}^{\prime} \\
u_{z}^{\prime}
\end{array}\right]
$$

where $u_{n}^{\prime}$ is the element of $\vec{u}^{\prime}$, and $\vec{u}^{\prime}$ is LOS vector in AR frame. $a_{n n}$ is the element of rotational matrix $\mathbf{A}$ used to transform into LOR frame from AR frame. Also, $u_{x}^{\prime}, u_{y}^{\prime}$ and $u_{z}^{\prime}$ is expressed as

$$
\begin{align*}
& u_{x}^{\prime}=\tan \left[\left(\psi_{y_{2}}-\frac{p_{l}-j}{p_{l}-1}\left(\psi_{y_{2}}-\psi_{y_{1}}\right)\right) \times \frac{\pi}{180}\right]  \tag{9}\\
& u_{y}^{\prime}=\tan \left[\left(\psi_{y_{2}}-\frac{p_{l}-j}{p_{l}-1}\left(\psi_{y_{2}}-\psi_{y_{1}}\right)\right) \times \frac{\pi}{180}\right]  \tag{10}\\
& u_{z}^{\prime}=-1 \tag{11}
\end{align*}
$$

where $\psi_{x_{1}}, \psi_{x_{2}}, \psi_{y_{1}}$ and $\psi_{y_{2}}$ are look angles of first and last point in x and y direction respectively, and $j$ is pixel count in IR frame.


Fig. 2. Orbit parameters definition; longitude of the ascending node $(\omega)$, inclination of the orbit plan(I), latitude argument of the satellite(W).

In order to calculate the local precision orbit, 4 orbit parameters are expressed by quadratic polynomials over time.

$$
\begin{align*}
& \rho(t)=\rho_{0}+\rho_{1} t+\rho_{2} t^{2}  \tag{12}\\
& I(t)=I_{0}+I_{1} t+I_{2} t^{2}  \tag{13}\\
& \omega(t)=\omega_{0}+\omega_{1} t+\omega_{2} t^{2}  \tag{14}\\
& W(t)=W_{0}+W_{1} t+W_{2} t^{2} \tag{15}
\end{align*}
$$

The sensor model in equation (6) and (7) has the observed image coordinates and the unknown parameters. We get the solution using the following linear model.

$$
\begin{equation*}
v+B_{s} \Delta_{s}+E=0 \tag{16}
\end{equation*}
$$

where, $v$ is misclosure vector, $B_{s}$ is design matrices composed of partial derivatives, $\Delta_{s}$ is vector of unknown parameters, and $E$ is residuals vector of the observations.

$$
\begin{equation*}
\Delta_{s}=N^{-1} U \tag{17}
\end{equation*}
$$

where, N and U is represented as

$$
\begin{equation*}
N=B_{S}^{T} B_{S}, U=-B^{T} E \tag{18}
\end{equation*}
$$

To obtain a better solution, we must use the iterations in equation (16), (17) and (18).

## 3. Test Data

We use the SPOT-5 data acquired 06/02/2003. SPOT imaging parameters are summarized in Table1.
Table 1. The imaging parameter of test data

| Parameter | Value |
| :--- | :--- |
| Center Time | $2003-02-06 \mathrm{~T} 02: 35: 30.202640$ |
| GRS | $308 / 279$ |
| Process Level | Scene 1A |
| Line Period | $0.000376(\mathrm{sec})$ |
| Look Angle $(\mathrm{x})$ | $0.533447,0.545598(\mathrm{deg})$ |
| Look Angle $(\mathrm{y})$ | $15.81262,19.94122(\mathrm{deg})$ |
| Ephemeris | Using 11 points |
| Attitude | Using 90 points |

## 4. Result

We tested the model using 20 GCPs (Ground Control Points) and 5 checking points. The RMSE acquired from 20 GCPs is 1.763 m and the RMSE about 5 checking points is 2.470 m . Table2 display the result about RMSE.

Table 2. RMSE of GCPs and checking points. Ex is RMSE in direction of pixel, Ey is RMSE in line direction and Et means Total RMSE.

|  | No | Ex | Ey | Et |
| :---: | :---: | :---: | :---: | :---: |
| GCPs | 20 | 0.591 m | 0.541 m | 0.705 m |
| Check points | 5 | 0.323 m | 0.984 m | 0.988 m |

## 5. Conclusions

We formulated a sensor model for satellite linear pushbroom images. The test results demonstrate that sensor model formulated in this study can be used to reconstruct the geometry of linear pushbroom images.

## References

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