# Segmentation of Millimeter-wave Radiometer Image via Classuncertainty and Region-homogeneity 

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Abstract: Thresholding is a popular image segmentation method that converts a gray-level image into a binary image. The selection of optimum threshold has remained a challenge over decades. Many image segmentation techniques are developed using information about image in other space rather than the image space itself. Most of the technique based on histogram analysis information-theoretic approaches.

In this paper, the criterion function for finding optimal threshold is developed using an intensity-based classuncertainty (a histogram-based property of an image) and region-homogeneity (an image morphology-based property). The theory of the optimum thresholding method is based on postulates that objects manifest themselves with fuzzy boundaries in any digital image acquired by an imaging device. The performance of the proposed method is illustrated on experimental data obtained by W-band millimeter-wave radiometer image under different noise level.

## 1. Introduction

Defining objects in an image commonly known as image segmentation. Pal and Pal [1] reviewed the various methods for gray level image segmentation. Thresholding is the most popular segmentation approach because of its simplicity. Automatic selection of threshold for better segmentation is remains challenged. Many methods for automatic threshold are reported in the literature, most of them based on histogram. Pal and Pal [2], [3], P.K. Saha and J. K. Udupa [4], [5] incorporate the spatial information for image segmentation. Many researchers also tried for spatial information in several nonthresholding image segmentation method [6], [7]. In histogram-based approach, the information that obtained from spatial distribution of intensity in image and image morphology is not utilized. Generally in practical situation it is difficult to select threshold directly from the histogram without knowledge of objects in image.

Most thresholding method utilized the some sort of an uncertainty or entropy criterion for the selection of optimal threshold criterion. If optimality criterion setup properly, then at optimal threshold, the image element in the vicinity of the boundaries should have high value of class uncertainty. But this is not possible without object knowledge in the image. P.K. Saha, J.K. Uduppa [4], [5] developed region-homogeneity criterion based on fuzzy connectedness principle to obtain a rough knowledge of object in image. Histogram based classuncertainty / entropy criterion is combined with region-
-homogeneity criterion to obtain better threshold.

## 2. Class-uncertainty

The generic formulation of intensity-based classuncertainty [5] is used in this. The idea behind intensity based class-uncertainty is to determine the uncertainty of classification of special element in to a certain object class based on its scene intensity and the prior knowledge of the intensity probability distribution of different classes. To determine the class-uncertainty it is assume that the prior knowledge of intensity probability distributions for the both the object and background and also the probability of spatial element belonging to the object.

Let $F_{o}$ and $F_{b}$ represent the object and background classes, respectively. Let $\theta$ denotes the probability of object belonging to the object class so $1-\theta$ is the probability of object belonging to the background class. Let $\mathrm{p}_{\mathrm{o}}(\mathrm{g})$ and $\mathrm{p}_{\mathrm{b}}(\mathrm{g})$ denote the priori probabilities that an object, background special element has the intensity value $g$ respectively.

$$
\begin{align*}
& p_{\mathrm{o}}(\mathrm{~g})=\mathrm{P}\left\{\mathrm{f}(\mathrm{c})=\mathrm{g} / \mathrm{c} \in \mathrm{~F}_{\mathrm{o}}\right\}  \tag{1}\\
& \mathrm{p}_{\mathrm{b}}(\mathrm{~g})=\mathrm{P}\left\{\mathrm{f}(\mathrm{c})=\mathrm{g} / \mathrm{c} \in \mathrm{~F}_{\mathrm{b}}\right\} \tag{2}
\end{align*}
$$

Where P denotes probability and c is special element. Probability that any spatial element has intensity value g is given by

$$
\begin{equation*}
\mathrm{p}(\mathrm{~g})=\theta \mathrm{p}_{\mathrm{o}}(\mathrm{~g})+(1-\theta) \mathrm{p}_{\mathrm{b}}(\mathrm{~g}) \tag{3}
\end{equation*}
$$

Now from Bayes rule, the posteriori probability that a spatial element having gray value $g$ belongs to the object class is

$$
\begin{equation*}
\mathrm{P}\left\{\mathrm{c} \in \mathrm{~F}_{\mathrm{o}} / \mathrm{f}(\mathrm{c})=\mathrm{g}\right\}=\theta \mathrm{p}_{\mathrm{o}}(\mathrm{~g}) / \mathrm{p}(\mathrm{~g}) \tag{4}
\end{equation*}
$$

Similarly posteriori probability that a special element having gray value $g$ belongs to the background class is given by

$$
\begin{equation*}
\mathrm{P}\left\{\mathrm{c} \in \mathrm{~F}_{\mathrm{b}} / \mathrm{f}(\mathrm{c})=\mathrm{g}\right\}=(1-\theta) \mathrm{p}_{\mathrm{b}}(\mathrm{~g}) / \mathrm{p}(\mathrm{~g}) \tag{5}
\end{equation*}
$$

From Shannon, after knowing intensity value at spatial element c is g , the uncertainty of classifying c into the object or background class is the entropy of above two posteriori probabilities. This is referred as intensitybased class- uncertainty or class-uncertainty [5]. The class- uncertainty at an intensity $g$ is:

$$
\begin{equation*}
\mathrm{H}(\mathrm{~g})=-\frac{\theta \mathrm{p}_{\mathrm{o}}(\mathrm{~g})}{\mathrm{p}(\mathrm{~g})} \log \left(\frac{\theta \mathrm{p}_{\mathrm{o}}(\mathrm{~g})}{\mathrm{p}(\mathrm{~g})}\right)-\frac{\theta \mathrm{p}_{\mathrm{b}}(\mathrm{~g})}{\mathrm{p}(\mathrm{~g})} \log \left(\frac{\theta \mathrm{p}_{\mathrm{b}}(\mathrm{~g})}{\mathrm{p}(\mathrm{~g})}\right) \tag{6}
\end{equation*}
$$

The probability $p_{o}, p_{b}$ can be taken as the probability density function of the underlying discrete random
variable ' g '. In many literature, $\mathrm{p}_{\mathrm{o}}$, $\mathrm{p}_{\mathrm{b}}$ taken Gaussian form and mean, standard deviation $m_{0}, m_{b}, \sigma_{0}, \sigma_{b}$ have been estimated from the given image as function of threshold [5].

## 3. Region-homogeneity

A fuzzy relation $\alpha$ on $Z^{n}$ is said to be fuzzy spatial element adjacency if it is reflexive and symmetric [5]. It is desirable that $\alpha$ be such that its membership value $\mu_{\alpha}(\mathrm{c}, \mathrm{d})$ for any $\mathrm{c}, \mathrm{d} \in \mathrm{Z}^{\mathrm{n}}$ is non -increasing function $\| \mathrm{c}$ $-\mathrm{d} \|$ between c and d , where $\|$.$\| denotes any L_{2}$ norm in $\mathrm{R}^{\mathrm{n}}$ (Here Euclidian Norm is used). For simplicity for any special element $\mathrm{c} \mu_{\alpha}(\mathrm{c}, \mathrm{c})=0$ and for any $\mathrm{c}, \mathrm{d} \in \mathrm{Z}^{\mathrm{n}}$, $\mu_{\alpha}(c, d)=1$ if they differ in exactly one co-ordinate by 1 otherwise $\mu_{\alpha}(c, d)=0$ the pair $\left(\mathrm{Z}^{\mathrm{n}}, \alpha\right)$ is called fuzzy digital space.

A scene over fuzzy digital space $\left(\mathrm{Z}^{\mathrm{n}}, \alpha\right)$ is a pair $\mathrm{C}=(C, f)$, where $C=\left\{\mathrm{c} /-\mathrm{b}_{\mathrm{j}}<=\mathrm{c}_{\mathrm{j}}<=\mathrm{b}_{\mathrm{j}}\right.$ for some $\left.\mathrm{b} \in \mathrm{Z}_{+}^{\mathrm{n}}\right\}$, f is function whose domain is $C$ is called scene domain and whose range is a set of intensities $\Gamma=[$ MIN, MAX $]$.

Fuzzy homogeneity $\mu_{\tau}: C \longrightarrow[0,1]$, it depends on spatial element adjacency and another fuzzy relation $\psi$ on $C$ called fuzzy affinity [3]. The strength $\mu_{\psi}(\mathrm{c}, \mathrm{d})$ of the relation between any two spatial element c and d in $C$ is greater when the scene intensities in vicinity of c and d are more similar. The homogeneity is given by

$$
\begin{equation*}
\mu_{\tau}(c)=\frac{\sum_{\mathrm{d} \in C} \mu_{\alpha}(\mathrm{c}, \mathrm{~d}) \mu_{\psi}(\mathrm{c}, \mathrm{~d})}{\sum_{\mathrm{d} \in C} \mu_{\alpha}(\mathrm{c}, \mathrm{~d})} \tag{7}
\end{equation*}
$$

Thus homogeneity at spatial element c is a weighted average of the affinities with c of spatial elements in $C$. Adjacency between spatial elements that are for apart is 0 , the spatial elements that actually matter in above eq. (7) are those that are in the close vicinity of c. Detail theory and Algorithm for fuzzy affinity is available in [4].

## 4. Threshold Selection

Here used threshold method utilizes both classuncertainty and region homogeneity. Main idea is "In any scene with fuzzy boundaries at optimum portioning of object classes, special element with high class uncertainty appear in the vicinity of object boundaries"[5]. Fuzzy boundary means intensity changes smoothly across the interface between object regions. This is true for radiometer image since it is highly blurred. Pixel near to the boundaries has intensities in between the average of the object intensities and hence has high-class uncertainty.

Criterion function for finding optimal threshold is such that when class-uncertainty and regionhomogeneity both low or high, assumes high value. Hence minimum of such criterion function produce the optimal threshold. The criterion function in functional form is defined for any threshold $t \in \Gamma_{\text {., }}$ where $\Gamma_{\text {. }}$
$=[\mathrm{MIN}-1, \mathrm{MAX}+2]$, for any scene $\mathrm{C}=(C, f)$ over fuzzy digital space $\left(\mathrm{Z}^{\mathrm{n}}, \alpha\right)$ as:

$$
\begin{equation*}
\mathrm{E}(\mathrm{t})=\sum_{\mathrm{c} \in \mathrm{C}} \mathrm{H}_{\mathrm{t}}(f(\mathrm{c})) \mu_{\tau}(\mathrm{c})+\left(1-\mathrm{H}_{\mathrm{t}}(f(\mathrm{c}))\right)\left(1-\mu_{\tau}(\mathrm{c})\right) \tag{8}
\end{equation*}
$$

When both class -uncertainty and region homogeneity are high, $E$ ( $t$ ) is high and the first term of above equation dominates. When both of these entities low then second term dominate. Analogously, when one of them is low and other is high $E(t)$ is low. Thus $E(t)$ is express the total disagreement of the spatial element values over scene domain to the underlying intensity distribution and to the homogeneity of the region. So optimal threshold is

$$
\begin{equation*}
\mathrm{t}_{\mathrm{opt}}=\arg \min \{\mathrm{E}(\mathrm{t}) / \mathrm{t} \in \Gamma\} \tag{9}
\end{equation*}
$$

## 5. Experimental Results and Discussion

The images are captured by W-band millimeter-wave radiometer having the antenna beamwidth of $0.8^{\circ}$ at 94 GHz .


Fig. 1 (a) Image of sky. (b) Binary Image resulting from thresholding of (a). (c) Distribution of criterion function $\mathrm{E}(\mathrm{t})$ for image (a), and (d) intensity histogram of (a).


(c)

(d)

Fig. 2 (a) Image of land containing road and grass. (b) Binary Image resulting from thresholding of (a). (c) Distribution of criterion function $E(t)$ for image (a). and (d) intensity histogram of (a).


Fig.3. Class-uncertainty image at optimal threshold of image in fig2. (a). Brightness at a pixel is proportional to its class-uncertainty value.

Generally people select threshold in valleys of the histogram. Histogram like Fig.2(d) is easy to select optimal threshold, but histogram like Fig.1(d) it is not possible to select threshold that produce better segmented image. Since there is no clear valley in histogram Fig.1(d), but corresponding criterion function distribution is very smooth and well behaved. From criterion function distribution very easily optimal threshold is obtained that correspond to the minimum value of energy criterion distribution.

Spatial element with high class-uncertainty appears in the vicinity of boundaries at optimal threshold. This can be easily verified by the class-uncertainty image at optimal threshold in Fig. 3

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