CDMA MIMO 시스템 용량 증가를 위한 시공간 처리 기법

Capacity Optimized Space-Time Processing for CDMA MIMO Antenna Systems

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Key Words : MIMO, CDMA, RAKE receiver, frequency-selective fading, precoder, decoder

목 차

Ⅰ. 서 론

The use of multiple transmit and receive antennas (MIMO) has recently been shown to provide the most effective way to increase transmission rate through the wireless fading channel. Investigation [1] on the capacity of MIMO channels observed that the frequency selectivity of the channel increases the capacity. Furthermore, in case of Orthogonal Frequency Division Multiplexing (OFDM)-based MIMO systems, delay spread boosts both the mean capacity and the outage capacity of multi-antenna systems [2].

For CDMA-based MIMO systems, the chip level equalization scheme [3] and the linear MMSE receiver [4] have been proposed to mitigate inter-symbol interference (ISI) as a result of multipath fading channels. To the best of the author's knowledge, unlike OFDM-based MIMO systems, there has been no attempt to increase the capacity by exploiting the frequency selectivity of the channel in case of CDMA-based MIMO systems.

Spatial multiplexing exploits orthogonal spatial signatures of transmitted signals from different antennas by multipath propagation. One important feature of CDMA is capability of resolving multipaths and the uses of RAKE receivers to achieve multipath diversity gain. From a different viewpoint independent fading of resolvable multipath provides each multipath with a different temporal signatures, which can be also exploited for temporal multiplexing. With spatiotemporal signatures provided by MIMO CDMA frequency-selective channel, the transmitted signals can multiplex by subchannels in space and time domain. In this paper, we propose the linear precoder and decoder for CDMA MIMO antenna systems converting a MIMO frequency selective channel to a set of multiplexing channels in space and time domain. Multiple transmit antennas and receive antennas each with a bank of RAKE receivers can multiplex by subchannels in space and time domain. The proposed CDMA-based two dimensional multiplexing system increases the number of subchannels, which leads to a significant increase in the mean capacity and the outage capacity.

In Section Ⅱ, the capacity optimized CDMA MIMO antenna system the precoder and the decoder of which are determined by the singular value decomposition (SVD) is proposed. Next, the capacity of the proposed system is analyzed in Section Ⅲ. Section Ⅳ gives the results of numerical evaluation, and discusses the influence of the various parameters on the capacity of the proposed system. A summary concludes this paper.

Ⅱ. System Model

This paper focuses on single-user communication and assumes that channel state information (CSI) is available both at the transmitter and receiver side. Fig.1 shows the configuration of the proposed CDMA MIMO system in L-path frequency selective channels. In the transmitter the information symbol stream is split up into K independent substreams, which are then individually precoded and simultaneously sent from n_T transmit antennas. Each signal undergoes multipath propagation, which provides transmitted signals with different spatiotemporal signatures. In the receiver, received signals by n_R receive antennas are despreaded and decoded to recover transmitted data.

At the Tx, symbols $s_k(t), k=1 \cdots K$ are precoded by $n_T \times 1$ $\tilde{v}_k$ and combined into an transmitted vector $\boldsymbol{x}(t)$ as follows

$$\boldsymbol{x}(t)=\sum_{k=1}^{K} s_k(t)\tilde{\boldsymbol{v}}_k \tag{1}$$

where K is the number of symbols simultaneously transmitted via parallel subchannels. The base-band equivalent transmitted signal for the kth subchannel is

$$s_k(t)=\sqrt{P_k}b_k(t)a(t) \tag{2}$$

where P_k denotes the transmitted signal power, $b_k(t)$ is the data signal for kth subchannel consisting of independent and identically distributed (i.i.d.) data bits with duration T, which takes on the value of ± 1 with equal probability. Likewise, $a(t)$ is the pseudo random sequence with ± 1 chips of duration T_c and code length $N=T/T_c$. The chip waveform is assumed as rectangular.

The transmit signal goes through the frequency-selective fading channel. A power delay profile (PDP) with maximum excess delay LT_c is assumed. By the tapped delay line model, we can write

$$\boldsymbol{H}(t)=\sum_{l=1}^{L}\boldsymbol{H}_l\delta(t-lT_c) \tag{3}$$

with the $n_R \times n_T$ complex valued random matrices $\boldsymbol{H}_l$ denoting lth tap of the stochastic matrix-valued channel impulse response. Throughout this paper we assume that the elements of the $\boldsymbol{H}_l$ are uncorrelated circularly symmetric complex Gaussian random variables. The received signal at the MS is

$$\boldsymbol{r}(t)=\sum_{l=1}^{L}\boldsymbol{H}_l\boldsymbol{x}(t-lT_c)+\boldsymbol{n}(t). \tag{4}$$

The $n_R \times 1$ output vector $\boldsymbol{y}_i(m)$ consisting of the output symbols of the n_R parallel PN correlators matched to the ith path becomes

$$\begin{aligned}\boldsymbol{y}_i(m) &= \left.\int_{(m-1)T+iT_c}^{mT+iT_c}\boldsymbol{r}(t)a(t-iT_c)\,dt\right) \\ &= NT_c\sum_{k=1}^{K}\sqrt{P_k}b_k(m) \\ &+\left(\boldsymbol{H}_i\tilde{\boldsymbol{v}}_k\sum_{l=1,l\neq i}^{L}\frac{\rho_k((l-i)T_c)}{NT_c}\boldsymbol{H}_l\tilde{\boldsymbol{v}}_k\right)+\boldsymbol{n}_i(m)\end{aligned} \tag{5}$$

where

$$b_k(m)=\int_{(m-1)T}^{mT}b_k(t)\,dt \tag{6}$$

and

$$\rho_k(sT_c)=\begin{cases}\displaystyle\int_0^{sT_c}\frac{b_k(m-1)}{b_k(m)}a(t-sT_c)a(t)\,dt+\int_{sT_c}^{T}a(t-sT_c)a(t)\,dt & s\geq 1\\ \displaystyle\int_0^{T+sT_c}a(t-sT_c)a(t)\,dt+\int_{T+sT_c}^{T}\frac{b_k(m+1)}{b_k(m)}a(t-sT_c)a(t)\,dt & s\leq -1\end{cases}$$

(7), respectively. The desired signal part of (5) is composed of two parts. The first is the desired signal of the matched RAKE finger, with the second the signal smeared from the other RAKE fingers, which will be considered as ISI for a conventional RAKE receiver.

$\rho_k(sT_c)$ is the autocorrelation of the PN sequences. Appearance of $b_k(m-1)/b_k(m)$ and $b_k(m+1)/b_k(m)$ is due to the asynchronism between the data symbol period and the integration limits, which arises from the multipath delay. Since $b_k(m-1)/b_k(m)$ and $b_k(m+1)/b_k(m)$ are unknown to the MS receiver, if the MS approximate $\rho_k(sT_c)$ to $\tilde{\rho}_k(sT_c)$ given by

$$\tilde{\rho}_k(sT_c)=\int_0^T a(t-sT_c)a(t)\,dt \tag{8}$$

, the actual PN code autocorrelation becomes

$$\rho_k(sT_c)=\tilde{\rho}(sT_c)+\Delta\rho_k(sT_c) \tag{9}$$

where $\Delta\rho_k(sT_c)=0$ only when $b_k(m-1)/b_k(m)=1$ or $b_k(m+1)/b_k(m)=1$. Both $\tilde{\rho}_k(sT_c)$ and nonzero $\Delta\rho_k(sT_c)$ can be approximated by zero mean Gaussian random variables with the variance of $2/3NT_c^2$ and $2/3|s|T_c^2$, respectively, for the concatenated orthogonal/PN spreading sequences when $|s|\geq 1$ [5]. $\boldsymbol{n}_i(m)$ is the $n_R \times 1$ noise vector of the additive white Gaussian noise with variance σ_n^2.

With the approximated PN code correlation $\tilde{\rho}_k(sT_c)$, we can write

$$\begin{aligned}\boldsymbol{y}_i(m) &= NT_c\sum_{k=1}^{K}\sqrt{P_k}b_k(m) \\ &+\left(\boldsymbol{H}_i\sum_{l=1,l\neq i}^{L}\frac{\rho((l-i)T_c)}{NT_c}\boldsymbol{H}_l\right)\tilde{\boldsymbol{v}}_k+\boldsymbol{n}_i(m)\end{aligned} \tag{10}$$

By stacking the vectors $\tilde{\boldsymbol{y}}_i(m)$, the perturbed $L_R \times 1$ PN correlator output matrix $\tilde{\boldsymbol{Y}}(m)$ of L RAKE fingers can be written as follows

$$\tilde{\boldsymbol{Y}}(m)=\begin{bmatrix}\tilde{\boldsymbol{y}}_1(m)\\ \tilde{\boldsymbol{y}}_2(m)\\ \vdots\\ \tilde{\boldsymbol{y}}_L(m)\end{bmatrix}=NT_c\tilde{\boldsymbol{H}}\tilde{\boldsymbol{V}}\boldsymbol{s}(m)+\boldsymbol{n}(m) \tag{11}$$

where $\tilde{\boldsymbol{H}}$ is the $Ln_R \times n_T$ symbol level perturbed channel transfer matrix given by

$$\tilde{H} = \begin{pmatrix} H_1 + \cdots + \frac{\tilde{\rho}((L-1)T_c)}{NT_c} H_L \\ \frac{\tilde{\rho}(-T_c)}{NT_c} H_1 + \cdots + \frac{\tilde{\rho}((L-2)T_c)}{NT_c} H_L \\ \vdots \\ \frac{\tilde{\rho}(-(L-1)T_c)}{NT_c} H_1 + \cdots + H_L \end{pmatrix} \quad (12)$$

And the $n_T \times K$ precoder matrix $\tilde{V}$ and $K \times 1$ symbol vector $s(m)$ are given by

$$\tilde{V} = (\tilde{v}_1 \ \tilde{v}_2 \cdots \tilde{v}_K) \quad (13)$$

and

$$s(m) = \begin{pmatrix} \sqrt{P_1} b_1(m) \\ \vdots \\ \sqrt{P_k} b_K(m) \end{pmatrix} \quad (14)$$

, respectively. The $Ln_R \times 1$ noise vector $n(m)$ is given by $n = [n_1^T n_2^T, \cdots, n_L^T]^T$.

The transmitter is assumed to have a perturbed symbol level channel matrix given by $\tilde{H}$, which requires feedback information from the MS including all delays to estimate PN sequence autocorrelations and L taps of the matrix-valued channel impulse response. At the transmitter, a precoder matrix $\tilde{V}$ optimal for $\tilde{H}$ is applied and transmission carried out accordingly over an equivalent actual channel matrix H given by

$$H = G\tilde{V} \quad (15)$$

where $Ln_R \times K$ G is given by (16). And the received and despread output matrix is decoded by an $Ln_R \times K$ decoder $\tilde{U} = [\tilde{u}_1, \tilde{u}_2, \cdots, \tilde{u}_K]$ also optimal for $\tilde{H}$ at the receiver. A precoder $\tilde{V}$ and a decoder $\tilde{U}$ are given by the singular value decomposition (SVD) of $\tilde{H}$. Let the SVD of $\tilde{H}$ be $\tilde{H} = \tilde{U}\tilde{D}\tilde{V}^H$ where $\tilde{D}$ is an $K \times K$ diagonal matrix consisting of nonzero square roots of eigenvalues $\tilde{\lambda}_k, k = 1, \cdots, K$, of $\tilde{H}^H\tilde{H}$. The columns of unitary matrix $\tilde{U}$ are K decoding vectors each for K subchannels encoded by the corresponding encoding vectors which are columns of unitary matrix $\tilde{V}$. The number of subchannels K is bounded by the rank of $\tilde{H}$ or H, which is $\min(n_T, Ln_R)$. $\Delta\rho_k(sT_c)$ is small enough to preserve the same rank of $\tilde{H}$ and H.

By applying $\tilde{V}$ and $\tilde{U}$ optimal for $\tilde{H}$ to the equivalent actual channel H, we can write

$$\begin{aligned} \tilde{U}^H Y(m) &= NT_c \tilde{U}^H H \tilde{V} s(m) + \tilde{U}^H n(m) \\ &= NT_c D s(m) + n'(m) \end{aligned} \quad (17)$$

where D has off-diagonal elements results from the discrepancy between the approximated channel matrix $\tilde{H}$ and the actual channel matrix H and can be written as follows

$$D = \begin{pmatrix} \sqrt{\lambda_1} & \sqrt{\Delta\lambda_{1,2}} & \cdots & \sqrt{\Delta\lambda_{1,K}} \\ \sqrt{\Delta\lambda_{2,1}} & \sqrt{\lambda_2} & \cdots & \sqrt{\Delta\lambda_{2,K}} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{\Delta\lambda_{K,1}} & \sqrt{\Delta\lambda_{K,2}} & \cdots & \sqrt{\lambda_K} \end{pmatrix}$$

Therefore (17) can be rewritten component-wise

$$\begin{aligned} b_k(m) &= NT_c \sqrt{\lambda_k P_k} b_k(m) \\ &+ NT_c \sum_{i=1, i\neq k}^{K} \sqrt{\Delta\lambda_{k,i} P_i} b_i(m) + n_k'(m) \end{aligned} \quad (18)$$

This clearly shows that the (n_T, n_R, L) MIMO CDMA system can be transformed into an equivalent system consisting of $\min(n_T, Ln_R)$ single-input single-output (SISO) subchannels, which are loosely coupled with each other. The proposed linear precoder and decoder increases the number of parallel subchannels up to $\min(n_T, Ln_R)$ by exploiting frequency selectivity of the channel. n_R being smaller than n_T, which is common situation in downlink communications because of the constraint on the physical size of mobile stations, the number of subchannels is always limited to n_R in case of previous CDMA MIMO systems in frequency selective fading channel [3],[4]. On the other hand, the proposed CDMA MIMO system with linear processing increase the number of parallel subchannels up to $\min(n_T, Ln_R)$ by exploiting frequency selectivity of fading channels. The increased number of multiplexing channels directly results in capacity increase in favorable SNR conditions. However, note that the coupling between the equivalent subchannels acts as inter-symbol interference (ISI) and thus limit the capacity increase at high SNR.

III. Capacity Analysis

The information capacity of the proposed CDMA MIMO system is the sum of the capacities of the K subchannels. The mutual information capacity of kth subchannel can be written as follows

$$I_k(b_k(m); \tilde{b}_k(m)) = \log_2\left(1 + \frac{TP_k\lambda_k}{T\sum_{i=1, i\neq k}^{K} \Delta\lambda_{k,i} P_i + \sigma_n^2}\right) \quad (20)$$

The sum capacity of one user with spatiotemporal K subchannels becomes [6]

$$\begin{aligned} C &= \frac{1}{N}\sum_{k=1}^{K} I_k(b_k(m); \tilde{b}_k(m)) \\ &= \frac{1}{N}\sum_{k=1}^{K}\left(1 + \frac{\gamma_k}{\sum_{i=1, i\neq k}^{K} \Delta\gamma_{k,i} + 1}\right) \end{aligned} \quad (21)$$

where the signal-to-noise ratio (SNR) of the kth subchannel γ_k and $\Delta\gamma_{k,i}$ are given by

$$\gamma_k = \frac{P_k T}{\sigma_n^2}\lambda_k = \gamma_0 \beta_k \lambda_k \tag{22}$$

and

$$\Delta\gamma_{k,i} = \frac{P_i T}{\sigma_n^2}\Delta\lambda_{k,i} = \gamma_0 \beta_i \Delta\lambda_{k,i} \tag{23}$$

, respectively. And $\gamma_0 = PT/\sigma_n^2$ and $\beta_j = P_j/P$. The γ_k are a function of transmit powers P_k which are under the overall transmit power constraint requiring that $\sum_{k=1}^{K} P_k = P$. Given λ_k, the power allocation that maximize the capacity is calculated through the water-filling (WF) algorithm [7]. The optimal power allocation by WF is derived as

$$\beta_k = (\epsilon - \frac{1}{\gamma_0 \lambda_k})^+ \tag{24}$$

where $(\cdot)^+$ denotes $\max(0, \cdot)$ and ϵ is determined so as to satisfy the power constraint. The effective number of simultaneously transmitted symbols, i.e., the effective number of subchannels, is the number of positive β_k's.

IV. Numerical Results and Discussions

In every simulation, 1000000 independent Monte Carlo runs were performed. In each Monte Carlo run an $Ln_R \times n_T$ $\tilde{\boldsymbol{H}}$ and $\boldsymbol{H}$ are generated according to (12) and (15), respectively, and the corresponding capacity C is computed according to (17) and (21). The resulting capacity random variable is used to estimate average capacity. An outage capacity of C_q is defined as $Prob[C > C_q] = 1 - q$. In this paper the case $q = 0.1$ is considered.

An exponentially decaying PDP model was employed. The average power of lth path is given by

$$\Omega_l = \frac{\exp(\delta(l-1))}{\Omega_0},\ l = 1, \cdots L \tag{25}$$

where $\Omega_l = E[trace(\boldsymbol{H}_l \boldsymbol{H}_l^H)] = E[trace(\tilde{\boldsymbol{H}}_l \tilde{\boldsymbol{H}}_l^H)]$ and δ is a decaying constant reflecting frequency-selectivity of the channel. Ω_0 is a normalizing factor that keeps the average total transmitted power constant regardless of the number of paths and given by $\Omega_0 = \sum_{l=1}^{L} \Omega_l$.

Fig.2 shows the average and the outage capacity of the proposed (n_T, n_R, L) CDMA MIMO system with $\delta = 0$ and $N = 16$. The result shows that fixing the number of transmit antennas $n_T = 4$ and increasing the number of receive sensors Ln_R to 4 leads to an increase in the number of multiplexing subchannels and hence an increase in the average and the outage capacity. The additional degrees of freedom obtained by increasing L are exploited to extend multiplexing domain to the space and time domain rather than to increase the diversity gain. As we increase SNR, the slope of outage capacity decreases because the coupling between multiplexing subchannels, that is, ISI also increases. Thus, a rise of ISI leads to an increase in the variance of the capacity random variables, which mainly affects the outage capacity.

Effects of ISI are also shown clearly in the c.c.d.f.s in Fig.3. Comparisons of the c.c.d.f.s of the $(4, 2, 2)$ and $(4, 2, 4)$ at $\gamma_0 = 30$dB with those at $\gamma_0 = 15$dB show a considerable increase in the variance of capacity at $\gamma_0 = 30$.

The outage capacity of the $(8, 2, L)$ is shown in Fig.4. As we increase the number of RAKE fingers L for $L \leqq 4$, we also increase the outage capacity by exploiting frequency-selective fading to increase multiplexing gain. Once the number of receive sensors Ln_R becomes larger than the number of transmit antennas n_T, the number of parallel subchannels is constrained by n_T and the increase of the outage capacity is stagnated.

According to the diversity-multiplexing tradeoff investigated in [8], the excess degrees of freedom $Ln_R - n_T$ are used to increase the diversity gain thus reducing the variance of the resulting capacity random variables. The c.c.d.f. plot in Fig.3 also provides us insight about the tradeoff between multiplexing and diversity gain. According to the c.c.d.f.s of the $(4, 2, 1)$ and $(4, 2, 2)$, the variance of the capacity also increases as L increases. The additional degrees of freedom by increasing L are exploited to increase multiplexing gain, which leads to an increase in the variance of the capacity, that is, a lower diversity gain. Comparison of the c.c.d.f. of the capacity of the $(4, 2, 2)$ with that of the $(4, 2, 4)$ shows an opposite tendency, namely, that an increase of L diminish the variance of the capacity, which is due to the fact that the excess degrees of freedom are used to increase the diversity gain thus reducing the variance of the capacity.

V. Conclusion

In this paper, we proposed a capacity optimized space-time linear precoder and decoder for CDMA MIMO antenna systems. We evaluated the average and the outage capacity of the proposed system with considering errors in the approximation of symbol level MIMO channel impulse response. Numerical results show that the proposed system exploits the additional degrees of freedom obtained by frequency-selective fading to increase the multiplexing gain and provides a gain in the average capacity and the outage capacity. Furthermore, we discussed the multiplexing-diversity tradeoff in the proposed CDMA MIMO system.

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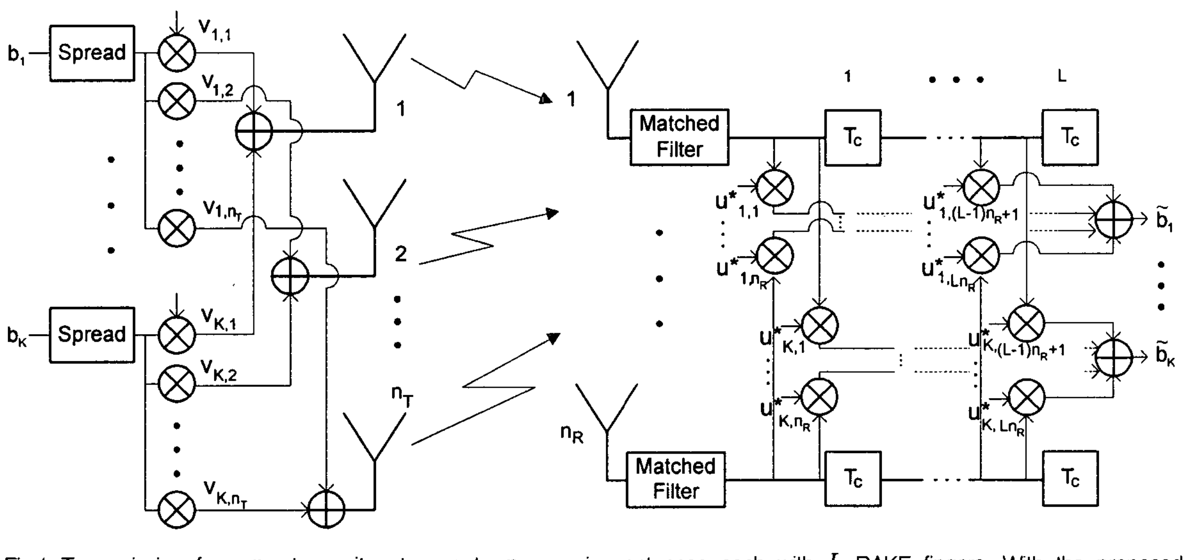

Fig.1 Transmission from n_T transmit antennas to n_R receive antennas each with L RAKE fingers. With the proposed linear precoder $\tilde{V}$ and decoder $\tilde{U}$, the proposed system can be transformed into an equivalent system consisting of $\min(n_T, Ln_R)$ SISO subchannels. The frequency selectivity of the fading channel is exploited in order to establish subchannels rather than provide multipath diversity gain.

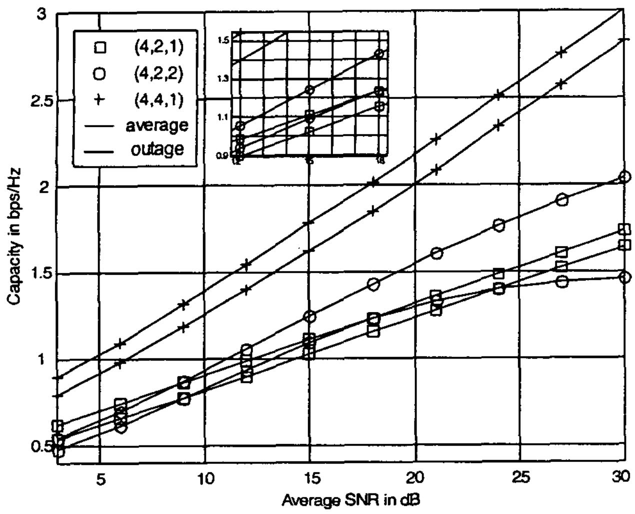

Fig.2 Average and outage capacity of (n_T, n_R, L) as a function of SNR γ_0 for different antenna configurations and channel delay spreads when $\delta = 0$ and $N = 16$.

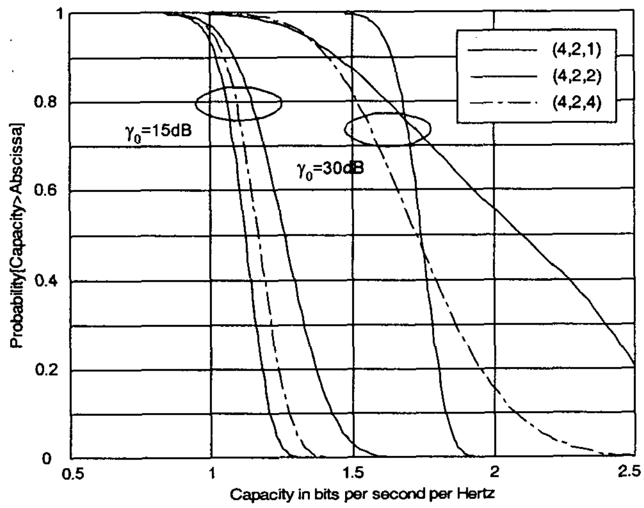

Fig.3 The complementary cumulative distribution function (c.c.d.f.) c the capacity of $(4, 2, L)$ when $\delta = 0$ and $N = 16$.

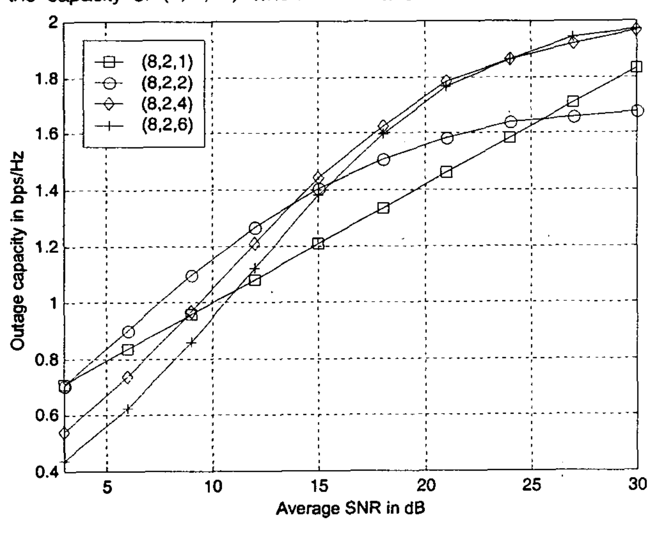

Fig.4 Outage capacity of $(8, 2, L)$ as a function of SNR when $\delta = 0$ and $N = 16$.