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Joint Optimal Production-Delivery Policy for Multiple Products with a Single Production Facility

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Abstract
The synchronization of production-delivery activities is one of crucial factors to get competitive collaboration benefits between the manufacturer and the retailer(s). There were several researches to study on the optimal delivery policy to minimize the total cost of integrated system of both manufacturer and retailer(s). In this research, we investigate the joint optimal shipment policy in case that a manufacturer produces multiple products sharing a single production facility in the manufacturer side and retailer(s) deploys JIT delivery pattern with equal-size shipment policy. We formulate this problem as a form of 'Common Cycle Approach' in classical ELSP (Economic Lot Scheduling Problem) and provide simple optimal solution procedure.

1. Introduction

For ideal production-delivery system, a manufacturer is expected to synchronize his/her production capacity with demand for multiple products to minimize or reduce the inventory in supply chain system. In most manufacturing environment, a manufacturer should utilize his/her single production facility to accommodate multiple products from retailer(s). In retailers' side, retailers request that a manufacturer guarantee stable production and delivery schedule without causing any trouble in retailers' material requirement planning. Recently, several studies have investigated the merit of split delivery for a batch order. Retailers would like to minimize their operating cost with reduced on-hand inventories through synchronization on ordering- production-delivery scheme. In this paper, we investigate joint optimal shipment policy in 'single-manufacturer and multiple-item system'; specifically, a manufacturer has a single production facility. There are several studies related to shipment policy from a manufacturer to retailers in terms of economic criteria within integrated scheme. There were several studies to find the optimal shipment policy in single-manufacturer and single(or multiple)-retailer system to minimize the overall system cost (Lu(1995), Goyal(1995), Hill (1997), Goyal and Nebebe(2000)). Hahm(1990) proposed three models in which multiple components are assemble at final assembly facility: two models are formulated by 'common cycle' approach and one by 'basic period' approach. Banerjee and Burton(1994) proposed the coordinated inventory replenishment policies for a vendor and multiple buyers. Nori and Sarker(1996) investigated the 'cyclic scheduling problem' for a multiple products with a single production facility under just-in-time delivery policy with the assumption that shipment quantity for products is predetermined.

Parija and Sarker(1999) developed solution procedure to find optimal procurement policy for raw materials and shipment policy for finished goods to multiple retailers. They assume that shipment quantity for each retailer is predetermined and only determined shipment frequency. David and Eben-Chaime(2003) analyzed the relationship between vendor and buyer with a JIT scheme in single-item, single-manufacturer and single-retailer model.

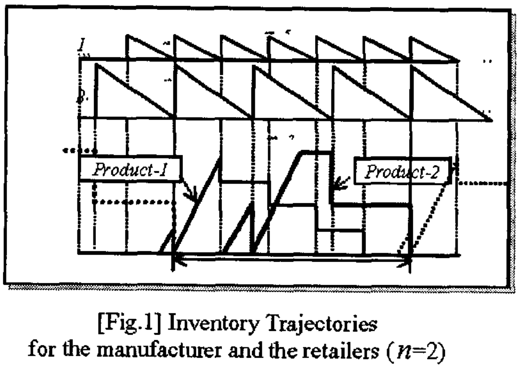
In this paper, we extend the optimization model proposed by Banerjee and Burton(1994) by relaxing the assumption that all products have the same replenishment cycle length and provide optimal procedure to find optimal production cycle length and replenishment cycle length for each product to minimize the overall cost of the system in which a manufacturer produces multiple products with a single production facility.

2. Joint Shipment Policy Model

Consider a supply chain consisting of one manufacturer and multiple retailers. Each retailer purchases its own product from the manufacturer and sells them to the customers. Demands at the retailers are assumed to be deterministic and constant over time. The retailers place orders their products to the manufacturer, and the manufacturer delivers the ordered quantities to the retailers. The following notation is used throughout the paper.

D_i : demand rate for product i in units (/ year)
P_i : production rate for product i in units (/ year)
A_i : retailer's order/shipment cost for product i (/ shipment)
S_i : manufacturer's setup cost for product i
h_i : retailer's holding cost for product i (/unit/ year)
H_i : manufacturer's holding cost for product i (/unit/ year)
m_i : shipment frequency for batch order of product i
$\mathbf{m}$: a vector of m_i, $\mathbf{m} = (m_1, m_2, \ldots, m_n)$
q_i : shipment quantity for product i (/shipment). A shipment quantity is assumed to be constant throughout all shipment period.

If each retailer adapts a classical EOQ model, then the optimal order quantity for product i is determined by $\sqrt{2A_iD_i/h_i}$ with its own cost function. However, in most cases, the manufacturer can't meet the all retailers' requirement with a single production facility without any dissatisfaction in retailer side. Hence, if each retailer determines its optimal order quantity independently without any synchronization with the manufacturer, the manufacturer cannot establish its production schedule without incurring shortages at the retailers. In order to avoid shortages, the manufacturer establishes the production schedule using a rotation cycle policy. In a rotation cycle, all products have the same cycle time, T, and during an interval of length T, a lot of each product is produced on the facility. The productions are run in a fixed sequence, which is repeated from cycle to cycle. In this paper, we assumed that deliveries of finished products to each retailer are executed in JIT pattern. In other words, deliveries can be executed several times within an ordering interval with constant delivery quantity for each retailer. Let the manufacturer's production lot size for product i be Q_i which can be set by an integer multiple of q_i ($Q_i = m_i q_i$). [Fig.1] shows inventory trajectories for the manufacturer and the two retailers.



[Fig.1] Inventory Trajectories
for the manufacturer and the retailers (n=2)

Since no shortages are permitted, the production lot size for product i must equal the demand during the cycle, i.e., $T=(Q_i/D_i)=m_i(q_i/D_i)$. The delivery cycle length for product i can be described by (q_i/D_i). Therefore, the average total cost for the manufacturer and the retailers can be described as follows:

$$TC(\mathbf{m},T)=\sum_{i=1}^{n}\left(\frac{S_i+A_i m_i}{T}+\frac{D_i T}{2m_i}\{H_i((1-D_i/P_i)m_i-(1-2D_i/P_i))+h_i\}\right) \quad (1)$$

$$=\frac{\sum_{i=1}^{n}(S_i+A_i m_i)}{T}+T\sum_{i=1}^{n}((\alpha_i-\gamma_i)/m_i+\beta_i)$$

,where

$$\alpha_i=\frac{D_i h_i}{2},\beta_i=\frac{D_i H_i(1-D_i/P_i)}{2},\gamma_i=\frac{D_i H_i(1-2D_i/P_i)}{2}$$

In Eq. (1), the first term represents the setup costs for the manufacturer and the retailers, and the second term is the holding costs for the manufacturer and the retailers. The average inventory for the manufacturer is calculated by the procedure proposed by Parija and Sarker (1999). It is our objective to find optimal shipment frequency m_i's for each product and production setup interval T to minimize the system cost described in Eq.(1). For an arbitrary product i, after finding optimal m_i^*'s and setup interval T^*, we get corresponding shipment quantity q_i^*'s from the equation of $q_i^*=(D_i T^*)/m_i^*$. It can be easily shown that $TC(T\,|\,\mathbf{m})$ is strictly convex in T. Thus, setting the first partial derivative of $TC(T\,|\,\mathbf{m})$ with T, at $T=T^*$, to 0. Then, the optimal setup interval $T^*(\mathbf{m})$ for all products is determined by

$$T^*(\mathbf{m})=\sqrt{\frac{\sum_{i=1}^{n}(S_i+A_i m_i)}{\sum_{i=1}^{n}((\alpha_i-\gamma_i)/m_i+\beta_i)}} \quad (2)$$

Substituting $T^*(\mathbf{m})$ into Eq.(1), we can describe the total cost as a function of m_i's as seen in Eq.(3):

$$TC(\mathbf{m},T^*(\mathbf{m}))=2\sqrt{\sum_{i=1}^{n}(S_i+A_i m_i)\left(\sum_{i=1}^{n}((\alpha_i-\gamma_i)/m_i+\beta_i)\right)} \quad (3)$$

Without considering the integrality condition of $\mathbf{m}$ in $TC(\mathbf{m},T^*(\mathbf{m}))$, we can get the following stationary point to minimize $TC(\mathbf{m},T^*(\mathbf{m}))$:

$$\mathbf{m}^0=(m_1^0,m_2^0,\ldots,m_n^0), \quad (4)$$

where

$$m_i^0=\sqrt{\frac{(\alpha_i-\gamma_i)}{A_i}\left(\frac{\sum_{j=1}^{n}S_j}{\sum_{j=1}^{n}\beta_j}\right)}\ ,\forall i$$

After then, we can get optimal delivery frequency by integerization of candidate integer sets neighboring $\mathbf{m}^0$. The proposed solution procedure can be easily applicable to find out joint optimal shipment policy. At first, we find the unique stationary point $\mathbf{m}^0$ to minimize $TC(\mathbf{m},T^*(\mathbf{m}))$ with Eq.(4) by considering $\mathbf{m}^0$ as continuous variables. There can be two integer values neighboring m_i^0 for product i by integerization step: $m_i=\lfloor m_i^0\rfloor$ or $\lceil m_i^0\rceil$. Hence, we select the best shipment policy $\mathbf{m}^*$ by evaluating the total cost with the maximum of 2^n candidate solution sets neighboring $\mathbf{m}^0$. After getting $\mathbf{m}^*$, we get the common cycle $T^*(\mathbf{m}^*)$ by Eq.(2) and the optimal shipment quantity with q_i^* and production quantity Q_i^* for product i with m_i^* and $T^*(\mathbf{m}^*)$ by Eq.(5).

$$q_i^*=T^*(\mathbf{m}^*)(D_i/m_i^*)\ ,Q_i^*=m_i^* q_i^*,\forall i \quad (5)$$

3. Examples Problems

To analyze the changes in joint optimal shipment policy, we suppose that a manufacturer produces three products with single production facility, such as Product-A, B and C. Basic data for numerical analysis is shown in [Table.1].

[Table.1] Basic data for illustrative examples

Product	D	P	S	A	H	h
A	3000	10,000	400	25	5	6
B	2000	5,000	400	25	4	5
C	4000	15,000	400	25	5	7

We find out and analyze joint shipment policy in single manufacturer and three-product case by changing setup cost for each product without altering basic data.

[Table.2] Effects of the changes in setup cost

S	T	Product-A	Product-B	Product-C
		(m,q,Q)	(m,q,Q)	(m,q,Q)
(400,400,40(	106	5,174,870	(4,145,580)	6,193,1158
(200,400,400)	97	(5,160,800)	(4,133,532)	(5,213,1065)
(200,200,400)	86	(4,177,708)	(3,157,471)	(5,189, 945)
(200,200,200)	76	(4,156,624)	(3,139,417)	(4,208, 832)

As seen in [Table.2], even though there is no change in cost factors at a certain retailer (or product), a joint shipment policy can be altered, even in changes in individual retailer's cost. It is shown that individual retailer's cost has a different sensitivity with respect to setup cost. From the numerical result as seen in [Table.2], 'Product-C' is relatively more sensitive to the changes in setup cost than other two products. This may be caused by asymmetric demand rate and cost parameters. To maintain more stable supply chain structure and ideally jointly optimized shipment policy, it may as well reduce the variance of sensitivity between multiple products (or retailers). If not, the potential disadvantage incurred by joint shipment policy in terms of individual cost structure can break the mutual cooperation scheme. To do this, a supplementary compensation or coordination mechanism should be provided to stabilize supply chain system by reducing the variance of sensitivity among multiple products.

A	T	Product-A	Product-B	Product-C
		(m,q,Q)	(m,q,Q)	(m,q,Q)
(25,25,25	106	(5,174,870	(4,145,58(	(6,193,1158)
(10,25,25)	105	(7,123,861)	(4,144,576)	(6,191,1146)
(25,10,25)	105	(5,173,865)	(6, 96,576)	(6,192,1152)
(25,25,10)	105	(5,173,865)	(4,144,576)	(9,128,1152)
(10,10,25)	104	(7,123,861)	(6, 95,570)	(6,191,1146)
(10,10,10)	104	(7,123,861)	(6, 95,570)	(9,126,1134)

[Table.3] Effects of the changes in shipment cost

In [Table.3], we compare the effects of the changes in shipment cost with fixed setup cost. From this result, we can see that the changes of shipment cost have little influence on the ordering cycle length itself. However, a shipment cost may alter optimal shipment frequency and shipment quantity simultaneously even though it has little effect in determining ordering cycle length. Additionally, since an ordering cycle length doesn't change, the changes of shipment cost for a certain product doesn't affect optimal policy for other products. A reduction in shipment cost definitely may increase the shipment frequency and decrease the shipment quantity for corresponding product(s) without altering ordering cycle. Accordingly, reduction of shipment quantity may decrease inventory holding cost in retailer side. However, the increment of shipment frequency may increase average inventory level and inventory holding cost in manufacturer side.

In summary, the changes of setup cost may have influence on the optimal policy for all products by altering ordering cycle length itself. To the contrary, the changes of shipment cost do nothing but affect the optimal policy for products in question with a little influence on other products.

4. Concluding Remarks

We propose simple optimal solution procedure to find joint shipment policy in single-manufacturer and multiple-product system. Our proposed model can be easily and usefully applied to meet the requirement of JIT delivery pattern for multiple products. It is necessary make a profound study of incorporating scheduling constraints for production/delivery and procurement (or material requirement) planning in both the manufacturer and the retailer more specifically.

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