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SIMULTANEOUS OPTIMIZATION OF TOLERANCE SYNTHESIS IN ASSEMBLY AND COMPONENT DIMENSIONS

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ABSTRACT

The majority of previous studies on tolerance synthesis have viewed the issue as a design methodology to determine optimal component tolerances on behalf of a manufacturer, while meeting given assembly tolerance requirements. Although a considerable amount of research has been done on this issue, a couple of important questions still remain unanswered. First, how can a design engineer quantitatively incorporate a customer's perception on a product quality into a tolerance synthesis scheme at the early design stage? Second, how can component tolerances and assembly tolerance be optimized in a simultaneous way? To answer these questions, this article presents the customer-driven concurrent tolerance synthesis which is facilitated by the notion of truncated distribution and the use of mathematical programming techniques, while adopting the major principles of Taguchi philosophy. The work presented in the article is an effort to gain insights, which can be useful in practice when setting up guidelines for an overall tolerance synthesis.

1. INTRODUCTION

1.1. *Background*

In engineering design, tolerances are intended to capture variations from an idealized nominal dimension as part of a process of realization in many manufacturing processes. As an integral part of engineering design, tolerance optimization has become the focus of increased activity as manufacturing industries strive to increase productivity and improve the quality of their products. The effects of tolerance optimization are far-reaching. Not only do the tolerances affect the ability to assemble a final product in terms of functional performance, but they also affect customer satisfaction, product quality, inspection, and manufacturing costs of the designed product. Consequently, the importance of the notion of tolerance has generated a strong demand for enhanced tolerances at a competitive cost by stimulating remarkable changes in the upstream product design and development processes. Yet in spite of its importance, tolerance optimization is one of the least understood engineering tasks.

1.2. *Related Literature*

There have been two parallel developments in tolerance optimization. The first development, the so-called 'screening procedures,' determines cost-effective tolerance limits for a product or assembled product, assuming that all products are subject to screening (i.e., 100% inspection). Screening may be an attractive industrial practice due to the rapid advancement of computerized inspection systems. However, this approach may be viewed as a short-term solution for improving product quality due to three main reasons. First, the screening procedures are usually performed at the end of manufacturing stage. Secondly, this approach will be less attractive if the inspection cost is very high. Finally, this approach optimizes tolerances for a product in the absence of components-assembly relations. Previous related works on this problem can be found in Tang [1,2], Tang and Tang [3], Bai et al. [4], Bai and Hong [5], Kapur and Cho [6], Cho et al.[7,8], Cho and Phillips [9], and Phillips and Cho [10]. In addition, Tang and Tang [11] provides an excellent discussion of the overall concept for the design of screening procedures within the framework of the various screening inspection environments including Deming's all-or-none rules, Taguchi's tolerance design, economic models for correlated variables, burn-in, and group testing.

In an assembly-manufacturing environment, considerable research has been done on the second development known as 'tolerance synthesis.' In practice, an assembly is composed of a number of component parts and tolerances of individual components are stacked up to the tolerance of the assembly. A 'tolerance stack-up' is the term to describe how individual processes or component tolerances can combine to affect a final assembly dimension. Tolerance synthesis is an optimization procedure that determines the resulting component tolerance allocations, given a required assembly tolerance. Currently, tolerance allocation is still performed largely on a trial-and-error basis using assigned default assembly tolerances. Although this ad hoc approach might be easy to implement, it does not lead to optimal tolerances due to the lack of systematic evaluation and optimization. Considerable research has been done on the tolerance optimization problem. See, for example, Speckhart [12], Parkinson [13], Chase and Greenwood [14], Dong and Soom [15], Lee and Woo [16], Zhang and Wang [17], Jeang [18], Soderberg [19], Feng and Kusiak [20], and Vasseur et al. [21] In addition, a survey of state-of-the-art tolerancing techniques can be found in Evans [22], and Zhang and Huq [23].

1.3. *Research Motivation*

Although a considerable amount of research has been done on the tolerance synthesis issue over a number of years, a couple of important questions still remain unanswered. This article attempts to provide a new perspective on this issue by answering the questions and rectifying existing problems in the current literature. First, the majority of previous studies on tolerance synthesis have been carried out based on an implicit relationship between tolerance and standard deviation, i.e., $t = 3\sigma$, where t and σ represent tolerance and standard deviation, respectively. The objective is to determine optimal tolerances on component dimensions, while meeting the stack-up tolerance so that the manufacturing cost would be minimized from a manufacturer's viewpoint. On the basis of the three-sigma relationship, the determination of component tolerances is basically equivalent to the selection of process precision (i.e., process variability). As pointed out in Vasseur et al. [21], however, there is no account for justifying the three-sigma relationship. From this perspective, the process precision levels as well as tolerances need to be determined by a systematic approach through quantitative modeling and design optimization. Furthermore, tolerance optimization by minimizing manufacturing costs solely on behalf of a manufacturer is unconvincing in the sense that customers must be foremost in a design engineer's mind from the beginning of tolerancing process. Thus, customer's perception on product quality needs to be taken into account in the process design stages. This article shows how the customer's perception on quality as well as the manufacturer's manufacturing costs can be quantitatively incorporated into the tolerance design process. Secondly, the current practice of tolerance synthesis aims to optimize component tolerances in such a way that the overall assembly cost function can be minimized, while meeting the assembly tolerance within design specifications. Although optimizing the component tolerances with the fixed assembly tolerance is a current practice, there is room for improvement. As pointed out by Zhang [24], ideal tolerance designs should take account of the determination of assembly tolerance as well as tolerance allocation among components, which requires a concurrent tolerance optimization scheme in assembly and components. In a practical sense, a customer's utmost interest does not lie in components themselves but an assembly or an assembled product. Thus, a design engineer needs to design and optimize the assembly tolerance along with component tolerances by incorporating customer's requirements and their perception as well as manufacturing costs. This article addresses how both component tolerances and assembly tolerance can be modeled and concurrently optimized within an integrated framework by combining mathematical programming techniques and Taguchi philosophy. With these two aims, this article lays out a modeling foundation and presents an optimization scheme for the simultaneous tolerance synthesis, which is, to the best of the authors' knowledge, the first attempt in the existing literature.

1.4. *Organization*

The remainder of this article is organized as follows: We first describe the validation of Taguchi quality loss function for an optimization purpose and develop a tolerance cost function in an assembly domain for encompassing the customer's perception on product quality. In Section 3, a manufacturing cost function associated with components is derived to determine the optimal process precision levels as well as tolerances. An optimization model is then proposed for determining both the assembly tolerance and component tolerances on the basis of economic considerations, and the conclusion is provided in the last section.

2. TOLERANCE COST IN ASSEMBLY DOMAIN

In a general design practice, a final assembly tolerance is usually given based on a three-sigma limit, or calculated in such a way that manufacturing costs are minimized from a manufacturer's point of view. Since the assembly is often viewed as a final product to the customer, this perception must be translated into the selection of tolerance design parameters such as the mean and standard deviation of the assembly. With this aim, Taguchi loss function may be a good alternative in modeling the quality cost associated with customer's perception on the assembled product. The assembly tolerance must be defined and optimized mathematically to avoid ambiguous interpretation and to provide a sound basis for an overall tolerance design process.

2.1. *Integrating a customer's perception*

One of the most important issues currently encountered in quality engineering is the selection of a proper quality loss function to relate the key characteristics of a product to its performance perceived by the customer. The quality loss function is a means of quantifying the quality loss of a product on a monetary scale, and is incurred when the product or its production process deviates from the customer-identified nominal dimension (target value) of the assembly. The functional relationship between the customer's dissatisfaction and assembly performance needs to be carefully defined since the choice of paths to performance enhancement for assembly depends heavily upon the type of quality loss function used at the tolerance design stage.

Among a number of quality loss functions proposed in the literature, Taguchi loss function may provide a good approximation to the quality loss, particularly over some range in the neighborhood of the nominal dimension of product performance. Taguchi loss function is given by $L(y)=k(y-\tau)^2$, where k and τ are a positive loss coefficient and nominal target dimension of the assembly, respectively. This loss function basically dictates that a loss is always incurred when a product performance deviates from its nominal dimension without regard to how small the deviation is. A look at the loss function reveals a very desirable characteristic. It can be shown that $E[L(y)]=k[(\mu-\tau)^2+\sigma^2]$, where $E[L(y)]$ is the expected value of $L(y)$. Thus, in order to minimize customer's loss, both the bias ($|\mu-\tau|$) and the variance (σ^2) must be reduced. Hence, one may employ Taguchi loss function as an approximation in situations where there is little or no information about the functional relationship between the assembly performance and the associated loss to the customer, or where there is no direct evidence to refute a quadratic representation. It is not our intention to provide detailed discussion on the loss function in this article. An extensive mathematical investigation on the loss function can be found in Cho and Leonard [25].

2.2. *Modeling Tolerance Cost*

Implementing a tight tolerance on an assembly may provide high outgoing quality (i.e., low loss to the customer), but it usually causes high manufacturing costs to the manufacturer. In contrast, implementing a loose tolerance may reduce the manufacturing costs to the manufacturer, but it may result in low outgoing quality (i.e., high loss to the customer). An immediate problem is then how to tradeoff these conflicting cost criteria to determine the most economical tolerance on the assembly.

Let Y be the functional dimension of an assembly, which is represented by a function of n component dimensions X_i's, $Y=f(X_1,X_2,\cdots,X_n)$. It can be shown through Taylor series expansion that the mean and variance of Y, which are denoted by μ and σ^2, respectively, can be approximated by

$$\mu=f(\mu_1,\mu_2,\cdots,\mu_n) \text{ and } \sigma^2=\sum_{i=1}^{n}\left(\frac{\partial f}{\partial X_i}\right)^2\sigma_i^2, \quad (1)$$

where μ_i and σ_i represent the mean and standard deviation of component dimension X_i, respectively. The loss $L(y)$ is incurred to the customer when $\tau-T\le y\le\tau+T$, where T denotes the assembly tolerance. Furthermore, if the deviation from the nominal target dimension is greater than T (i.e., $y<\tau-T$ or $y>\tau+T$), the assembly is assumed to be rejected (e.g., reworked or scrapped in a practical sense, depending upon the manufacturing environment) and excluded from shipment. These rejection costs would be incurred to the manufacturer. Let R_U and R_L represent rejection costs when

$y > \tau + T$ and $y < \tau - T$, respectively. The expected total cost associated with the assembly manufacturing, which is denoted by $E[TC_{ASM}]$, is then expressed as

$$
\begin{aligned}
&E[TC_{ASM}] \\
&= \int_{\tau-T}^{\tau+T} k(y-\tau)^2 f_Y(y)dy + R_U \cdot P(Y \geq \tau+T) + R_L \cdot P(Y \leq \tau-T) \quad (2) \\
&= \int_{\tau-T}^{\tau+T} k(y-\tau)^2 f_Y(y)dy + R_U \cdot [1 - F_Y(\tau+T)] + R_L \cdot F_Y(\tau-T),
\end{aligned}
$$

where $F_Y(\cdot)$ and $f_Y(\cdot)$ represent the cumulative distribution and probability density functions of the assembly dimension Y, respectively.

3. TOLERANCE COST IN COMPONENT DOMAIN

3.1. *Modeling Tolerance Cost*

In addition to the assembly tolerance, tolerances on component dimensions need to be optimized so that the associated costs would be minimized. Two cost factors are considered in this article: operating cost and rejection cost. First, requiring better operating devices and more trained personnel, a manufacturing process with a higher precision (or lower variability) usually results in an increased operating cost. On the other hand, a manufacturing process with a lower precision may require a less operating cost, but the outgoing products may reveal a poor product quality due to high variability. Secondly, nonconforming products falling outside the tolerance limits need to be reworked or scrapped. With a given process precision, a tight tolerance limit results in a high outgoing quality accompanied by high rejection costs. By the same token, implementing a loose tolerance limit may reduce rejection costs at the expense of product quality. Thus, there is a need to find the optimum tolerance allocation scheme minimizing the sum of operating cost and rejection cost while meeting given design constraints.

Suppose that there are m alternative manufacturing processes for a component which have different levels of process precision (or variability). Let u_{ij} be the indicator variable, which takes 1 if the j^{th} process is selected for producing the component dimension X_i. The operating cost and standard deviation of the j^{th} process for dimension X_i are denoted by c_{ij} and σ_{ij}, respectively. Denoting the operating cost and standard deviation of dimension X_i by OC_i and σ_i, respectively, $OC_i = \sum_{j=1}^{m} c_{ij}u_{ij}$ and $\sigma_i = \sum_{j=1}^{m} \sigma_{ij}u_{ij}$. It is assumed that rejection costs are incurred when X_i falls outside the tolerance limits. Letting r_{Ui} and r_{Li} denote the unit rejection costs when X_i falls above the upper tolerance limit and below the lower tolerance limit, respectively, the expected rejection cost for component dimension X_i, denoted by $E[RC_i]$, can then be written as

$$E[RC_i] = P(X_i \geq \mu_i + t_i) \cdot r_{Ui} + P(X_i \leq \mu_i - t_i) \cdot r_{Li}, \quad (3)$$

where t_i represents the tolerance of component dimension X_i. The expected cost associated with the allocation of assembly tolerance among component dimensions can then be written as the sum of $E[RC_i]$ and operating cost for all components as follows:

$$
\begin{aligned}
&\sum_{i=1}^{n} (E[RC_i] + OC_i) \\
&= \sum_{i=1}^{n} [(1 - F_i(\mu_i + t_i)) \cdot r_{Ui} + F_i(\mu_i - t_i) \cdot r_{Li}] + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}u_{ij},
\end{aligned}
$$

where $F_i(\cdot)$ is the cumulative distribution function of component dimension X_i. The expected total cost, denoted by $E[TC]$, can now be written as

$$
\begin{aligned}
E[TC] &= E[TC_{ASM}] + \sum_{i=1}^{n} (OC_i + E[RC_i]) \\
&= \int_{\tau-T}^{\tau+T} k(y-\tau)^2 f_Y(y)dy + R_U \cdot [1 - F_Y(\tau+T)] \\
&\quad + R_L \cdot F_Y(\tau - T) + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij}u_{ij} \qquad (4) \\
&\quad + \sum_{i=1}^{n} [(1 - F_i(\mu_i + t_i)) \cdot r_{Ui} + F_i(\mu_i - t_i) \cdot r_{Li}].
\end{aligned}
$$

Note that $E[TC]$ is a function of process parameters, such as means, variances, and rejection costs, as well as component tolerances. In many practical situations, a normal distribution is often assumed to describe the process distribution, and then $E[TC_{ASM}]$ and $E[RC_i]$ are given by

$$E[TC_{ASM}] = 2k\sigma^2\left[\Phi\left(\frac{T}{\sigma}\right)-\frac{T}{\sigma}\phi\left(\frac{T}{\sigma}\right)-\frac{1}{2}\right] + (R_U + R_L)\cdot\left[1-\Phi\left(\frac{T}{\sigma}\right)\right],$$

$$\text{and } E[RC_i] = (r_{Ui} + r_{Li})\cdot\left[1-\Phi\left(\frac{t_i}{\sigma_i}\right)\right], \quad (5)$$

where $\Phi(\cdot)$ and $\phi(\cdot)$ represent the cumulative distribution and probability density functions of the standard normal distribution, respectively.

3.2. *Desirable cost functions*

The main concern of tolerance synthesis is to determine the optimal tolerance allocation scheme to minimize the manufacturing cost. Some existing objective cost functions used in the previous studies are summarized in Table 1, where t, $h(t)$, and c_i's denote tolerance, manufacturing cost to produce a product with the tolerance of t, and constant coefficients, respectively. The cost models shown in Table 1 are functions of component tolerances only and there is no consideration on process characteristics. As pointed out in Vasseur et al. [21], however, a better model should be formulized to incorporate process parameters and component tolerances into tolerance synthesis. To support this argument, suppose that the standard deviation of component dimension X_i increases as tools wear. In this case, costs associated with tolerance allocation to components tend to increase since more nonconforming components would be realized. This forces a design engineer to change his or her current tolerance allocation scheme. Hence, to capture tolerance-cost relationships more efficiently, inclusion of the process parameters in the modeling phase is believed to be crucial for an enhanced tolerance design scheme. In this sense, the proposed cost function has advantages over the existing functions. Comparing the cost model expressed in equation (5) with the existing models of previous studies, it can be seen that the objective cost functions in the existing models contain component tolerance terms only, while the objective cost function in the proposed model contains the terms of both component tolerances and process parameters such as process means and variances.

4. OPTIMIZATION MODEL

To establish an optimization model for tolerance synthesis, one first needs to consider the stack-up of component tolerances to the assembly dimension. Suppose that components falling outside the tolerance limits are rejected. In this case, only the fraction of components within the tolerance limits is passed into the assembly process, which results in a truncated distribution. That is, the components are passed into the assembly process only when $\mu_i - t_i \le X_i \le \mu_i + t_i$. Thus, the corresponding truncated distribution represents the actual population of components passed into the assembly manufacturing and consequently its associated parameters, such as mean and variance of the truncated distribution, need to be obtained to describe the component dimensions realized in the assembly process. The notion of truncated distribution is depicted in Figure 1, where $f_i(x_i)$ and $\tilde{f}_i(x_i)$ represent the original and truncated probability density functions of X_i, respectively. It is well known that

Table 1. Existing manufacturing cost-tolerance models.

Cost model	Mathematical form	Author
Reciprocal	$h(t) = c_0 + c_1 t^{-1}$	Chase and Greenwood [18]
Reciprocal squared	$h(t) = c_0 + c_1 t^{-2}$	Parkinson [16]
Reciprocal power	$h(t) = c_0 + c_1 t^{-c_2}$	Lee and Woo [23]
Exponential	$h(t) = c_0 + c_1 e^{-tc_2}$	Speckhart [12]
Polynomial	$h(t) = \sum_{k=0}^{n} c_k t^k$	Dong and Soom [22]

$$\tilde{f}_i(x_i) = \frac{f_i(x_i)}{\int_{\mu_i - t_i}^{\mu_i + t_i} f_i(x_i)dx_i}.$$

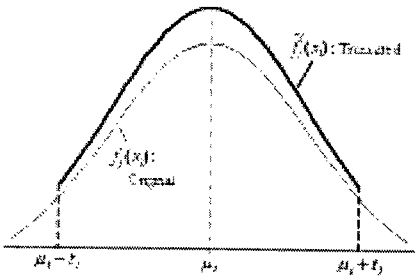

Figure 1. Truncated distribution.

The mean of the truncated distribution corresponds to μ_i since the original distribution is symmetrically truncated about the mean μ_i. Let $\tilde{\sigma}_i^2$ be the variance of the truncated distribution, which can be obtained by

$$\tilde{\sigma}_i^2 = \frac{\int_{\mu_i - t_i}^{\mu_i + t_i} (x_i - \mu_i)^2 f_i(x_i)dx_i}{\int_{\mu_i - t_i}^{\mu_i + t_i} f_i(x_i)dx_i}. \qquad (6)$$

For example, assuming a normal distribution for the component dimension X_i, the truncated variance of component dimensions realized in the assembly process is found to be

$$\tilde{\sigma}_i^2 = \sigma_i^2 \left[1 - \left(\frac{t_i}{\sigma_i} \right) \frac{\phi\left(t_i / \sigma_i \right)}{\Phi\left(t_i / \sigma_i \right) - 1/2} \right]. \qquad (7)$$

Since only the fraction of components within the tolerance limits is passed into the assembly process (see Figure 2), the variance of assembly dimension is now

$$\sigma^2 = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial X_i} \right)^2 \tilde{\sigma}_i^2. \qquad (8)$$

Furthermore, it is a common practice to specify the desirable level of process capability for the assembly dimension which is denoted by C_P. The standard deviation of the assembly can then be written as $\sigma = T/(3C_p)$ to achieve the process capability of C_P for a given assembly tolerance T. From equation (8), the tolerances on component dimensions consequently need to be selected to satisfy

$$\sum_{i=1}^{n} \left(\frac{\partial f}{\partial X_i} \right)^2 [\tilde{\sigma}_i(t_i; \sigma_i)]^2 = \left(\frac{T}{3C_p} \right)^2.$$

In addition, the process means of component dimensions need to be set at the target to minimize quality loss, i.e., $\mu = f(\mu_1, \mu_2, \cdots, \mu_n) = \tau$.

Summing up these observations, the optimization model can then be written as

Minimize

$$E[TC] = E[TC_{ASM}] + \sum_{i=1}^{n} \left(E[RC_i] + \sum_{j=1}^{m} c_{ij} u_{ij} \right)$$

Subject to

$$\sum_{i=1}^{n} \left(\left. \frac{\partial f}{\partial X_i} \right|_{X_i = \mu_i} \right)^2 \tilde{\sigma}_i^2 = \left(\frac{T}{3C_p} \right)^2 \qquad (9)$$

$$f(\mu_1, \mu_2, \cdots, \mu_n) = \tau$$

$$t_i^{\min} \le t_i \le t_i^{\max}, \qquad \text{for } i = 1, 2, \cdots, n$$

$$\sum_{j=1}^{m} \sigma_{ij} u_{ij} = \sigma_i, \qquad \text{for } i = 1, 2, \cdots, n$$

$$\sum_{j=1}^{m} u_{ij} = 1, \qquad \text{for } i = 1, 2, \cdots, n$$

$$T \le T^{\max},$$

where $t_i^{\min}$ and $t_i^{\max}$ represent the minimum and maximum of the component tolerance t_i, respectively, and $T^{\max}$ is the maximum allowable assembly tolerance. Examining a traditional tolerance synthesis problem, two observations are drawn. First, only the component tolerances are to be determined to minimize the manufacturing costs for a given fixed assembly tolerance. Second, the precision levels of component dimensions are accordingly adjusted on the basis of the three-sigma relationship. Note that decision variables in the proposed optimization model include the assembly tolerance, process means, and precision levels (variability) of component dimensions as well as the component tolerances. Thus, tolerancing processes in assembly and component domains are concurrently determined in a single optimization model.

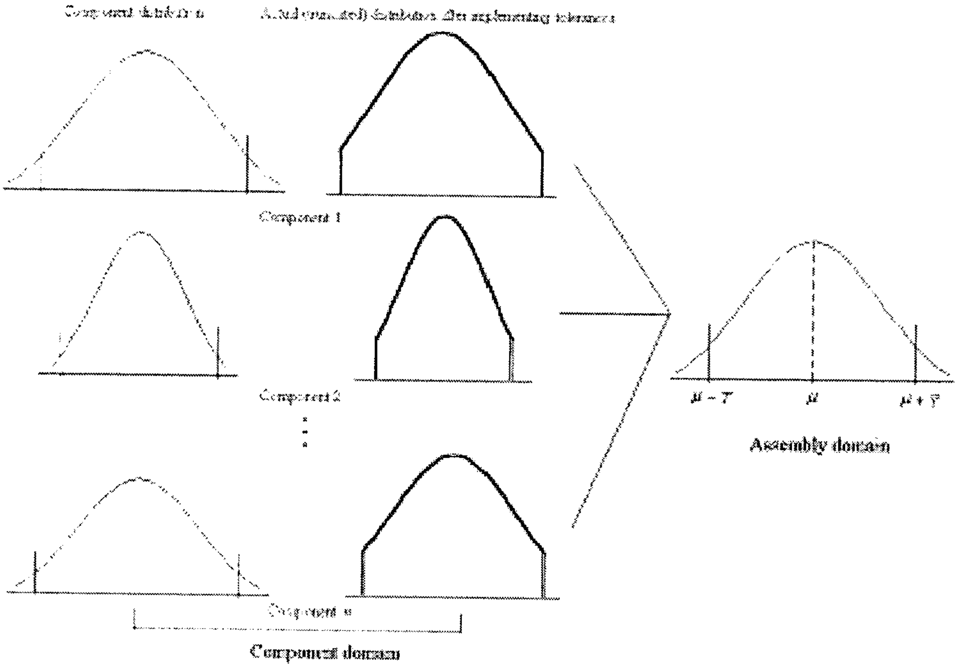

Figure 2. Graphical representation of component and assembly domains.

5. CONCLUSION

This article proposes a new method for tolerance synthesis in which assembly tolerance and component tolerances are concurrently optimized within an integrated framework. In the traditional tolerance synthesis problem, the component tolerances are determined to minimize the manufacturing costs for a given fixed assembly tolerance. In the proposed model, the assembly tolerance is viewed as a decision variable to minimize the customer's quality cost as well as manufacturing cost while meeting functional requirements. In addition, the process precision levels of component dimensions as well as component tolerances are to be determined in the optimization model. Consequently, the determination of assembly tolerance and its allocation among component dimensions are integrated in a single optimization model while incorporating the customer's perception on product quality. The proposed method may help a design engineer incorporate the customer's voice in a quantitative mechanism early at the design stage, and may be a more visible set of tools to practitioners in an assembly manufacturing environment who seek cost-effective high quality products.

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