한국경영과학회/대한산업공학회 2003 춘계공동학술대회
2003년 5월 16일-17일 한동대학교(포항)

Group Technology Cell Formation Using Production Data-based *P*-median Model

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Abstract

This study is concerned with the machine part grouping in cellular manufacturing. To group machines into the set of machine cells and parts into the set of part families, new *p* median model considering the production data such as the operation sequences and production volumes for parts is proposed. Unlike existing *p* median models relying on the classical binary part machine incidence matrix which does not reflect the real production factors which seriously impact on machine part grouping, the proposed *p* median model reflects the production factors by adopting the new similarity coefficient based on the production data based part machine incidence matrix of which each non binary entry indicates actual intra cell or inter cell flows to or from machines by parts. Computation test compares the proposed *p* median model favorably.

1. Introduction

During the last three decades, cellular manufacturing(CM) has received considerable attention since CM has been proved a very effective approach for improving the productivity of batch type manufacturing systems. The fundamental step toward designing CM system is to create part families and associated machine cells or vice versa, which is known as the machine part grouping (MPG) problem in literature. Part family is a collection of parts that have similar operations and require a similar set of machines for the completion of these operations. A set of machines grouped to produce the parts in a specific part family is called the machine cell.

The objective of MPG is to find independent machine cells with minimum interaction between cells so that a set of part family can be completely produced in a cell. However, MPG problem is often made complicated by exceptional parts and/or exceptional machines. An exceptional part is a part that requires processing in another machine cell. An exceptional machine is a machine that processes parts from a different family. Both exceptional parts and exceptional machines cause inter cell movement of parts. An effective CM system needs an MPG approach that produces part families and machine cells to minimize inter cell moves.

As a solution methodology for solving MPG problem, mathematical programming approach attempts to find machine cells and part families by formulating an MPG problem as the linear or nonlinear programming model. Kusiak[5] suggested a linear integer programming model, called the *p* median model, seeking to maximize the similarity coefficients defined between pairs of parts and several authors proposed the modified versions over Kusiak's formulation([12],[13]). As a solution methodology for MPG, the *p* median approach has been proved very effective on intermediate size MPG problem[1].

As the basic input to analysis of MPG the binary part machine incidence matrix (PMIM) $\mathbf{A}(=[a_{ij}])$ where the element a_{ij} is 1 or 0 depending on whether or not part i requires processing on machine j has been widely used. Given a binary PMIM, similarity coefficients defined between machines or parts are used to group machines and parts in most MPG algorithms.

However, the classical binary PMIM does not capture shop floor realities such as the followings[6]:

- it fail to recognize that an intermediate operation in an external cell generates two inter cell moves,
- disregard multiple visits by a part to the same machine, and
- lack of a provision to incorporate weights that reflect the production volume.

To overcome the limitations of classical binary PMIM in the formation of machine cells and part families, some authors suggested some variants over binary PMIM called type I and and type II production data based PMIM's[14]. The type I and II PMIM's, which are generated for a given binary PMIM, reflect the actual flows by parts considering the operation sequences and production volumes for the parts to be manufactured. But the type II production data based PMIM has a limitation that it is not uniquely determined unlike the classical binary PMIM.

In this study, a new *p* median model considering the production data such as operation sequences and production volumes for the parts is proposed. The proposed *p* median model adopts new similarity coefficient based on the type I production data based PMIM of which each non binary entry indicates actual intra cell or inter cell flows to or from machines by parts. Basically, the proposed formulation extends Viswanathan's 1996 model[12] with production data. In addition, new measure for evaluating the goodness of block diagonal solution is proposed. The formulation is solved on properly intermediate size sample problems taken from the literature.

2. New similarity measure

To construct the *p* median model considering the production data such as the operation sequences and demand for the parts to be manufactured, similarity measure reflecting those production factors is needed. However, most of similarity measures that have been proposed to cluster machines and parts in cellular manufacturing are typically based on the binary PMIM which only considers the routing information as to whether or not a specific part requires operation on a machine without incorporating the production data into the similarity measures.

To develop new similarity measure which overcomes the drawbacks of existing measure discussed above, we adopt type I production data based PMIM from Won and Lee[14] that captures production data to reflect the real inter cell or intra cell moves by parts. Type I production data based PMIM $\mathbf{B}(-[b_{ij}])$ is given by

$$b_{ij} - \sum_{r \in R_{ij}} f_{ijr} d_i$$

where

$R_{ij}-$ set of the operation sequence number along which part i visits machine j

d_i- production volume of part i

$$f_{ijr} - \begin{cases} 1 & \text{if } r \text{ is } 1 \ or \ n_i \\ 2 & \text{if } r \text{ is neither } 1 \ or \ n_i \\ 0 & \text{otherwise} \end{cases}$$

n_i- total number of operations for part i.

Given an $n \times m$ type I production data based PMIM, new similarity coefficient between pair of machines j and k is defined as

$$s_{jk} - \sum_{i}^{n} \Gamma(b_{ij}, b_{ik})$$

where

$$\Gamma(b_{ij}, b_{jk}) = \begin{cases} +2\min(b_{ij}, b_{jk}) & \text{if } b_{ij}>0 \text{ and } b_{jk}>0 \\ -\max(b_{ij}, b_{jk}) & \text{if either } b_{ij}>0 \text{ or } b_{jk}>0 \\ 0 & \text{otherwise.} \end{cases}$$

The calculation of the proposed similarity coefficient is illustrated with an example. Table 1 showing the operation sequences and production volumes for 5 part types processed on 5 machine types is adopted from Won and Lee[14]. Figure 1 shows the binary PMIM corresponding to the processing requirement on machines given in table 1. Figure 2 shows the corresponding type I production data based PMIM and figure 3 shows the symmetric lower triangular machine to machine similarity coefficient matrix. As can easily be seen from the figures, the classical binary PMIM contains no sufficient information of production characteristics of the parts to be manufactured and this validates the adoption of nonbinary PMIM such as type I production data based PMIM for the design of cellular manufacturing system in the real field.

Part no.	Operation sequence	Production volume
1	2 4 2 4 5	20
2	1 3	10
3	1 3 1 5	50
4	4 2 4	40
5	2 1 5 1 2 1 5 1	30

Table 1. The operation sequences and production volumes for the parts

Machines

Parts	1	2	3	4	5
1		1		1	1
2	1		1		
3	1		1		1
4		1		1	
5	1	1			1

Figure 1. binary PMIM

Machines

Parts	1	2	3	4	5
1		60		80	20
2	10		10		
3	150		100		50
4		80		80	
5	210	90			120

Figure 2. type I production data based PMIM

Machines

Machines	1	2	3	4	5
1					
2	120				
3	10	340			
4	530	190	270		
5	310	90	50	210	

Figure 3. similarity coefficient matrix

3. Mathematical model

Given an $m \times m$ machine to machine similarity coefficient matrix, the production data based p median mathematical model (PP) of machine cell formation is as follows:

(PP)

$$\text{Maximize} \quad \sum_{j=1}^{m}\sum_{k=1}^{m} s_{jk} x_{jk} \qquad (1)$$

$$\text{subject to} \quad \sum_{k=1}^{m} x_{jk} = 1, \quad j = 1, \cdots, m \qquad (2)$$

$$\sum_{j=1}^{m} x_{jk} \geq L x_{kk}, \quad k = 1, \cdots, m \qquad (3)$$

$$\sum_{j=1}^{m} x_{jk} \leq U x_{kk}, \quad k = 1, \cdots, m \qquad (4)$$

$$x_{jk} = 0 \text{ or } 1, \quad j, k = 1, \cdots, m \qquad (5)$$

where

$$x_{jk} = \begin{cases} 1 & \text{if machine } j \text{ is clustered into cell } k \\ 0 & \text{otherwise.} \end{cases}$$

The objective function (1) maximizes the sum of similarity coefficients between machine pairs. Constraint (2) specifies that each machine needs to be assigned to one and only one cell. Constraint (3) ensures that at least L machines should be assigned to cell k only if that cell k is generated and constraint (4) guarantees that at most U machines can be assigned to cell k only if that cell k is generated. Constraint (5) ensures the binary solution for machine allocation.

Like in Viswanathan's 1996 formulation, the constraint for the prespecified number of cells is omitted since the objective function seeks to find the optimal solution of machine allocation so that it maximizes the sum of similarities between machine pairs.

Once the initial machine cell is found with the above p median model, its corresponding part family is formed by assigning parts to its most proper cell with the following part assignment rule:

- Find a cell in which each part has most flows on machines and assign that part to the cell.
- If ties occur, assign that part to the cell in which it visits most machines.

4. Performance measure

To evaluate the effectiveness of MPG method, several measures have been proposed. The recent papers by Sarker[8] and Sarker and Khan[9] provide extensive reviews and discussions of the measures of goodness based on the binary PMIM for clustering machines and parts in cellular manufacturing.

In this study we propose a new measure that considers those production factors. This measure extended from GCI[4] is called the weighted group capability index(WGCI) and is defined from type I production data based PMIM as follows:

$$WGCI = 1 - \frac{\text{sum of exceptional } b_{ij}}{\text{sum of } b_{ij}}$$

The sum of exceptional b_{ij}'s in a type I production data based PMIM represents the actual flows incurred by the operations performed outside the diagonal blocks and, hence the WGCI measures the proportion of the actual flows incurred by the operations performed within the main diagonal blocks. Note that the WGCI measure reduces to the GCI measure if binary PMIM is considered.

5. Computation experiment

To show the merit of the proposed p median model in designing cellular manufacturing system of which the fundamental objectives is to minimize inter cell part moves, different model that compares the goodness of solution is needed. In this study, the mathematical model in Won and Lee[14] is selected for comparative purpose since the authors linear integer programming model seeks to directly minimize the sum of exceptional b_{ij}'s.

Table 2 shows the list of ten problem sets available in the literature. All the problems were tested on an Pentium PC with 1 GHz using an extended version of HYPER LINDO which can solve the integer linear programming problem with 1,000 or less binary variables. Limit on the computation time is taken as the number of pivot iterations automatically preset by the HYPER LINDO program on an instance of optimization problem. Two columns in the rightmost side of the table show the number of pivot iterations taken by the p median model and Won and Lee's model, respectively, until the optimal solution is found on each instance of optimization problems. Note that the HYPER LINDO stops running Won and Lee's model after the number of iterations preset in problems 2 to 5 are completes while implementing the model. Especially, in implementing Won and Lee's model the HYPER LINDO stops running problem 9 due to lack of space requirement even before the number of iterations preset on that problem is complete. It can be noticed that the proposed p median model takes much less time to find the optimal solution in all cases as compared with Won and Lee's model.

		# of pivot iterations	
Source	size	p median	Won&Lee
1. Selvam	10×5	370	561
2. Gupta	43×16	3,643	219,203(*)
3. Harhalakis	20×20	7,718	129,261(*)
4. Seifoddini	41×30	164,973(*)	638,127(*)
5. Seifoddini	41×30	164,973(*)	638,127(*)
6. Nair(1)	7×7	23	748
7. Nair(2)	20×8	153	2,623
8. Nair(3)	12×10	139	12,911
9. Nair(4)	40×25	11,323	**
10. Wu	13×13	1,541	18,330

* denotes the HYPER LINDO program stops running iteration after the number of iterations preset for an instance of optimization problem is complete.

** denotes the HYPER LINDO program stops running iteration due to lack of space requirement.

Table 2. Test problems taken from the literature and the number of pivot iterations taken until the optimal solution is found.

Table 3 shows the computational result in terms of the proposed WGCI measure. First of all, the proposed p median model yields high WGCI values in all problems within the number of pivot iterations preset by the HYPER LINDO while Won and Lees model does not even yield the integer feasible solutions in problems 2,4,5, and 9 after the number of pivot iterations preset on those problems are complete. In problems 1,6,7,8,and 10, the proposed p median model yields the solutions with identical values of WGCI within much shorter computer runtime compared with Won and Lees model. It is interesting to note that for problem 3 the proposed p median model even yields better solution than Won and Lee's model because the latter takes further pivot iterations to find the optimal solution. This reveals the competitiveness of the proposed p median model.

Problem No.	p	WGCI(%)	
		p median	Won&Lee
1.	3	94.74	94.74
2.	7	73.54	***
3.	6	81.36	78.81
4.	10	82.44	***
5.	10	85.98	***
6.	3	76.92	76.92
7.	3	80.49	80.49
8.	4	78.95	80.70
9.	10	72.34	***
10.	4	90.81	90.81

*** denotes the WGCI measure corresponding to that problem is not available since the integer feasible solution can not even be found at the end of the preset number of pivot iteration.

Table 4. Comparison of the WGCI measure
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6. Concluding remarks

In this study we propose new p median model considering the production data such as the operation sequences and production volumes for parts. The proposed p median model adopts new similarity coefficient based on the production data based PMIM of which each non binary entry indicates actual intra cell or inter cell flows to or from machines by parts. In addition, new measure for evaluating the goodness of block diagonal solution is proposed. The formulation is solved on properly intermediate size sample problems taken from the literature. Computation test compares the proposed p median model favorably. This encourages development of algorithmic approaches based on the proposed new similarity coefficient and it is the future research to be done.

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