Iterative search for a combined pricing and (S-1,S) inventory policy in a two-echelon supply chain with lost sales allowed

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Abstract

This paper considers a continuous-review two-echelon inventory control problem with one-to-one replenishment policy incorporated and with lost sales allowed where demand arrives in a stationary Poisson process. The problem is formulated using METRIC-approximation in a combined approach of pricing and (S-1,S) inventory policy, for which an iterative solution algorithm is derived with respect to the corresponding one-warehouse multi-retailer supply chain. Specifically, decisions on retail pricing and warehouse inventory policies are made in integration to maximize total profit in the supply chain. The objective function of the model consists of sub-functions of revenue and cost (holding cost and penalty cost). To test the effectiveness and efficiency of the proposed algorithm, numerical experiments are performed. The computational results show that the proposed algorithm is efficient and derives quite good decisions.

1. Introduction

Traditional inventory planning models for supply chains assume that all associated demand processes and revenue streams are exogenously determined. As a consequence, such models focus on operation cost minimization in the associated supply chains based on demand forecasts which are usually determined by marketing models. On the other hand, marketing models focus on determining pricing strategies and analyzing their impact on sales volumes and revenues, typically by rudimentary and simplistic treatment of operation cost in their supply chains [2]. However, it has been noticed that simultaneous decision on pricing and inventory policy leads to more profit in a single-echelon inventory system [7, 19]. Motivated by this notice, the combined approach of pricing and inventory policy has been intensively investigated for many single-echelon systems [1, 6, 10, 11, 15]. It has also been shown that the combined approach works better than any separated approach of treating the two problems individually. However, there are few studies about the combined approach for multi-echelon inventory systems. This provides the motivation for this paper to integrate the pricing and inventory control issues together in a two-echelon inventory system with stochastic demand processes incorporated. The inventory system consists of a central warehouse and multiple retailers, where the warehouse distributes a single type of products to multiple retailers who will sell it to consumers. The retailers serve geographically dispersed and heterogeneous markets. Demands at each retail market arrive continuously but in a fashion of forming a non-linearly decreasing function of retail price in the market. The warehouse replenishes its inventory from an external supplier with ample capacity. Each retailer and the warehouse use the same (S-1,S) policy.

Generally, the (S-1,S) inventory policy is applied to situations where demand loss is not allowed or penalty changes are severe as in military service. Accordingly, the approach of combining the pricing issue and the issue of (S-1,S) inventory policy adaptation with lost sales allowed may be applied to dealing with expensive goods like jewelry or deluxe cars whose demand rates are subject to their price changes.

It has been reported in the literature that if the price and multi-echelon inventory decisions are made together at the same time, then the associated supply chain will get more profits due to increased sales and decreased operation cost, and so make the supply chain customers satisfied. Thereupon, this paper will focus on constructing a model that combines both the multi-echelon inventory decision and the pricing decision in the associated supply chain.

The exact cost for a single-echelon lost sales inventory system having Poisson-distributed demands and fixed leadtime has been derived in Hadley and Whitin [3]. Sung and Yang [18] have considered (s,S) inventory policy with limited backlogging and stochastic leadtime. Smith [17] has demonstrated how to evaluate and find the optimal (S-1,S) inventory policy for an inventory system with lost sales allowed but without any replenishment cost allowed and with generally distributed stochastic leadtime allowed. A METRIC-model has been suggested as one of the most widely known multi-echelon inventory models in Sherbrooke [16].

Nahmias and Smith [13] have considered a lost sales case for a multi-echelon system via the METRIC-model which has specifically considered periodic review batch order policies with partial lost sales allowed.

There are some marketing literatures studied on supply chain coordination between retailer and supplier which focuses on pricing. For example, Jeuland and Shugan [5] have considered a simple pricing issue for a single-supplier and single-retailer system. Their model did not consider any inventory replenishment. The single-retailer part has been extended to multi-retailer settings by Ingene and Parry [4]. Monahan [12] has determined prices subject to the restriction that both supplier and retailer use identical order intervals. Lal and Staelin [8] have considered a pricing problem with non-identical retailers, under the assumption that all demand processes are not exogenously given and inventory replenishment is made infrequently.

The approach of integrating inventory control and pricing issues together was first advocated by Whitin [19]. Both Whitin [19] and Mills [10], [11] have addressed a single-period, single-location model to determine the associated single-price and supply quantity. Karlin and Carr [6] have considered an infinite horizon model for a single item, under the assumption that a single price needs to be specified at the beginning of the planning horizon. Chen et al. [2] have considered both coordination (power-of-two) mechanism and non-coordination mechanism for multi-retailer systems under a periodic review inventory policy. Lee and Hong [9] have integrated the pricing issue and the (r, Q) policy adaptation issue in a single-warehouse and single-retailer system with stochastic demand processes incorporated.

2. Problem description

The proposed problem considers a two-echelon inventory system with single central warehouse and multiple retailers as depicted in Fig.1. The retailers, which have different market sizes, serve to satisfy customer demands and replenish the associated stocks from the central warehouse. The warehouse, in turn, replenishes its stock from an outside supplier. The customer demand rate decreases exponentially as the price increases, while the retail price at each market is the same. The objective of the problem is to find the combined policy for inventory replenishment and pricing that maximizes long-run average profit in the associated two-echelon supply chain. There is a central planner who makes the pricing and inventory replenishment decisions. That is, the central planner makes both decisions simultaneously to maximize the total average profit of the two-echelon supply chain.

Demand process at each retailer follows a stationary Poisson process with constant arrival rate. When a retailer is out of stock, any arriving demand at the retailer will be lost. However, when a stockout occurs at the warehouse, all demands from the retailers are fully backlogged and the backorders are filled according to a FIFO-policy. Each retailer and the warehouse use the same (S-1,S) policy.

The transportation time from the warehouse to any retailer is assumed to be constant. The transportation time from the external supplier to the warehouse is also constant. The external supplier is assumed to have infinite capacity, which means that the replenishment leadtime for the central warehouse is constant. The replenishment and backorder costs are assumed to be negligible, compared to the holding and stockout costs. Any units held in stock at the warehouse and the retailers incur holding costs per unit per time. A fixed shortage cost per lost customer is incurred at the retailers.

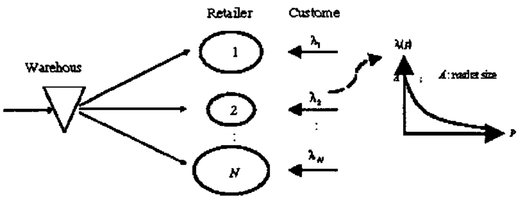

Figure 1. Two-echelon inventory system (1:1:N)

The objective of the problem is to maximize the long-run total average profit of the associated supply chain, which is defined as the difference between the associated total revenue and cost. The total revenue function can be easily defined by multiplying total sales by retail price. However, the cost function is more complex, so that some detailed analysis on the warehouse and the retailers should be made before deriving the associated cost function. The total cost consists of inventory holding costs at the warehouse and all the retailers, and penalty costs at all the retailers.

A queueing system analogy will be used when evaluating costs at the retailers, which has been adapted successfully in the analysis of inventory systems as in Sherbrooke [16]. It is noted that the demand process at the retailers follows a stationary Poisson process and the replenishment leadtime is stochastic, since orders from the retailers can be delayed at the central warehouse due to stochastic stockouts. According to Palm [14], it is also noted that the steady state occupancy level is Poisson distributed with mean λL, where λ is the mean arrival rate and L is the mean service time, which holds for i.i.d. service times. However, the stochastic leadtimes in the proposed problem are evidently not independent to each other. By the way, by disregarding any associated correlation, the number of outstanding orders will be approximated in this paper as to follow a Poisson distribution, as adapted in the METRIC-approximation in Sherbrooke [16].

The demand rate from retailer i can be represented as $\lambda_i = A_i e^{-\alpha p}$, where A_i is the market size of retailer i, $i = 1,2,\ldots,N$ and α is the elastic coefficient of retail price p. In the situation where lost sales are allowed, the corresponding queueing system of interest can be modeled as an M/G/S/S queue with S servers, each with generally distributed service time and no queueing allowed, where S is the order-up-to level. If the service times are independent random variables with mean $\overline{L}$, then the associated Erlang's loss formula can be viewed as stating the steady-state distribution of occupancy level at the retailer as $q^s(j) = \frac{(A_i e^{-\alpha p}\overline{L})^j / j!}{\sum_{n=0}^{S}(A_i e^{-\alpha p}\overline{L})^n / n!}$ $0 \le j \le S$ where $q^s(j)$ is the probability that j servers (out of S) are occupied in steady state. Based on the METRIC approximation explained above, the number of outstanding orders at the retailer can be modeled as $q^s(j)$.

Let the mean replenishment leadtime at retailer i be $\overline{L_i}$ and let $q_i^{s_i}(j)$ be the steady-state probability of j outstanding orders, given a desired-stock level S_i. Then, the expected number of lost sales per unit time is derived as $\lambda_i q_i^{s_i}(S_i) = A_i e^{-\alpha p} q_i^{s_i}(S_i)$ and the expected number of units in stock is derived as $\sum_{j=0}^{S_i}(S_i - j)q_i^{S_i}(j) = S_i - [1 - q_i^{S_i}(S_i)]A_i e^{-\alpha p}\overline{L_i}$. The demand rate from retailer i without loss allowed is derived as $(1 - q\,(S))Ae$. Let h_i be the unit holding cost per unit time at retailer i and π_i be the unit shortage cost per lost sale at retailer i. Therefrom, the associated total cost function at retailer i can be derived as

$$C_i(S_i, \overline{L_i}, p) =$$
$$A_i e^{-\alpha p}\pi_i q_i^{S_i}(S_i) + h_i(S_i - [1 - q_i^{S_i}(S_i)]A_i e^{-\alpha p}\overline{L_i}) \quad (1)$$

In the backorder case, the demand process at the warehouse follows the same stationary Poisson process as at the retailers. However, in the lost sales case, the warehouse may not have the same Poisson process, because any demand from customers may be lost during leadtime intervals for the orders of the retailer placed to the warehouse. For example, if the basestock level at a retailer is one, then the retailer leadtime will be included in the inter-arrival time interval between two successive demand arrivals at the warehouse from the retailer so that all the demands from customers arrived during the leadtime will be lost due to stockout at the retailer. Therefore, the associated demand process at the warehouse does not remain as the original Poisson process any longer. The remaining demand process will be rather complex to characterize. Therefore, in this paper the demand process at the warehouse will be approximated as a stationary Poisson process but with adjusted arrival rate. The arrival rate is assumed to be Λ where Λ depends on how much demand is lost at all the retailers, which is determined as

$$\Lambda = \sum_{i=1}^{N} A_i e^{-\alpha p}(1 - q_i^{S_i}(S_i)) \quad (2)$$

Now, given a fixed deterministic leadtime L_0, we can find the average holding cost incurred at the warehouse as a function of demand intensity Λ and order-up-to level S_0, where h_0 is the unit holding cost per unit time at the warehouse,

$$C_0(S_0, \Lambda) = h_0 \sum_{j=0}^{S_0}(S_0 - j)\frac{(\Lambda L_0)^j}{j!}\exp(-\Lambda L_0) \quad (3)$$

where Λ is the function of S_i and p, so that Eq. (2) can be incorporated as
$$C_0(S_0, \bar{S}, p) =$$
$$h_0 \sum_{j=0}^{S_0}(S_0 - j)\frac{(\sum_{i=1}^{N} A_i e^{-\alpha p}(1 - q_i^{S_i}(S_i))L_0)^j}{j!}\exp(-\sum_{i=1}^{N} A_i e^{-\alpha p}(1 - q_i^{S_i}(S_i))L_0) \quad (4)$$

The mean delivery delay can also be derived in consideration of stockout at the warehouse by using B_0, the average number of backorders at the warehouse, which can be calculated as

$$B_0 = \sum_{j=S_0+1}^{\infty}(j - S_0)\frac{(\Lambda L_0)^j}{j!}\exp(-\Lambda L_0) \quad (5)$$

Then, the Little's formula is applied to obtain the average delivery delay, B_0 / Λ. Therewith, the mean leadtime for retailer i is derived as

$$\overline{L_i} = L_i + B_0 / \Lambda \quad (6)$$

The total revenue function can be represented as the function of order-up-to level at retailer S_i and retail price p, where c is the unit purchasing cost at the warehouse

$$G_i(S_i, p) = A_i e^{-\alpha p}(1 - q_i^{S_i}(S_i))(p - c) \quad (7)$$

The total cost function is obtained in the associated supply chain as

$$TC(\cdot) = C_0(S_0, \bar{S}, p) + \sum_{i=1}^{N} C_i(S_i, \overline{L_i}, p) \quad (8)$$

As mentioned above, a central planner makes the pricing and inventory replenishment decisions together so as to maximize the long-run average profit in the two-echelon supply chain, which is equal to the total revenue minus cost, expressed as

$$TP(S_0,\vec{S},p)=\sum_{i=1}^{N}G_i(S_i,p)-\left[C_0(S_0,\vec{S},p)+\sum_{i=1}^{N}C_i(S_i,p)\right] \quad (9)$$

It can be represented as the function of S_0, $\vec{S}$, and p which is given as

$$\begin{aligned}&TP(S_0,\vec{S},p)\\&=\sum_{i=1}^{N}A_i e^{-\alpha_i p}(1-q_i^{S_i}(S_i))(p-c)\\&-h_0\sum_{j=0}^{S_0}(S_0-j)\frac{(\sum_{i=1}^{N}A_i e^{-\alpha_i p}(1-q_i^{S_i}(S_i))L_0)^j}{j!}\exp(-\sum_{i=1}^{N}A_i e^{-\alpha_i p}(1-q_i^{S_i}(S_i))L_0) \quad (10)\\&-\sum_{i=1}^{N}A_i e^{-\alpha_i p}\pi_i q_i^{S_i}(S_i)+h_i(S_i-[1-q_i^{S_i}(S_i)]A_i e^{-\alpha_i p}\overline{L_i})\end{aligned}$$

3. Solution Search

The objective of the proposed problem is to find the optimal inventory positions S_0^* and $\overrightarrow{S^*}$ and the optimal price p^* together which maximize the total supply chain profit. The total profit is calculated by subtracting the total cost at the warehouse and all the retailers from the total revenue of the retailers. Inventory holding costs at the warehouse and all the retailers and penalty costs at all the retailers for lost sales cases are considered. To maximize the total profit, the total revenue must be maximized, while the total cost is minimized. As shown in Eq.(10), the total profit function is non-linear so that each involved decision variable is hard to clearly find its mathematical feature. Therefore, a near-optimal solution will be found by using an iterative search algorithm.

Given $\overline{L_i}$ and p, the optimal order-up-to-level S_i, which minimizes the cost of the retailers, is obtained by a local search procedure. $\overline{L_i}$ is a function of Λ and S_0 as shown in Eqs. (5) and (6). This implies that the optimal order-up-to-level S_i can be found. The retail price p and order-up-to-level at the warehouse S_0 are decision variables with which Λ can be determined from Eq. (2). As seen in Eq. (1), once p and $\overline{L_i}$ are known, S_i can be determine and so Λ can be updated as in Eq. (2). Therewith, given p and S_0, the optimal value of S_i can be calculated.

In summery, if p and $\overline{L_i}$ values are given(fixed), then S_i value can be determined. These values lead to determine the associated best S_0 value from Eq. (8) which minimizes the total cost $TC(\cdot)$ in the supply chain. However, it cannot be guaranteed that this value maximizes the total revenue. Therefore, it may be needed to try with an iterative procedure for finding the optimal solutions S_0^* and S_i^* by varying the retail price p which maximize the total profit. The iterative procedure is given below as the solution procedure.

The following notation will be used throughout the rest of this paper.

$C_i^{\min} \equiv \min_{S_i} C_i(S_i, L_i)$: minimum cost per unit time for retailer i in steady state when the leadtime, L_i, is substituted for $\overline{L_i}$, $i=1,2,...,N$, given fixed p.

$S_i(k)$: order-up-to-level at retailer i iteration k.

Solution Procedure:

Step 1 : Assign the lower bound and upper bound of retail price (P_{LB}, P_{UB}) and the step length(δ).

i) Set $p := P_{LB}$, P_{UB}, δ, goto Step2.

Step 2 : Find the total cost function value, min TC^*, under given p.

i) Set $S_0^{\max}=0$ and $TC_{lb}(S_0^{\max})=\infty$.

ii) Set $S_0=0$ and $TC_{\min}=\infty$.

iii) Set $k=0$ and $\Lambda=\lambda_0$.

iv) For each $i = 1,2,\ldots,N$, calculate $\overline{L_i}$ by Eqs. (5) and (6) and set $S_i(k)=\arg\min_s(C_i(S,\overline{L_i},p))$

v) If $k>0$, and $S_i(k)=S_i(k-1)$ for all $i = 1,2,\ldots,N$, then goto vi), else calculate by (2), set $k:=k+1$ and goto iv).

vi) Set $TC^*(S_0)=C_0(S_0,S_i(k),p)+\sum_{i=0}^{N}C_i(S_i(k),\overline{L_i},p)$.

If $TC^*(S_0)<TC_{\min}$, then $TC_{\min}=TC^*(S_0)$ and let $S_0^{opt}=S_0$ and $S_i^{opt}=S_i(k)$ for $i = 1,2,\ldots,N$. If $TC_{\min}\le TC_{lb}(S_0^{\max})$, then STOP, else set $S_o=S_o+1$, and check if $S_0\ge S_0^{\max}$, then set $S_0^{\max}=S_0^{\max}+1$ goto ii), else goto iii).

Step 3 : Calculate the total profit $TP^{\max}$

i) Set $TP=G-TC^*$, calculate TP .If $p\ge P_{UB}$ then STOP. Else set $p=p+\delta$, goto Step2.

ii) Find $p^*=\arg\min_p(TP(p))$.

Then $TP^{\max}=TP(S_0^{opt},\overrightarrow{S^{opt}}\mid p^*)$

To search for the lower and upper bounds of retail price p, the step length (δ) should be determined. To determine the step length of retail price, experiments with step lengths (δ) =0.2, 0.5, 1, 1.5 are performed with the parameter set given in Section 4. As the step length (δ) increases, the gap between the optimal solution and the solution of the proposed iterative solution algorithm increases, and the calculation time decreases, as shown in Figs.4 and 5. Therefore, considering the trade-off between the solution gap and the calculation time, a reasonable value of the step length (δ) must be found. As shown in Figs.4 and 5, it is certain that the step length (δ) has a reasonable value at the value 1.

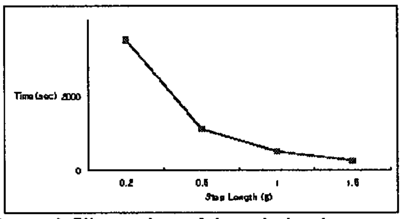

Figure 4. Illustration of the relation between calculation time and step length

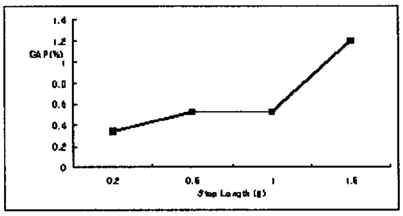

Figure 5. Illustration of the relation between solution gap and step length

4. Computational results

In order to examine the efficiency of the proposed iterative solution algorithm, 6 different problems are considered and the number of retailers is 2 for each problem. For each test problem, the best order-up-to level and retail price are obtained using the proposed iterative solution algorithm. The experimental problem set provides 98% service level which means a very reliable set.

The boundary of the retail price p in the full search has the same range as the proposed iterative solution algorithm. The range of the order-up-to-level at the warehouse S_0 and the order-up-to-level at the retailer S_i is set at 4 times the range of the proposed iterative solution algorithm. The calculation time of the full search is set at the time for single unit iteration multiplied by the number of iterations. In this paper, the retail price is considered as a discrete function, since the retail price is determined by unit value on each customer arrival.

6 problems are considered when the retailers receive demands arrived at the same demand rate (i.e. same market size). It is assumed that the market size is 1000 for the first 3 problems. And the market size of the last 3 problems is set at the value 1200. On each case, the transportation time from the warehouse to each retailer is varied from 1 to 2.

The average gap of total profit between the heuristic solution and full search solution is about 0.814303%. The order-up-to-level of the retailer increases, while the total profit decreases, as the transportation time gets larger. This indicates that the possibility of lost sales grows as the transportation time becomes larger. Therefore, to prevent the increase of the cost, the stock level at the retailers must be larger. As the stock level at the retailers increases, the holding cost increases, which results in increase in the total cost and decrease in the total profit. The order-up-to-level of the retailers changes much due to the change of the market size and the transportation time. It can be interpreted that the change of the market size and the transportation time influences much more on the order-up-to-level of retailers than on the retail price. The problem data and results are presented as in Table 1.

#		λ	L	Heuristic Algorithm				Full Search Algorithm				GAP(%)
				S_0^{hev}	S_i^{hev}	p^{hev}	TP^{hev}	S_0^{opt}	S_i^{opt}	p^{opt}	TP^{opt}	
	R1											
1	R2	1000	1	6	18	50	114.99486	6	17	61	115.96677	0.857249
	R1				21				24			
2	R2	1000	2	4	21	52	111.81844	6	24	61	113.04188	1.082595
	R1		1		17				18			
3	R2	1000	2	5	23	61	113.94612	5	24	61	114.91446	0.847961
	R1				19				20			
4	R2	1200	1	5	19	61	140.78186	6	20	61	140.97874	0.158168
	R1				27				29			
5	R2	1200	2	5	27	61	136.21141	5	28	61	137.77732	1.136548
	R1		1		19				20			
6	R2	1200	2	6	25	61	138.1246	6	29	61	139.29962	0.849517
AVG												0.814303

Table 1. The identical retailers case

Table 2 indicates that the iterative solution algorithm takes about 3%~4% off from the full search calculation time. From computational point of view, the proposed algorithm is efficient and simple.

Instance	Heuristic	Optimal
1	425	29359
2	1121	40083
3	822	32541
4	773	32956
5	2589	39138
6	1622	36213
AVG	1225.333	35048.33

Table 2. The numerical results of calculation time

5. Conclusions

This paper considers a combined model of pricing and (S-1,S) policy for a single-warehouse, multi-retailer inventory system with lost sales allowed. For the model evaluation, the well-known

METRIC-approximation is used. Most of the multi-echelon models consider inventory policy only, while this paper integrates retailer pricing and inventory control issues together to maximize long-run total profit in the associated supply chain. The objective function of the integrated model consists of sub-functions of long-run total revenue and total cost (consisting of holding cost and penalty cost). Concerned with combining the pricing issues and the lost sale issue together, the proposed algorithm may be applied to dealing with expensive products like jewelry or deluxe cars whose demand rates are subject to their price changes.

An iterative solution algorithm is derived to search for each approximate retail price. A guideline for the associated search boundary and step size determination is also provided as required in the search.

From the computational results, it is found that the performance of the proposed algorithm is quite effective and simple to use.

The model may be applied to supply chain systems dealing with expensive products like jewelry or deluxe cars where demands are subject to price changes.

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