An integrated one-vendor multi-buyer production-inventory model with shipment consolidation policy incorporated

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Abstract

This paper considers an integrated one-vendor multi-buyer production-inventory model where the vendor manufactures multiple products in lot at their associated finite production rates. In the model, it is allowed for each product to be shipped in lot to the buyers before the whole product production is not completed yet. Each product lot is dispatched to the associated buyer in a number of shipments. The buyers consume their products at fixed rates. The objective is to the production and shipment schedules in the integrated system, which minimizes the mean total annual cost per unit time. The mean total annual cost consists of production setup cost, inventory holding cost and shipment cost. For the model, an iterative optimal solution procedure with shipment consolidation policy incorporated is derived. It is then tested through numerical experiments to show how efficient and effective the shipment consolidation policy is.

1. Introduction

With growing focus on supply chain management, firms realize that inventories across the entire supply chain can be more efficiently managed through greater cooperation and better coordination.

Many researches associated with multi-echelon inventory system have been reported as in the literature. The multi-echelon inventory system was introduced by Clark and Scarf (1960), and has been extensively investigated since 1960. In recent years, better information flow and greater cooperation between companies have been realized as providing the motivation for concentrating on studying multi-echelon inventory system in the supply chain, particularly between a main manufacturer and its component buyers.

Monahan (1984) has developed a model offering quantity discount so as to induce the buyer to order in a quantity that maximizes the vendor's profit under infinite production rate assumption. Joglekar (1988) has extended Monahan's model by incorporating a finite production rate assumption. Banerjee (1986) has generalized Monahan's model by incorporating a vendor's inventory carrying cost. Lee and Rosenblatt (1986) have also considered the model, by removing the assumption of Monahan (1984) and Banerjee (1986) that the vendor performs his part of negotiation based on a lot-for-lot policy.

Goyal (1977) has considered a joint economic lot size model where the objective is to minimize total relevant cost for a system of one vendor and one buyer. In the lot size model, the lot-for-lot policy is incorporated, and everybody can contribute himself to the whole system rather than his own benefit. Banerjee (1986b) has generalized Goyal's model by incorporating a finite production rate for the vendor. Goyal (1988) has also generalized Banergee's model by removing the lot-for-lot policy for the vendor, and showed that his model provides a lower (or equal) joint total relevant cost. Goyal and Szendrovits (1986) have considered a shipment policy which combines a number of increasing shipment sizes followed by a number of equal shipment sizes. A review of published work on buyer-vendor coordination models up to the year 1988 has been given by Goyal and Gupta (1989).

More recently, Lu (1995) has relaxed the lot-for-lot assumption of Goyal (1988) and suggested the shipment policy with equal shipment size. Goyal (1995) has suggested a shipment policy with unequal shipment sizes allowed, which involves successive shipment sizes within a lot consecutively increasing by a factor equal to the ratio between the vendor' s production rate and the demand rate on the buyer. This was again based on an earlier idea from Goyal (1977). Goyal (1995)'s shipment policy can result in a lower joint total cost than the Lu's equal shipment size policy in one-vendor and one-buyer situation.

Hill (1997) has suggested a shipment policy with unequal shipment sizes allowed which increases by a general fixed factor in one-vendor and one-buyer situation. He has suggested that the ith shipment size should be determined by evaluating the product term, (first shipment size) $\cdot\, y^{i-1}$, where $1 \le y \le$ *(Production rate/Demand rate)*. The resulting policy obtained by Hill (1997) would provide a lower total cost policy as compared to the policy obtained by Lu (1995) and Goyal (1995). It is not surprising that this more general class of policy gives rise to lower joint total cost solutions than either of the special cases, but this is at the expense of producing solutions that are less likely to be of practical interest.

Goyal (2000) has suggested a shipment policy with unequal shipment sizes allowed in one-vendor and one-buyer situation that the first shipment will be of small size followed by (n-1) equal sized shipments, which is computed at the size; (first shipment size) • (Production rate/Demand rate). This type of policy ensures a quick delivery of the first shipment to the buyer and avoids excessive inventory levels of higher order shipments at the buyer's end.

Lu (1995) has suggested a shipment policy for a one-vendor multi-buyer multi-product problem with infinite production rate allowed. In the problem, the lot-for-lot policy is not incorporated either, but only the vendor can have market power (negotiation power). Therefore, the problem has the objective of minimizing the total cost of the vendor. In other words, the problem didn't consider coordination between vendor and buyers.

There are many system issues which need to be considered in the multi-buyer case. To the best knowledge of the authors, however, most of the work done in the literature has concentrated on proposing one-vendor one-buyer case. This provides the motivation of considering a production planning and shipment policy for an integrated one-vendor multi-buyer inventory model concerned with multiple products.

In a manufacturing setting, it is assumed that each time a machine is set up, a major setup cost is incurred independent of which product type is produced. Furthermore, when each product is produced, a minor setup cost, namely an additional fixed setup cost is charged, which depends on the product type. By coordinating production cycle and replenishments of different product types, the manufacturer can reduce the associated average major setup costs.

In this paper, it is assumed that each buyer can purchase only one type of product. This paper considers the issue associated with consolidation of products. Consolidation is considered whenever different products are to be carried together in the same vehicle (Randolph W. Hall, 1987). Consolidation involves picking-up and dropping-off products at different origins and destinations. In this paper, consolidation means that a vehicle of infinite capacity delivers ("drops off", or "peddles") all the required products to all the buyers through a single route, which will be called *shipment consolidation policy* in the rest of this paper. Whenever shipment consolidation occurs, it incurs a fixed cost, such as driver wages, fuel and vehicle maintenance, etc. While shipment by use of the shipment consolidation policy becomes complex, one of the benefits associated with the shipment consolidation policy is that average transportation cost can be reduced.

This paper compares three shipment policies (Policy_1, Policy_2 and Policy_SC) among themselves to manage the integrated one-vendor multi-buyer integrated inventory model concerned with multiple products. Policy_SC is the shipment policy considering consolidation for multi-buyers. Poicy_1 and Policy_2 are not considering any consolidation. In other words, these policies are direct shipment policies for buyers.

The remainder of this paper is organized as follows. Section 2 describes the detail assumptions and notation, and presents the model formulation for the proposed integrated multi-product one-vendor multi-buyer inventory problem. Section 3 analyzes the solution properties and Section 4 proposes an algorithm based on the solution properties. Section 5 gives computational results to verify the effectiveness of the proposed shipment consolidation policy (Policy_SC). Section 6 states some concluding remarks.

2. Problem description

The proposed integrated multi-product inventory system considers a single vendor (manufacturer) supplying N buyers with N different types of products. Each buyer is assumed to purchase one type of product. The vendor manufactures multi-products at finite rates. It is assumed that each time a machine is set up, a major setup cost is incurred independent of which product type is produced. Furthermore, when each product is produced, a minor setup cost, namely an additional fixed setup cost, is charged that depends on the product type. In this paper, the lot-for-lot restriction is removed; in other words, each production lot is dispatched to the buyer in a number of shipments and some of which may be made, while production is still taking place.

It is assumed that demand rates, production rates and inventory costs are known and constant, and also that no shortage is allowed for finished products. Moreover, all replenishment lead times are assumed to be zero.

Let index i denote the i-the buyer. Buyer i purchases product type i from the vendor at an annual demand rate D_i, ordering cost A_i and annual inventory holding cost H_i. Let T be the vendor's time interval between two consecutive major setups, which is the time length of one production cycle, and let the vendor's annual inventory holding cost be h_i ($H_i > h_i$) and the vendor's annual production rate of product type i be P_i such that $P_i > D_i$ for $\forall i$. There are two kinds of setup costs associated with production; a major setup cost S incurred when each production cycle is started and a minor setup cost s_i is incurred when product i is produced. It is further assumed that all the minor setup costs associated with each product type can be treated with negligible setup cost in each production cycle.

This paper considers shipment consolidation policy of products, which means that a vehicle with infinite capacity delivers ("drops off", or "peddles") all the required products to all the buyers through a single route. Thus, all the shipments for each buyer occur at the same time and shipment size for each buyer is constant, respectively. This paper introduces *consolidation routing cost, CR*, which denotes the fixed transportation cost for shipment consolidation.

Assuming that the production of a lot is started as late as possible, the dispatching of the first shipment will return the vendor stock level to be zero. Because the shipment sizes of each lot are non-decreasing, the amount of time to be spent for consuming the last shipment of one lot will be greater than the amount of time to be spent for producing the first shipment of the next lot. Therefore, the dispatching of the last shipment of one lot takes place before the production of the next lot starts.

Essentially, the decision issues of the decision makers (vendor and buyer) are as follows;

1. production cycle for the vendor,
2. economic number of shipments in which a lot will be sent to the buyer.

Thus, the objective is to determine the production cycle and shipment schedule which minimize the mean total annual cost per unit time. Individual shipment sizes are calculated naturally when the production cycle and the number of shipments are calculated.

2.1 Problem Formulation

The cost of manufacturing set ups and the cost of shipments for all policies are represented together by $\frac{1}{T}\left\{S+\sum_{i=1}^{N}(s_i+n_iA_i)\right\}$.

2.1.1 Policy_1 and consolidation policy

(a) Average stock level of buyer i

Let's consider a production lot of cycle time T which is made up of n_i shipments. Since we are assuming that $h_i > H_i$, the optimal solution must involve the vendor sending a shipment only when the buyer is just about to run out of stock.

The size of the j-th shipment within a lot is $\lambda_i^{j-1}q_i$. This shipment will allow for the buyer to last during the period of $\lambda_i^{j-1}q_i/D_i$, and during this time the average stock level of buyer i is $\frac{1}{2}\lambda_i^{j-1}q_i$. Thus, the time-weighted stockholding for buyer i during a complete production cycle is

$$\sum_{j=1}^{n_i}\frac{1}{2}\lambda_i^{j-1}q_i\times\frac{\lambda_i^{j-1}q_i}{D_i}=\sum_{j=1}^{n_i}\frac{(\lambda_i^{j-1}q_i)^2}{2D_i}=\frac{q_i^2(\lambda_i^{2n_i}-1)}{2D_i(\lambda_i^2-1)}$$

The total lot production size for product i (sum of the n_i shipments) is

$$\sum_{j=1}^{n_i}\lambda_i^{j-1}q_i=\frac{q_i(\lambda_i^{n_i}-1)}{(\lambda_i-1)}=D_iT$$

and the production cycle time for product i is equal to the time duration for the demand process to consume the lot quantity which is $\frac{q_i(\lambda_i^{n_i}-1)}{D_i(\lambda_i-1)}=T$. Hence, the average stock level of buyer i is the buyer's time-weighted stock hold during a cycle divided by the cycle time T;

$$\frac{q_i^2(\lambda_i^{2n_i}-1)}{2D_iT(\lambda_i^2-1)}$$

(b) Average stock level of vendor for the product i

The total stock for product i in the system is at a minimum when the production of a lot is just about to start. At this point, the vendor's stock level is zero and the buyer's stock level is just enough to satisfy the demand until the first shipment of the next lot arrives which corresponds to the stock amount q_iD_i/P_i. The total stock increases at the rate of (P_i-D_i) during the time duration when it takes to manufacture the lot quantity of D_iT at the rate of P_i, and reaches the maximum of $\frac{D_iq_i}{P_i}+(P_i-D_i)\times\frac{D_iT}{P_i}$ at the point when the production of a lot finishes. Thus, the average total stock for product i in the system is

$$\frac{D_iq_i}{P_i}+\frac{(P_i-D_i)D_iT}{2P_i}$$

and the average stock level of the product i held by the vendor is the average total stock minus the average stock level of product i;

$$\frac{D_iq_i}{P_i}+\frac{(P_i-D_i)D_i}{2P_iT}-\frac{q_i^2(\lambda_i^{2n_i}-1)}{2D_iT(\lambda_i^2-1)}$$

(c) The total annual cost of the vendor-buyers

Now, the mean total annual cost per unit time, $TC(\vec{n},\lambda_i,T)$, can be derived as

$$TC(\vec{n},T) =$$

$$\frac{1}{T}\left\{S+\sum_{i=1}^{n}(s_i+n_iA_i)\right\}+\sum_{i=1}^{n}H_i\times\left[\frac{D_i^2T(\lambda_i-1)}{P_i(\lambda_i^{n_i}-1)}+\frac{(P_i-D_i)D_iT}{2P_i}\right]$$

$$+\sum_{i=1}^{N}(h_i-H_i)\times\left[\frac{D_iT(\lambda_i-1)(\lambda_i^{n_i}+1)}{2(\lambda_i^{n_i}-1)(\lambda_i+1)}\right]$$

2.1.2 Policy_2

The first shipment will be of a small size followed by (n-1) equal sized shipments which is computed at the size; (First shipment size) • (Production rate/Demand rate).

This policy ensures a quick delivery of the first shipment to the buyer and avoids excessive inventory levels of higher order shipments at the buyer's end.

The vendor ships the entire lot quantity, D_iT, in n_i shipments as given below;

First shipment quantity of q_i, followed by (n-1) shipments each having equal quantity $\lambda_i q_i$. Hence, the lot quantity is at $D_iT = q_i(q+(n_i-1)\lambda_i)$

(a) Average stock level of buyer i

The time weighted inventory for buyer i in a cycle is given by

$$\text{Total inventory} = \frac{q_i^2}{2D_i} + \frac{(n_i-1)(q_i\lambda_i)^2}{2D_i}$$

$$\text{Average inventory} = \frac{q_i^2(1+(n_i-1)\lambda_i^2)}{2D_iT}$$

(b) Average stock level of vendor for product i

The total stock for product i in the system is at the minimum when the production of a lot is just about to start. At this point, the vendor's stock level is zero and the buyer's stock level is just enough to satisfy the demand until the first shipment of the next lot arrives which corresponds to the stock amount of q_iD_i/P_i. The total stock increases at the rate of $(P_i - D_i)$ during the time duration when it takes to manufacture the lot quantity of D_iT at the rate P_i and reaches the maximum of $\frac{D_iq_i}{P_i} + (P_i - D_i)\times\frac{D_iT}{P_i}$ at the point when the production of a lot finishes. Thus, the average total stock for product i in the system is

$$\frac{D_iq_i}{P_i} + \frac{(P_i - D_i)D_iT}{2P_i}$$

and the average stock level of product i held by the vendor is the average total stock minus the average stock level of product i;

$$\frac{D_iq_i}{P_i} + \frac{(P_i - D_i)D_iT}{2P_i} - \frac{q_i^2(1+(n_i-1)\lambda_i^2)}{2D_iT}$$

(c) The total annual cost of the vendor-buyers

Now, the mean total annual cost per unit time, $TC(\vec{n},\lambda_i,T)$, can be derived as

$$TC(\vec{n},T) = \frac{1}{T}\left\{S+\sum_{i=1}^{n}(s_i+n_iA_i)\right\}+\sum_{i=1}^{n}H_i\times\left[\frac{D_iT}{\lambda_i(1+(n_i-1)\lambda_i)}+\frac{(P_i-D_i)T}{2\lambda_i}\right]$$

$$+\sum_{i=1}^{N}(h_i-H_i)\times\left[\frac{D_iT(1+(n_i-1)\lambda_i^2)}{2(1+(n_i-1)\lambda_i)^2}\right]$$

3. Analysis

Now, the mean total annual cost per unit time, $TC(\vec{n},T)$, can be derived as

$$TC(\vec{n},T) = \frac{1}{T}\left\{S+\sum_{i=1}^{n}(s_i+n_iA_i)\right\}+\sum_{i=1}^{n}H_i\times\left[\frac{D_i^2T(\lambda_i-1)}{P_i(\lambda_i^{n_i}-1)}+\frac{(P_i-D_i)D_iT}{2P_i}\right]$$

$$+\sum_{i=1}^{N}(h_i-H_i)\times\left[\frac{D_iT(\lambda_i-1)(\lambda_i^{n_i}+1)}{2(\lambda_i^{n_i}-1)(\lambda_i+1)}\right]$$

Given all the values of $\vec{n}$, the value of T, denoted by T^*, which minimizes TC (obtained by differentiating TC with respect to T and setting the result to the value 0) is derived as

$$T^* = \left[X(\vec{n})/Y(\vec{n})\right]^{1/2}$$

where $X(\vec{n}) = S+\sum_{i=1}^{N}(s_i+n_iA_i)$

$$Y(\vec{n}) = \sum_{i=1}^{n}H_i\times\left(\frac{D_i^2T(\lambda_i-1)}{P_i(\lambda_i^{n_i}-1)}+\frac{(P_i-D_i)D_i}{2P_i}\right)+\sum_{i=1}^{n}(h_i-H_i)\times\left(\frac{D_i(\lambda_i-1)(\lambda_i^{n_i}+1)}{2(\lambda_i^{n_i}-1)(\lambda_i+1)}\right)$$

and the corresponding mean total cost per unit time is derived as

$$TC(T^*) = 2\left[X(\vec{n})Y(\vec{n})\right]^{1/2}.$$

3.1 Policy_1

For policy_1 ($\lambda_i = P_i/D_i$), the results are reduced to the function

$$TC(\vec{n},T) = \frac{1}{T}\left\{S+\sum_{i=1}^{N}(s_i+n_iA_i)\right\}+\sum_{i=1}^{N}(H_i+\lambda_ih_i)\times\frac{D_iT(\lambda_i-1)(\lambda_i^{n_i}+1)}{2\lambda_i(\lambda_i^{n_i}-1)(\lambda_i+1)}$$

Given all the values of $\vec{n}$, the value of T, denoted by T^*, which minimizes TC (obtained by differentiating TC with respect to T and setting the result to the value 0) is derived as

$$T^* = \left[X(\vec{n})/Y(\vec{n})\right]^{1/2}$$

where $X(\vec{n}) = S+\sum_{i=1}^{N}(s_i+n_iA_i)$

$$Y(\vec{n}) = \sum_{i=1}^{N}(H_i+\lambda_ih_i)\times\frac{D_i(\lambda_i-1)(\lambda_i^{n_i}+1)}{2\lambda_i(\lambda_i^{n_i}-1)(\lambda_i+1)}$$

and the corresponding mean total annual cost per unit time is derived as

$$TC(T^*) = 2\left[X(\vec{n})Y(\vec{n})\right]^{1/2}.$$

3.2 Policy_2

For policy_2 ($\lambda_i = P_i/D_i$), the results are reduced to the function

$$TC(\vec{n},T) = \frac{1}{T}\left\{S+\sum_{i=1}^{n}(s_i+n_iA_i)\right\}+\sum_{i=1}^{n}H_i\times\left[\frac{D_iT}{\lambda_i(1+(n_i-1)\lambda_i)}+\frac{(P_i-D_i)T}{2\lambda_i}\right]$$

$$+\sum_{i=1}^{N}(h_i - H_i)\times\left[\frac{D_i T(1+(n_i-1)\lambda_i^2)}{2(1+(n_i-1)\lambda_i)^2}\right]$$

Given all the values of $\vec{n}$, the value of T, denoted by T^*, which minimizes TC (obtained by differentiating TC with respect to T and setting the result to the value 0) is derived as

$$T^* = \left[X(\vec{n})\big/Y(\vec{n})\right]^{1/2}$$

where $X(\vec{n}) = S + \sum_{i=1}^{N}(s_i + n_i A_i)$

$$Y(\vec{n}) = \sum_{i=1}^{N} H_i \times\left[\frac{D_i}{\lambda_i(1+(n_i-1)\lambda_i)} + \frac{(P_i - D_i)}{2\lambda_i}\right] + \sum_{i=1}^{N}(h_i - H_i)\times\left[\frac{D_i(1+(n_i-1)\lambda_i^2)}{2(1+(n_i-1)\lambda_i)^2}\right]$$

and the corresponding mean total cost per unit time is derived as

$$TC(T^*) = 2\left[X(\vec{n})Y(\vec{n})\right]^{1/2}.$$

3.3 Consolidation policy

The number of shipments is adjusted to deliver each product to the corresponding buyer at the same time. For this case, we eliminate the subscript i of n_i. Also, each routing cost is replaced by the associated shipment costs.

For the case of consolidation shipment policy ($\lambda = 1$), the results are reduced (using L'Hôpital's Rule) to the function

$TC(n,T)$ =

$$\frac{1}{T}\left\{S + n\times R + \sum_{i=1}^{N} s_i\right\} + \sum_{i=1}^{N} H_i \times\left[\frac{D_i^2 T}{nP_i} + \frac{(P_i - D_i)D_i T}{2P_i}\right]$$

$$+\sum_{i=1}^{N}(h_i - H_i)\times\left[\frac{D_i T}{2n}\right].$$

Given the value of n, the value of T, denoted by T^*, which minimizes TC (obtained by differentiating TC with respect to T and setting the result to 0) is derived as

$$T^* = \left[X(n)\big/Y(n)\right]^{1/2}$$

where $X(n) = S + n\times R + \sum_{i=1}^{N} s_i$

$$Y(n) = \sum_{i=1}^{N} H_i \times\left(\frac{D_i^2}{nP_i} + \frac{(P_i - D_i)D_i}{2P_i}\right) + \sum_{i=1}^{N}(h_i - H_i)\times\left(\frac{D_i}{2n}\right)$$

4. Algorithm

In this section, an iterative optimal search procedure for the integrated multi-product inventory problem is proposed. For consolidation policy ($\lambda = 1$), $TC(n,T)$ is jointly convex in n and T (see Appendix) so that the optimal solution can be obtained by finding one parameter with the other parameter fixed in alternating manner. First for a particular integer value of n > 0, we find T', and then, for the fixed T', we find n' next. This fashion of alternating procedure continues, iteratively.

Search Procedure;

Step 0. Finding T' with particular parameter n

Step 1. $TC(n,T)$ is a convex function of n. Given T', T is substituted by T' in $TC(n,T)$ and then differentiated with respect to n.

$$n^2 = \frac{T}{R}\sum_{i=1}^{N}\left\{\frac{H_i D_i^2 T}{P_i} + (h_i - H_i)\times\left(\frac{D_i T}{2}\right)\right\}$$

Using Schwarz's result (1973), when n is found as an integer, the optimal solution of $TC(n,T')$ is n itself, which is an integer such that

$$n(n-1) < \frac{T}{R}\sum_{i=1}^{N}\left\{\frac{H_i D_i^2 T}{P_i} + (h_i - H_i)\times\left(\frac{D_i T}{2}\right)\right\} \le n(n+1),$$

from which n' is found.

Step 2. Finding $T' = \left[X(n)\big/Y(n)\right]^{1/2}$ with parameter n'.

Step 3. If T' is equal to its preceding T', then stop. If else, Go to *Step 1.*

The mean total annual cost converges to the optimal solution.

5. Computational Results

This chapter consists of three parts. The first part is the basic data part to demonstrate the total cost and the shipment frequency of the three policies. Specifically, the consolidation policy is compared with policy_1 and policy_2 in terms of production lot size and the number of shipments. The second part solves numerical examples and depicts the trend of how the cost function moves with changing parameters. The last part describes the efficiency of consolidation policy by comparing it with two types of policies (policy_1 and policy_2).

In Tables 5-1, a variety of different problems with a variable major setup cost, minor setup costs, shipment costs and vendor's holding costs, buyer's holding costs, demand rates and production rates are considered to compare two types of inventory systems one against the other and explain the effectiveness and efficiency of the consolidation policy. For the efficiency investigation, 30 problems are generated with 5, 7, and 9 retailers. In order to evaluate the performance of the consolidation policy, the optimal solutions obtained with each of the ratios are compared with the associated optimal solutions which are found through the other policies. Column "B.E.P" means the ratio, "Break even percent," of the total shipment cost to the routing cost when the smaller value between Policy_1 and Policy_2's total costs meets the consolidation policy's total cost.

It can not be claimed that one policy is always better than the other policy, between Policy_1 and Policy_2's policy. The lower mean total annual cost is obtained by the consolidation policy when routing cost is lower than 75.3% of the total shipment cost on average.

	# of buyer	shipment cost					Policy_1	Policy_2	Consolidation Policy			
		1	2	3	4	5	Total cost	Total cost	60% cost	70% cost	80% cost	B.E.P.
1	5	95	118	205	298	137	19488.7	19111.8	18013.4	18694.8	19237.7	77.7
2	5	181	313	181	331	248	20347.4	20290.9	18679.3	19359.5	20016.6	84.3
3	5	67	259	308	177	51	19246.4	19108.1	18039.1	18666.2	19431.5	75.8
4	5	297	319	335	124	219	19697.5	19728.3	18312.9	18945.9	19558.4	81.4
5	5	310	202	291	180	257	20461.7	20517.4	19163.7	19857.9	20867.2	76
6	5	333	312	217	172	75	19757.7	19606.3	18242.5	19010.5	19583.6	80.4
7	5	121	55	312	227	194	19158.2	19206	18015.6	18651.6	19351.7	77.2
8	5	269	202	72	163	262	17924	17878.1	17299.9	18061.4	18580.9	67.6
9	5	61	212	90	265	215	19132.3	19328.5	18541.1	19166.8	19925.1	69.4
10	5	55	90	173	279	323	20450.3	20096.5	19296.7	19949.1	20580.8	72.3

Table 5-1. Computational Results in the 1:N inventory system with

$N=5$, $S\sim U(800\sim1300)$, $s_i\sim(300,500)$, $A_i\sim(50,350)$, $H_i\sim(3,6)$, $h_i\sim(2,5)+H_i$, $D_i\sim(1000,2000)$, $P_i\sim(2500,3500)$

6. Conclusions

This paper deals with an integrated multi-product inventory model with shipment consolidation policy incorporated. This model is appropriate when major setup cost is heavy. The proposed policy is compared with two other policies; Policy_1 is the policy that each shipment consecutively increases by the factor λ and Policy_2 is the policy that only the second shipment size increases by the factor λ. The above two policies are direct shipment policies. The proposed C_policy is a consolidation policy that all products are consolidated together whenever shipment is executed. Thus, the associated routing cost is substituted for the associated shipment cost. The objective of the model is to determine that production and shipment schedules, in the integrated system, which minimize the mean total annual cost per unit time for a vendor manufacturing products to supply to multi buyers, under the assumption that each product lot is dispatched to the associated buyer in a number of shipments, some of which may be made while production is still taking place. Three policy models are constructed to find the lower mean total annual cost for which an iterative optimal solution procedure is derived and tested for its performance by comparing the consolidation policy with policy_1 and policy_2. Also numerical examples are presented to show how the models behave with parameter change and 30 instances are solved to show that the consolidation policy can perform efficiently by coordinating the replenishments of all the different product types. The lower mean total annual cost is obtained by the consolidation policy when the routing cost is lower than 75.3% of the total shipment cost on average. Based on the computational experiments, it is concluded that the proposed consolidation policy is efficient and effective when the total shipment cost is heavy.

As a further study, the following interesting issues may be considered to extend the policy; the routing policy which divides buyers into several groups and applies consolidation or direct shipment according to each group. Other possible extensions include incorporation of delivery constraint on the maximum quantity which can be shipped at any time or storage constraint on the maximum stock which can be held at any time by the buyer.

Acknowledgement

This work was partly supported by grant No. (R01-2002-000-00023-0) from the Basic Research Program of the Korea Science & Engineering Foundation.

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