

A Dual-Time Stepping Method for Unsteady Computations of Incompressible Flows

S. GOKALTUN¹, H. SAYGIN², M. MURADOGLU³

1. Informatics Institute, Istanbul Technical University, Maslak, 34469 Istanbul, TURKEY, gokaltunse@itu.edu.tr

2. Energy Institute, Istanbul Technical University, Maslak, 34469 Istanbul, TURKEY, sayginh@itu.edu.tr

3. Department of Mechanical Eng, KocUniversity Sariyer, 34450 Istanbul, TURKEY, mmuradoglu@ku.edu.tr

Corresponding author S. GOKALTUN

Abstract

Time accurate simulations of unsteady flows are of great practical importance since most engineering flows are inherently time dependent, such as flows involving fluid-structure interactions. Apart from spatial resolution, temporal resolution is also required in unsteady flow simulations. Since the time scale is large compared to that governing the stability of the numerical method used to solve the equations of motion, the biggest difficulty in time-accurate solutions of unsteady flows is the long computation time. Explicit methods have been used traditionally for time-accurate calculations but in these methods the time-step set by stability restrictions becomes prohibitively small for long time simulations. In order to overcome this difficulty, implicit methods have been developed which have a larger stability region compared to explicit methods. However the resulting linear system of equations in implicit schemes are generally CPU and memory demanding. Alternating Directional Implicit (ADI) scheme introduced by Beam and Warming [1] and Briley and McDonald [2] is usually used to remove these deficiencies in implicit methods. ADI scheme has been successfully applied to steady flow computations [3]. However, recently it has been found useful to use dual-time stepping method for computations of unsteady flows [7],[6],[3]. The dual-time stepping method suggested first by Merkle and Athavale has some advantages over the conventional algorithms including elimination of the factorization error in factored implicit schemes, elimination of errors due to approximations made in the implicit operator to remove numerical efficiency, elimination of errors due to lagged boundary conditions and ability to use preconditioned iterative methods for more efficient convergence. Dual-time stepping method can also incorporate other convergence acceleration techniques such as multigrid and local time-stepping methods in order to further improve the computational efficiency in solving the system of equations resulting from the implicit formulation of the governing equations. Artificial compressibility method introduced by Chorin [5] has proven to successfully accelerate the convergence rate and improve the numerical accuracy of steady computations [10],[8],[7]. This method was initially developed for incompressible flow equations [5] but it was then generalized by Turkel for incompressible and compressible flow computations. Time-accuracy is destroyed by the artificial compressibility in the conventional time-stepping algorithms so it is not applicable for time-accurate computations. However, the artificial compressibility method can be safely used in dual-time stepping method since we are free to modify the pseudo time derivative terms to remove stiffness of the governing equations as long as a steady state is reached in each inner iteration in pseudo time.

In the present paper an efficient numerical method based on the dual-time stepping approach is presented for computations of unsteady incompressible flows. The base method is the multigrid ADI scheme developed by Caughey [4] for the compressible Euler equations. Pseudo-time derivative terms are added to the incompressible Navier-Stokes equations and Turkel's type preconditioning method is used to improve the efficiency of the numerical algorithm. The resulting equations are solved by an alternating direction implicit (ADI) method developed by Beam and Warming [1] and Briley and McDonald [2]. Convergence in pseudo time is further accelerated by a multigrid technique. The spatial derivatives are approximated by a finite volume method using central differences and fourth order numerical dissipation terms are added to stabilize the scheme. The physical time derivative terms are

discretized by using a three-point second order implicit method. Stability of the scheme is investigated by using the Von Neumann analysis applied to a model scalar equation both in real and pseudo times.

The method is implemented for solving 2-D plane and axisymmetric incompressible laminar flows and is validated for an axisymmetric unsteady oscillating pipe flow [9] and flows around oscillating circular cylinder.

Keywords: *Unsteady flow, dual time stepping, preconditioning, incompressible.*

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- Ahn ByungJae, 1-2D-4
 Ahn Byungjin, 3-2A-3
 Ahn Chang Su, 2-3A-4
 Ahn MuYoung, 2-1C-3
 Aoyama Takashi, 2-3A-5
 Arai Norio, 2-3A-1
 Aristov Vladimir, 3-3B-1
- Badri M. A., 3-1D-2
 Baek Jehyun, 2-3A-5
 Bai KwangJune, 2-3D-1
 Baranov N.A, 2-2A-1
 Belotserkovsky A.S., 2-2A-1
 Biolan Tian, 1-2A-1
 Boo KyungTae, 2-1D-2
- Cao Handing, 3-1C-1
 Cao Renjing, 1-3D-1
 Chang Keun Shik, 3-2A-4, 2-1C-3, 3-1A-3
 Chen Chaosong, 3-1C-1
 Chen Shia-Chung, 2-1B-1
 Cho InSung, 2-2A-4
 Cho Seong Gee, 3-2D-1
 Cho Song Pyo, 2-3D-1
 Choe HiJun, 2-1C-3
 Choi Dohyung, 2-1D-2
 Choi Haecheon, 2-1A-1, 1-3A-4, 1-1D-2, 3-2B-4,
 Plenary Lecture
 Choi HyoungGwon, 1-3C-1
 Choi Jeong-Yeol, 1-2B-4
 Choi Jin, 1-3A-4
 Choi SeokKi, 1-1A-3
 Chong QI Gansu, 3-1A-2
 Chu W. kwang-Hua, 1-3B-2
 Chun Ho Hwan, 2-2D-1
 Chun Suh Jung, 1-1D-4
 Chun Tae-Hyun, 1-2A-3, 1-3A-3
 Chung DongKyu, 2-1C-3
 Chung KyungNam, 2-1D-4
 Chung Y. M., 1-2A-4
 Ciheng Zhang, 3-1C-2
 Cooper-White Justin, 1-1B-2
 Cummings Russ, Plenary Lecture
- Dartzi Pan, 1-1B-1
 Davidson Malcolm, 1-1B-2
 Ding Shifa, 3-1C-1
- Feng Shi-De, 1-3D-2
 Forsythe Jim, Plenary Lecture
 Fu Dexun, 1-2A-1
 Fu L.-M., 2-1B-5
- Gokaltun S., 3-3D-4
 Guzev Mihail, 2-1D-3, 2-2D-3
- Ha ManYeong, 3-3C-5
 Hai Duong Ngoc, 3-2D-3, 3-1D-4
 Han Cho Young, 3-2C-3
 Hahn Seonghyeon, 1-3A-4
 Hassan Mohammah, 2-2C-2
 Heo HyeungSeok, 2-1B-4
 Hideo Kashimuro, 2-2A-2
 Himeno R., 2-2B-1
 Honami Shinji, 1-3D-5
 Hong Jun Ho, 2-3A-1
 Hong SeungKyu, 2-2A-3, 2-3A-2
 Horitani Yoshiki, 3-1C-3
 Hsu Kun-Lin, 2-1B-1
 Huh Kang, 2-1B-2
 Hui W.H., 1-1C-3, 3-1D-1
 Hwang In Ju, 3-1D-3
 Hwang Jong Yeon, 1-2D-1
- Ido Masato, 2-3A-3
 Inamuro Takaji, 2-1B-3
 Inoue Osamu, 1-3B-1
 Ito Shigeyuki, 3-1B-2
 Iton Yuichi, 3-2C-2
- Jang Dong Soon, 3-2D-2, 3-2C-1
 Jang KyoungSik, 2-1A-3
 Jeong Giho, 3-2A-3
 Jeong SooIn, 3-2A-3
 Jeong Woo-Pyung, 1-3A-4
 Jin June Ha, 3-2D-2
 Joo Jae-Hyun, 1-2C-3
 Joo Jin, 3-3A-4
 Jung Namkyun, 2-1D-2
 Jung Sung Hee, 3-2D-2
- Kajishima T., 1-1A-1
 Kamiyo Takuma, 1-3D-5
 Kamiunten Shouji, 1-3D-5
 Kan Makiko, 3-1B-4
 Kang HoKeun, 1-3B-4
 Kang Kuk-Jin, 2-3D-2, 2-3D-3
 Kang Sung Mo, 3-1C-4
 Kang Sungwoo, 1-3C-1
 Kashimura Hideo, 2-3C-2
 Katsuhiko SAKAI, 2-3B-2
 Kaqamura Tetuya, 3-1B-4
 Kawamura Tetuya, 3-2A-1, 3-2A-2, 3-2D-4, 3-3A-2, 3-3D-2
 Kawasaki Yasukazu, 1-3D-2
 Kim B.J., 1-3D-4
 Kim Beomjun, 1-3C-1
 Kim ChongAm, 2-3C-1, 3-3D-2
 Kim Dehee, 1-1C-2
 Kim Dong-Hoon, 3-3A-3
 Kim Dong Won, 3-3A-1

- Kim Eui Kwang, 1-1A-3
 Kim Eun Chan, 2-3D-2
 Kim Heuy Dong, 2-3C-2, 3-3B-2
 Kim HeySuk, 3-2D-2, 3-2C-1
 Kim Hong-Min, 3-2B-2
 Kim HongSik, 3-1D-3
 Kim JaeSoo, 2-1C-2
 Kim JaeWook., 2-3B-5
 Kim Jeong Soo, 3-2C-3
 Kim Jeonglae, 1-3A-4
 Kim Jinsung, 1-1D-2
 Kim JooSung, 2-3B-1
 Kim Justin, Plenary Lecture
 Kim Insun, 1-2B-3
 Kim K.Y., 1-3C-4
 Kim Kuisoon, 3-2A-3
 Kim Kwang Su, 1-1D-4
 Kim KwangYong, 3-2B-2
 Kim Kyu Hong, 2-3C-1
 Kim SaYup, 2-2B-3
 Kim Se-eun, 2-1D-2
 Kim Seong O, 1-1A-3
 Kim SeongLyong, 1-2B-3
 Kim Sung Kyun, 1-3D-3
 Kim Tae-Ho, 3-3B-2
 Kim TaeKook, 1-3C-3
 Kim WooSeung, 2-2D-2
 Kim Y.I., 2-1D-4
 Kim Yoonsik, 1-1C-2
 Kim YongMo, 3-1C-4
 Kim Yongsik, 2-1C-3
 Kim Youn J., 1-2D-4, 3-3A-1
 Kim Youn Jea, 3-1D-3
 Kim YoungHo, 2-2B-2, 2-2B-3
 Kim Youngtae, 1-3C-3
 Kimura Isao, 2-3B-2
 Kmiyama Kohji, 1-1D-3
 Ko K., 1-3A-3
 Kobayashi Kazuaki, 2-1B-3
 Kobayashi Seijiro, 2-3C-3
 Kobayashi Toshio, 3-2C-2
 Kondo Natsuki, 3-3A-5
 Koo Hyung Mo, 2-3C-1
 Koshel Konstantin, 2-2D-3
 Kouta T., 1-3D-5
 Krishnan L., 3-3D-3
 Kuang Jianghong, 3-1C-1
 Kulkarni PS, 1-2B-2, 2-2C-1, 3-3C-3
 Kulkarni V.N., 1-2B-2
 Kwon Jang Hyuk, 1-3A-1, 1-1C-2
 Kwon OhJoon, 3-3A-4, 3-2B-3, 2-3B-1
 Kyoung JoHyun, 2-3D-1
- Lee ChangHoon, 3-2D-1, 1-3A-2
 Lee Dong-kon, 1-3A-4
 Lee Duck Joo, 2-3A-4, 2-3B-5
- Lee G.-B., 2-1B-5
 Lee Hee Dong, 3-3A-4
 Lee Jaeyoung, 3-3C-5
 Lee Jun Hwa, 3-3A-1
 Lee Jung Sang, 3-3D-2
 Lee Kwang Seop, 2-2A-3
 Lee Kyun Ho, 3-2C-3
 Lee MiYoung, 3-2A-1
 Lee Sang Ill, 3-2D-2
 Lee Sang Wook, 3-2B-3
 Lee Seung Jae, 1-1D-4
 Lee ThongSee, 3-3C-1
 Li Ting Wen, 2-1D-2
 Li Xinliang, 1-2A-1
 Lim Seokhyun, 3-2B-4, Plenary Lecture
 Lin R. K., 1-1D-1
 Lin Yan-Ji, 2-1B-1
 Lee Young Ho, 1-3B-4
 Liu Mingyu, 1-2A-1
 Liu NY, 3-1A-1
 Liu Pingyuan, 3-1C-1
 Liu Shuhong, 3-1A-4
 Liu Y.Z., 1-3D-4
 Liu Zai lun, 3-1A-2
 Liou Meng-Sing, Plenary Lecture
 Liou Tong-Miin, 2-1B-1
- Ma Yanwen, 1-2A-1
 Matsuno Kenichi, 1-3C-2
 Matsuo Shigeru, 2-3C-2
 Matsuuchi Kazuo, 3-2B-1
 Mazarehi Kiumars, 1-2B-1
 Min K.S. 2-1D-4
 Mirzaei Masoud, 2-2C-2
 Miyashita Kazuko, 3-3A-2
 Mizumot N., 3-2D-4
 Mochimaru Yoshiro, 1-2C-2
 Mohd Zulkiefly Abdul, 1-3D-5
 Moon YoungJune, 1-3B-3
 Morton Scott, Plenary Lecture
 Muradoglu M., 3-3D-4
 Muramoto Kazunao, 1-1B-3
 Myong Hyon Kook, 1-2A-3, 1-3A-3
 Myasnikov Veniamin, 2-1D-3
- Nakahashi K., 1-2C-1
 Nakanishi Tameo, 2-1A-2
 NEJATI Vahid, 3-2B-1
 NG Eddie YK, 3-1A-1
 Niiyama Daisuke, 2-3B-3
 NISHIDA Hidetoshi, 1-2D-2
 Nguyen The Duc, 3-1D-4, 3-2D-3
 Nks Rajan , 3-3C-3
 Noda Susumu, 3-1C-3
- Ogino Fumimaru, 2-1B-3

- Oh JaiHo, 1-3C-3
 Oh Se-Jong, 1-2B-4
 Ohm Tae In, 3-2C-1
 Ohwada Taku, 2-3C-3
 Okajima Atsushi, 1-1D-3
 Oshima K., Plenary lecture
 Oshima Yuko, 3-1B-2
- Pan Cun-Hong, 3-1D-1
 Park Chul, Plenary lecture
 Park Hyung-Koo, 3-3A-3
 Park Il Ryoung, 2-3D-2, 2-3D-3, 2-2D-1
 Park JaeHyun, 1-3D-3
 Park Jaewan, 2-1B-2
 Park Jong-hun, 2-2D-1
 Park Nam Eun, 2-1C-2
 Park Noma, 2-1A-1
 Park Seung O, 2-2A-3, 2-1A-3
 Park Sung Kwan, 3-3A-1
 Park Soo Hyung, 1-1C-2, 1-3A-1
 Park Warn Gyu, 3-1D-4, 3-2D-3
 Popov Gennadiy, 2-1D-3
- Ra Seung-Ho, 1-2B-3
 Rahimian Mohammad Hassan, 3-3C-4
 Rajan S C, 3-3D-3
 Ramakrishna M. 3-3D-3
 Reddy K.P.G., 1-2B-2
 Rhee Seung Wu, 3-2C-3
 Rho Oh Hyun, 2-3C-1, 3-3D-2
 Ro Ki Deok, 1-3B-4
 Roe P.L., Plenary lecture
 Rokugou Akira, 1-1D-3
 Ruey-Hor Yen, 1-1C-1
 Rummler B., 1-2D-3
- Sah JongYoub, 1-2C-3
 Saito Shigeru, 2-3A-5, 3-3A-5
 Sato Yuko, 3-1B-4, 3-2A-2
 Saygin H., 3-3D-4
 Sereshk Reza Hosseini, 2-2C-2
 Seo Jeong Il, 3-3C-2
 Seo Junghee, 1-3B-3
 Setoguchi Toshiaki, 2-3C-2
 Seyed Reza, 2-2C-2
 Shabani Mohammad Reza, 3-3C-4
 Shigeru Itoh, 3-1B-3
 Shao Qi, 3-1A-4
 Shim InBo, 2-3A-4
 Shimura Tsutomu, 1-1B-4
 Shin Byeng Rog, 2-3B-3, 2-3B-4
 Shin Jae-Ryul, 1-2B-4
 Shin Kuan-Chen, 2-1B-1
 Shin Mi Soo, 3-2D-2, 3-2C-1
 ShiuWu Chau, 2-1B-1
 Sim Bok-Cheol, 2-2D-2
- Sohn Myoung Hwan, 3-3B-2
 Son Young Rak, 1-3D-3
 Song Dong Joo, 3-3C-2
 Song Hae Hwan, 1-2C-2
 Song Inhang, 2-1D-2
 Squires Kyle, Plenary lecture
 Srinarayana N., 2-2C-1
 Steponov Dmitriy, 2-2D-3
 Suh Yong Kweon, 2-1B-4
 Sun H.I., 3-1B-1
 Sun Mingyu, 2-2C-3
 Sung H.J., 1-2A-4, 1-3C-4, 1-3D-4
 Sung Chun-ho, 1-1C-2, 1-3A-1
 Sung Woongle, 2-3A-2
 Sutthiphong Srigrarom, 3-3D-1
- Takamiya Toshiyuki, 3-1B-2
 Takashi Aoyama, 3-3A-5
 Takayama Kazuyoshi, 2-2C-3
 Takeshi Matsuoka, 2-2A-2
 Tan SY, 3-1A-1
 Tanaka Masanori, 2-3C-2
 Taniguchi Nobuyuki, 3-2C-2
 Terada Yayoi, 1-1B-3, 1-1B-4
 Tokuyama Michio, 1-1B-3, 1-1B-4
 Tony W. H. Sheu, 1-1D-1
 Toshiaki Setoguchi, 2-2A-2
 Tsutahara Michihisa, 1-3B-4, 1-3D-2
 Tupake Ravindra, 3-3C-3
 Turchak L.I., 2-2A-1
- Vai Kuong Sin, 3-1B-1
 Van Suak Ho, 2-3D-3
- Wang Jian-Ping, 2-1D-1
 Wang Kee In, 1-3A-3
 Wu Yu-Lin, 1-1A-2, 3-1A-4
 Wu Z.N., 1-1C-3, 3-3B-3
- Xu Jinyuan, 3-1C-1
 Xue-yi QI, 3-1A-2
- Yamakawa Masashi, 1-3C-2
 Yamamoto Makoto, 1-3D-5
 Yamamoto Satoru, 2-3B-3, 2-3B-4
 Yamamuro Kunihiko, 3-1C-3
 Yamaoka Yuki, 2-1B-3
 Yamazaki Hiroyuki, 1-1B-4
 Yasunobu Tsuyoshi, 2-2A-2, 2-3C-2
 Yang Choongmo, 2-3A-5
 Yang Jaw-Yang, 1-1C-1
 Yang KyungSoo, 1-2D-1
 Yang Na, 1-2A-2
 Yang R.-J., 2-1B-5
 Yeo Kyongmin, 1-3A-2
 Yeu-Ching Perng, 1-1C-1

Yoo Jung Yul, 1-3C-1, 2-1A-1
Yoon Bok-Hyn, 3-3B-2
Yoon JuSig, 3-2A-4
Yoon J. S., 3-1A-3
Yu-Lin Wu, 2-1A-4
Yun Sangho, 2-2B-2

Zabelok Serguei, 3-3B-1
Zebib Abdelfattah, 2-2D-2
Zheng Chen, 2-1C-1
Zhong MENG Fan, 3-1A-2
ZhongDong QIAN, 1-1A-2
Zhang Ruyan, 3-1B-4
Zi-Niu WU, 2-1C-1

Edited by

Keun Shik Chang

Seung Kyu Hong

Dong Joo Song

Yong Kwon Suh

Korea Society of Computational

Fluids Engineering

ISBN 89-7581-237-5 93550

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Printed in Korea