A Dual-Time Stepping Method for Unsteady Computations of Incompressible Flows

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Abstract

Time accurate simulations of unsteady flows are of great practical importance since most engineering flows are inherently time dependent, such as flows involving fluid-structure interactions. Apart from spatial resolution, temporal resolution is also required in unsteady flow simulations. Since the time scale is large compared to that governing the stability of the numerical method used to solve the equations of motion, the biggest difficulty in time-accurate solutions of unsteady flows is the long computation time. Explicit methods have been used traditionally for time-accurate calculations but in these methods the time-step set by stability restrictions becomes prohibitively small for long time simulations. In order to overcome this difficulty, implicit methods have been developed which have a larger stability region compared to explicit methods. However the resulting linear system of equations in implicit schemes are generally CPU and memory demanding. Alternating Directional Implicit (ADI) scheme introduced by Beam and Warming [1] and Briley and McDonald [2] is usually used to remove these deficiencies in implicit methods. ADI scheme has been successfully applied to steady flow computations [3]. However, recently it has been found useful to use dual-time stepping method for computations of unsteady flows [7],[6],[3]. The dual-time stepping method suggested first by Merkle and Athavale has some advantages over the conventional algorithms including elimination of the factorization error in factored implicit schemes, elimination of errors due to approximations made in the implicit operator to remove numerical efficiency, elimination of errors due to lagged boundary conditions and ability to use preconditioned iterative methods for more efficient convergence. Dual-time stepping method can also incorporate other convergence acceleration techniques such as multigrid and local time-stepping methods in order to further improve the computational efficiency in solving the system of equations resulting from the implicit formulation of the governing equations. Artificial compressibility method introduced by Chorin [5] has proven to successfully accelerate the convergence rate and improve the numerical accuracy of steady computations [10],[8],[7]. This method was initially developed for incompressible flow equations [5] but it was then generalized by Turkel for incompressible and compressible flow computations. Time-accuracy is destroyed by the artificial compressibility in the conventional time-stepping algorithms so it is not applicable for time-accurate computations. However, the artificial compressibility method can be safely used in dual-time stepping method since we are free to modify the pseudo time derivative terms to remove stiffness of the governing equations as long as a steady state is reached in each inner iteration in pseudo time.

In the present paper an efficient numerical method based on the dual-time stepping approach is presented for computations of unsteady incompressible flows. The base method is the multigrid ADI scheme developed by Caughey [4] for the compressible Euler equations. Pseudo-time derivative terms are added to the incompressible Navier-Stokes equations and Turkel's type preconditioning method is used to improve the efficiency of the numerical algorithm. The resulting equations are solved by an alternating direction implicit (ADI) method developed by Beam and Warming [1] and Briley and McDonald [2]. Convergence in pseudo time is further accelerated by a multigrid technique. The spatial derivatives are approximated by a finite volume method using central differences and fourth order numerical dissipation terms are added to stabilize the scheme. The physical time derivative terms are

discretized by using a three-point second order implicit method. Stability of the scheme is investigated by using the Von Neumann analysis applied to a model scalar equation both in real and pseudo times.

The method is implemented for solving 2-D plane and axisymmetric incompressible laminar flows and is validated for an axisymmetric unsteady oscillating pipe flow [9] and flows around oscillating circular cylinder.

Keywords: *Unsteady flow, dual time stepping, preconditioning, incompressible.*

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