Aspect Ratio Effect of the Natural Convection in Horizontal Enclosure with an Array of Square

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Abstract

The physical model considered here is a horizontal layer of fluid heated below and cooled above with a periodic array of evenly spaced square cylinders placed at the center of the layer, whose aspect ratio here varies from unity to six. Periodic boundary condition is employed along the horizontal direction to allow for lateral freedom for the convection cells. Two-dimensional solution for unsteady natural convection is obtained using an accurate and efficient Chebyshev spectral multi-domain methodology for a given Rayleigh numbers of 10^6.

Keyword: *natural convection, heat transfer, aspect ratio*

1. Introduction

Rayleigh-Bénard convection in a horizontal layer of fluid confined between two parallel plates, with the bottom plate heated and the top one cooled is well studied for over a century. It has been well stablished that the horizontal layer of fluid becomes unstable above a Rayleigh number of 1708 and convective motion sets in the form of steady convective rolls of aspect ratio (width to height) of about two[1,2]*. Recently Ha* et al.[3,4] *considered the problem of natural convection in a square enclosure with isothermal top and bottom boundaries, and adiabatic side walls. A square cylinder was placed centered inside the enclosure and four different boundary conditions were considered for this internal cylinder: adiabatic, hot isothermal, cold isothermal, and neutral isothermal. The influence of the internal square body and its thermal boundary condition on the nature of convection and overall heat transfer was investigated.*

Here we consider a horizontal layer of fluid heated below and cooled above with a periodic array of evenly spaced square cylinders placed at the center of the layer. We consider a wide layer of fluid of aspect ratio (width to height) of the range of unity to six and employ periodic boundary condition along the horizontal direction to allow for lateral freedom for the convection cells.

2. Numerical Methodology

We solve the continuity, Navier-Stokes and energy equations in their non-dimensional forms defined as

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1a}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + Pr \frac{\partial^2 u_i}{\partial x_j \partial x_j} + RaPr\theta\delta_{i2} \tag{1b}$$

$$\frac{\partial \theta}{\partial t} + u_j \frac{\partial \theta}{\partial x_j} = \frac{\partial^2 \theta}{\partial x_j \partial x_j} \tag{1c}$$

In the simulations to be reported here the Prandtl number, Pr, has been taken to be 0.7 corresponding to that of air. The Rayleigh number, Ra has been taken to be 10^6.

A spectral multi-domain methodology is used for the spatial discretization along the x and y directions. In this technique the overall computational domain is subdivided into a number of smaller rectangular subdomains. Within each subdomain a local spectral Chebyshev discretization is defined. The Computational geometry consists of 8 subdomains in the x-y plane as the aspect ratio is increased one by one, with each subdomain resolved by up to 15x15 points. The grid points are the Gauss-Lobatto points corresponding to Chebyshev expansion within each subdomain and are therefore non-uniformly distributed.

Once the velocity and temperature fields are obtained, the local, surface-averaged, time-averaged, and time-and-surface-averaged Nusselt number at the top and bottom walls are defined as

$$Nu = \left.\frac{\partial\theta}{\partial n}\right|_{wall} \qquad \overline{Nu} = \int_0^1 Nu\, dS$$
$$< Nu > = \frac{1}{t_p}\int_0^{t_p} Nu\, dt \qquad < \overline{Nu} > = \frac{1}{t_p}\int_0^{t_p} \overline{Nu}\, dt \qquad (2)$$

3. Result and Discussion

For the aspect ratio of *2L*, a wavelength is about 2 and the roll cell circulates around the bodies but not satisfies x – symmetry and correspondingly the secondary vortices occur in between bodies. The secondary vortices does not vanish nor merge into a primary roll cell, that is, they move around square cylinder with clockwise/counterclockwise direction. They have relatively high speed of rotation around the bodies. For the given aspect ratio of *3L* the time-averaged isothermal and velocity vectors are similar to them of *6L*. upwelling hot plume in between bodies produces the main driving forces to make primary roll cell. In the figure of the case of *4L*, cell bifurcation undergoes a transient process apparently. The wavelength of a pair of cell is larger than 2 but each component of the pair has unbalanced shape. For the aspect ratio of *5L*, the time-averaged distributions of isotherms and velocity vectors is similar to those of *6L* described at previous section. The wavelength is almost 3. An instantaneous flow regime is almost same as one of *6L* but the sequence of surface-averaged Nusselt number is not. In this case, the lateral motion over the body occurs at one point, that is, an upwelling hot plume around the middle body moves back and forth, whereas cold plume fixed at its region in between bodies as shown in figure. The frequency of this case is about 52 with harmonics and power spectrum has just one outstanding value. It is related with the fact that the only one heat plume moving back and forth with lateral motion in the middle region behaves with primary one frequency valued about 52.

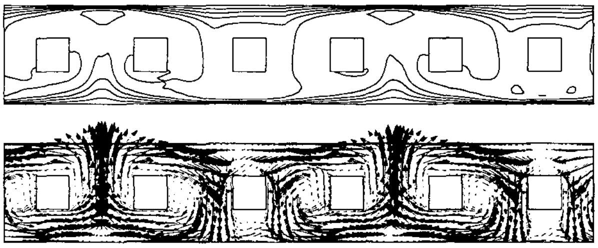

Figure 1. Time-averaged isotherms and velocity vectors for the aspect ratio of six

4. Conclusion

Here we solved the unsteady natural convection problem in various wide aspect ratio of horizontal layer of fluid with equi-spaced square bodies in the interior using an accurate and efficient Chebyshev spectral approach. Compared with the result of unity, As aspect ratio increases, the wave length of convecting cell is free to extend toward horizontal direction.

5. References

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