Mixed analytical/numerical method applied to the low Reynolds number k-epsilon turbulence model

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Abstract

A mixed analytical/numerical method is developed here to solve the low Reynolds number k-epsilon turbulence model. In this method the advection-diffusion part is solved numerically, while the source terms are split into two parts: one part is solved analytically and the next is solved numerically.

Keyword: *Mixed analytical/numerical method, k-epsilon turbulence model, explicit method, source term*

Because of the recent enormously progress in the capability of computers, low Reynolds two-equation turbulent models become more and more welcome in engineering fluid computation. In the past, various forms of low-Reynolds-number k - ε turbulent models have been proposed. The detail of two-equation models and low Reynolds corrections are presented in Chapter 14 (Turbulence Modeling and Simulation) of handbook [1].

Though mathematically the $k-\varepsilon$ model is well posed[2], the strong nonlinearities may interact with numerical errors in such a way that computation may break down easily.. A typical behavior of unstable computations involves the loss of positivity of k or ε, though the original differential equations have positive solution [3]. The appearance of negative values changes the sign of several terms in the models, so that turbulent quantities may increase unboundedly [4]. Even through the two turbulence equations can be solved exactly in the same manner as the mean flow equations, it has been found that such a method often leads to an unstable solution, or even incorrect solutions [5]. The damping functions in the low Reynolds turbulent models improve the model prediction capability for near wall flow, but also introduce more severe numerical stiffness for the source terms.

There are a huge amount of numerical methods for compressible Navier-Stokes equations coupled with two equation models [6-9]. The two-equation turbulent model is a typical example of partial differential equations with source terms. Great progress has been made in efficient treatment of the source terms [10-12]. Helzel, LeVeque, and Warnecke [12] treated chemical reacting flow with an Arrhenius law for the source term by mixed method in detonation waves computation. In [13], a mixed analytical/numerical method for oscillating source terms has studied. In this method the advection-diffusion part and the source terms are treated separately through operator splitting. The advection-diffusion part (PDE) is integrated numerically while the source term part (ODE) is integrated analytically. Hence this method is called mixed analytical/numerical method. The mixed method performs well for partial differential equations with source terms, in which the time scale of source term (S-scale, denoted T_S) is much smaller than mean flow scale (M-Scale, denoted T_M) inherent to the advection-diffusion part. Furthermore, the mixed method has extended to the implicit solution of high Reynolds number and compressible turbulent flows [14]. Numerical results show the mixed method can give the robust, steady and fast convergence solution.

In this paper, we extend the mixed method to the low Reynolds number turbulent model with explicit solver. The source term form of low Reynolds number turbulent model is very complicated. It is hardly to get the analytical solution of source term equation. Moreover, there are several dozens of well-known low Reynolds models, so we need universal numerical method. The general form of low Reynolds number turbulent model can be denoted as:

$$\frac{D\rho k}{Dt} = \frac{\partial}{\partial x_k}\left[\left(\frac{\mu_t}{\sigma_k}+\mu\right)\frac{\partial k}{\partial x_k}\right] + P - \beta^{*}\tilde{\varepsilon} + \chi \qquad (1)$$

$$\frac{D\rho\tilde{\varepsilon}}{Dt} = \frac{\partial}{\partial x_k}\left[\left(\frac{\mu_t}{\sigma_\varepsilon}+\mu\right)\frac{\partial \tilde{\varepsilon}}{\partial x_k}\right] + C_{\varepsilon 1} f_1 \frac{\tilde{\varepsilon}}{k} P - C_{\varepsilon 2} f_2 \frac{\tilde{\varepsilon}^2}{k} + \xi \qquad (2)$$

Te source terms are split two parts, the standard part $S_S(U)$ (without damping function χ and ξ) and damping function part $S_D(U)$ in the mixed method, which are treated respectively. The standard part $S_S(U)$ is solved with analytical method (f_1, f_2 and β^* is regarded as constant coefficient in analytical solution), and the other complex additional damp function part $S_D(U)$ is discreted with numerical method. For the negative part, the implicit treatment is still needed. In this way, new method would be universal and can be applicable to general low Reynold turbulence model. We solve the Hwang-Lin low-Reynold number $k-\tilde{\varepsilon}$ turbulence model with new mixed method in this paper. The new method gives the satisfying results.

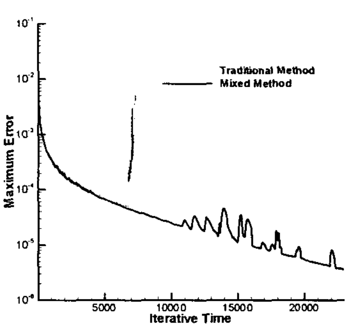

Fig.1

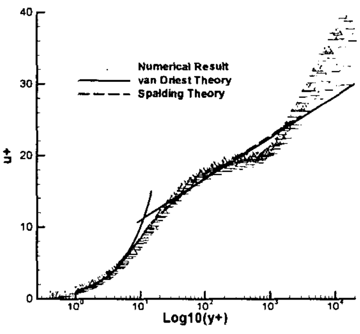

Fig.2

Fig.1. Comparative convergence history for flat plane

Fig.2. Velocity profile of flat plate flow

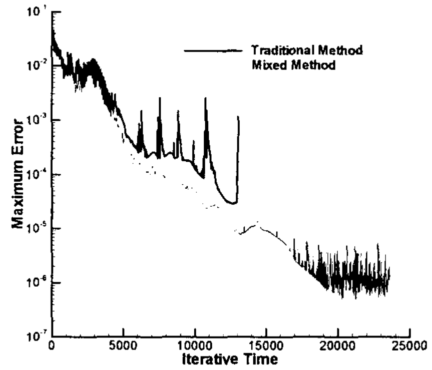

Fig.3

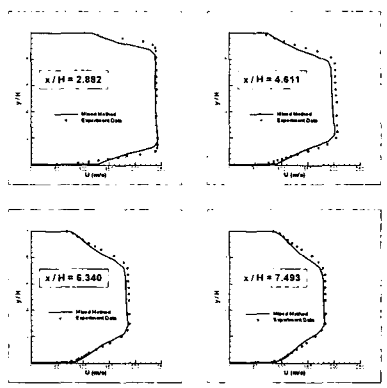

Fig.4

Fig.3. Comparative convergence history for transonic diffuser

Fig.4 Velocity profiles at four axial locations for transonic diffuser

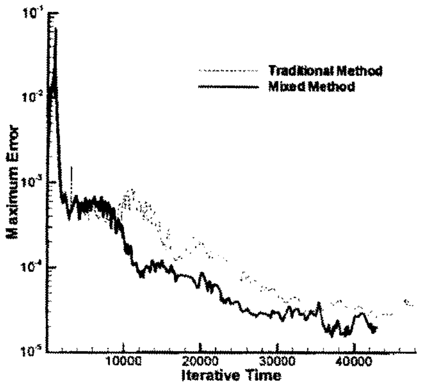

Fig. 5

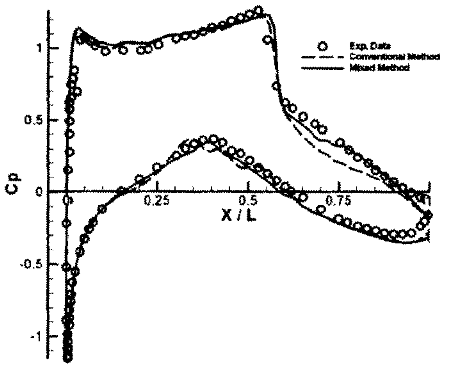

Fig.6

Fig. 5. Comparative convergence history for RAE2822 airfoil

Fig. 6. The pressure coefficient profiles of RAE2822 airfoil

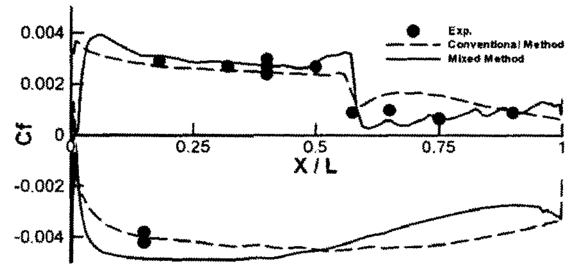

Fig. 7. The skin friction coefficient profiles of RAE2822 airfoil

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