Direct Solving the Boltzmann Equation for Supersonic Jet Problems with Instabilities

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Abstract

The Boltzmann kinetic equation is solved directly by means of the conservative splitting method. Underexpanded supersonic free jet flows with small Knudsen numbers are studied. In this numerical simulation features intrinsic to appropriate experiments are observed. Streamwise vortices in a mixing layer and chaotic downstream temporal-spatial fluctuations of microscopic quantities with large amplitude are obtained.

Keyword: *The Boltzmann equation, unstable jet flow, turbulent pulsations in mixing layer, Taylor-Goertler instability*

1. Introduction

We developed direct method for the kinetic equation [1]. This approach can applied for mathematical modeling flows from free-molecular to continuum limits. The Boltzmann equation is assumed to describe adequately unstable turbulent flows therefore we expect to simulate the appropriate unstable zones in flow.

2. The main equation and formulation of the problem

The Boltzmann equation for a monoatomic gas without external forces is written in Cartesian coordinates. 2D and 3D problems are considered (unstable processes with vortex structure require 3D geometry). Monte-Carlo and deterministic methods for evaluation of collision integrals are applied. Rigid sphere molecular model is used.

A free supersonic underexpanded jet is an interesting object for study both from theoretical and practical points of view. At small Knudsen numbers there exist different regimes up to nonstationary turbulent pulsation downstream. The formulation of the problem for the Boltzmann equation is relatively simple. The computational domain with fixed conditions (which is an equilibrium distribution function of a gas at rest) on free boundaries is considered. For the equilibrium distribution function in the nozzle exit the Mach number set to 1. Conditions in this jet flows are such that there is only one barrel wave structure, namely the aspect ratio is large and equals to 12.2, the characteristic Reynolds number is varied from 0 to 100000.

3. Method of solution

The conservative splitting method is applied. This method allows us to compute a complex flow where there are regions with different rarefaction because the scheme approximates asymptotically the Euler equations. Moreover one can simulate instabilities at supercritical regimes. From the mathematical point of view the discrete velocity procedure provides an approximation of the discrete number of the differential equations with a linear convective left-hand side and the nonlinear quadratic form for the right-hand side. The mathematical model with a quadratic nonlinearity fundamentally differs from the mathematical model of the Navier-Stokes equations. In particular, a small scheme viscosity does not play an important role (the nonlinearity appears in velocity space) therefore instabilities are observed for coarse spatial meshes.

4. Results

Numerical results are compared with experiments [2] which showed Taylor-Goertler type of instability and [3] where turbulent fluctuations going from a mixing layer were observed. An important item is a recognition of mechanism of a real transition to instability. The structure of the streamwise vortices was obtained in a mixing layer in direct solutions [4] for the Boltzmann equation. In numerical solutions the characteristics of stochastic oscillations are similar to the experimental data. Density pulsation is of the order of 10% (an amplitude) and of slow frequencies (from 0.1 to 1 of the reference frequency). New detail computations with small time and spatial parameters are carried out on parallel processors with the aim to test results and to obtain new information about these complex jet flows. Verification is provided with time step being of the order of Knudsen number, e.g. at Kn=3/100000 the time step is equal to 1/100000. These test calculations demonstrate that fluctuations are of the same amplitude and frequency.

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