Numerical Simulation of the Tidal Bores on the Qiantang River

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Abstract

The tidal bores of the Qiantang River on the East coast of China are simulated numerically based on the shallow water theory. The governing equations, which were traditionally formulated using water depth, are formulated in terms of water surface level, and the fractional-step method is applied in conjunction with a Godunov-type scheme. In addition, the source terms due to bottom gradient are discretized centrally to exactly balance the flux terms. Our numerical simulation produces tidal bores in excellent agreement with field measurements.

1. Introduction

Tidal bores are spectacular phenomena and are unique to estuaries. The most famous tidal bores are the ones on the Qiantang River on the East coast of China which can reach four meter high. Tidal bores can be studied using shallow water theory for which the governing equations are traditionally written in terms of the water depth, $h(x, y, t)$, from the bottom $b(x, y)$ to the free surface $\zeta(x, y\ t)$:

$$\begin{cases} \dfrac{\partial h}{\partial t}+\dfrac{\partial hu}{\partial x}+\dfrac{\partial hv}{\partial y}=0 \\ \dfrac{\partial hu}{\partial t}+\dfrac{\partial}{\partial x}\left(hu^2+\dfrac{1}{2}gh^2\right)+\dfrac{\partial huv}{\partial y}=-\left(g\dfrac{\partial b}{\partial x}+m\right) \\ \dfrac{\partial hv}{\partial t}+\dfrac{\partial huv}{\partial x}+\dfrac{\partial}{\partial y}\left(hv^2+\dfrac{1}{2}gh^2\right)=-h\left(g\dfrac{\partial b}{\partial y}+n\right) \end{cases} \quad (1)$$

Here, t is time and xy are Cartesian coordinates in the horizontal plane, g is acceleration due to gravity, $u(x, y, t)$ and $v(x, y, t)$ are x- and y-component of fluid velocity, respectively, and m and n are the corresponding components of the bottom friction force due to its roughness.

In the special case when the bottom is horizontal and the friction force negligible, the governing equations simplify to the conservation law equations and can be readily solved using shock-capturing methods [1, 2]. However, with this formulation there are inherent difficulties in using the fractional step method to cope with the bottom topography source terms, especially in computing stationary flow and in computing tidal bores when the tide is receding. Recently, many ideas have been suggested [3-10], but the problem had not been solved.

2. Present Approach

In this paper, we formulate the problem of shallow-water flow in terms of the water surface level $\zeta = h + b$ and solve it by the fractional step method together with a Godunov-type shock-capturing method.

In this formulation, the resulting Riemann problem is solved with a simple approximation, which amount to coarsening the grid for the bottom topography by doubling its size locally, whilst the source terms due to bottom gradient are discretized centrally so as to exactly balance the flux terms. We show that this formulation avoids the difficulties mentioned above while giving accurate numerical solutions to shallow-water flow in all cases: stationary, steady and unsteady flow.

3. Results

The Qiantang River and the Hangzhou Bay runs from West to East and flows into the Pacific Ocean on the East coast of China. When the tide comes in from the Pacific Ocean, it is enhanced by up to 75% due to the converging delta shape of the Bay. From Ganpu on, it is the Qiantang River whose bottom, $b(x, y)$, grows higher upstream (Fig. 1). This shallow-water effect distorts the shape of the tidal wave and ultimately leads to formation of tidal bores. A grid of 182 × 18 cells is laid over a region from Ganpu to Hangzhou over a distance of about 100km (Fig. 2). The purpose of our computation is to replicate the tidal waves and bores recorded on 16-17 September, 2000. In equation (1) the friction force components are $m = gu\sqrt{u^2 + v^2} / C^2(\zeta - b)$ and $n = gv\sqrt{u^2 + v^2} / C^2(\zeta - b)$, where the Chezy coefficient is $C = (\zeta - b)^{\frac{1}{6}} / N$. For the Manning constant, a value $N = 0.004 - 0.013$ is used in our computation. At the upstream and downstream boundary, the observed water levels were imposed as boundary conditions. This is a time-dependent, almost periodic flow.

Computed time series of water levels at two locations, the Cao'e River outlet and Yanguan, are plotted in Figs. 3 & 4 for 36 hours and are compared with field measurements at these locations. The agreements at both locations are excellent, despite the coarse grid used.

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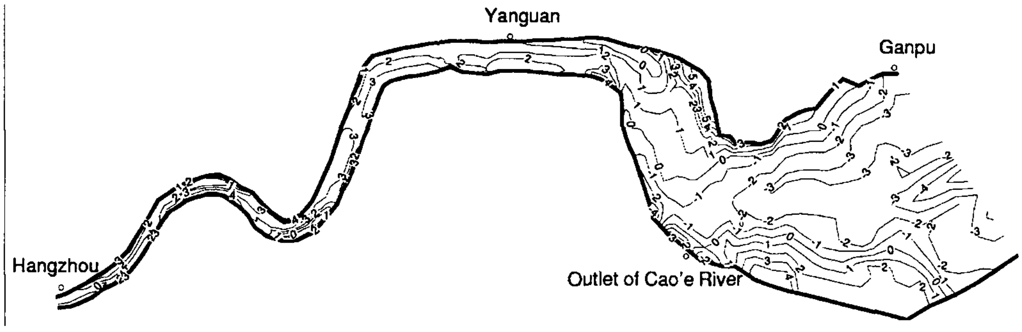

Fig. 1: Contours of Qiantang River bottom (meters).

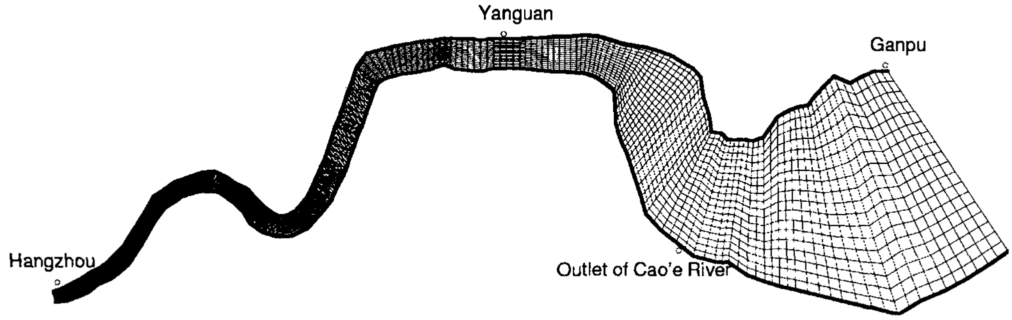

Fig. 2: The 2-D grid over the Qiantang River: 182 × 18 cells.

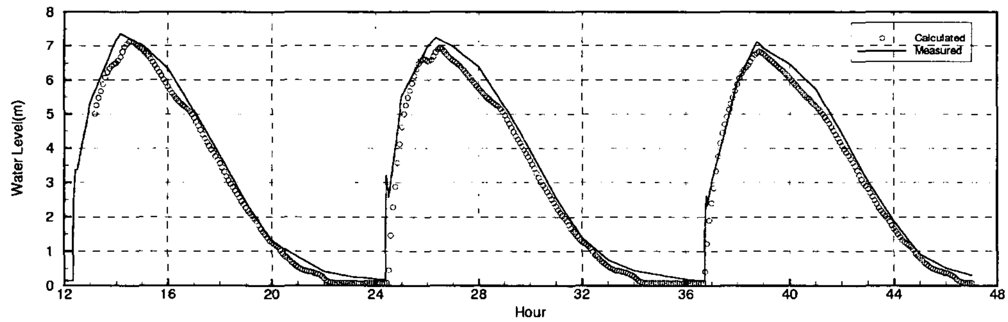

Fig. 3: Water level at outlet of Cao'e River on the Qiantang River.

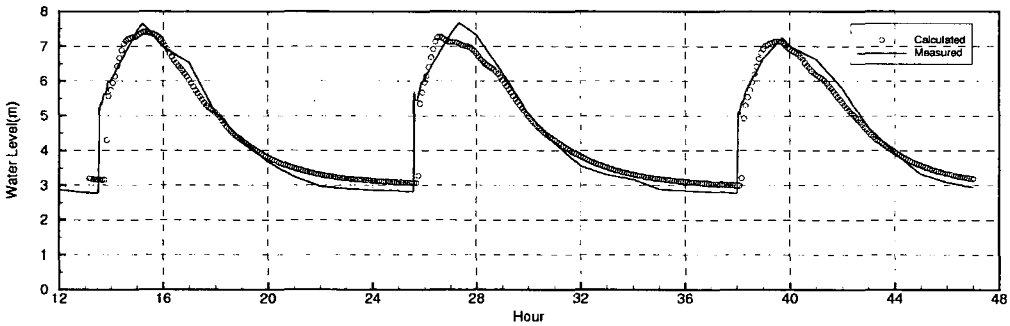

Fig. 4: Water level at Yanguan on the Qiantang River.