Study of Greitzer's B-Parameter Model Using ANOVA & Taguchi Method

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Abstract

In this work, the Greitzer's B-parameter model is applied for analyzing the stall and surge characteristics. The four parameters in the model are highlighted in order to establish the influence of each parameter on the system.

First of all, the governing equations of stall and surge behavior are solved numerically using fourth-order Runge-Kutta method. The Taguchi method is then used to analyze the results generated to obtain the extent of effects of the parameters on the system by varying the parameters in a series of combinations. Finally, a thorough analysis is carried out on the results generated from the Taguchi method and the graphs.

Keyword: *compressor system, stall, surge, Taguchi method, Greitzer's B-parameter model.*

1. Introduction

There are two types of instabilities that could be encountered in axial compressor systems. They are rotating stall and surge respectively. Both of them occur as the flow through an axial compressor system that is throttled from the design point to the stall limit.

The surge and rotating stall are serious problems for the axial compressor system. It is thus important to know the characteristics of these two phenomena. In addition, as their consequences are different, it is useful to be able to predict which of these problems will occur for a given situation.

In the current work, the authors solve numerically the governing equations of stall and surge behavior using fourth-order Runge-Kutta method [1]. Then applying Taguchi method [2][3] we analyze the results generated to obtain the extent of effects of the parameters on the system by varying the parameters in a series of combinations. Finally, a thorough analysis is performed and the conclusion about the deciding factor for compressor characteristics is worked out.

2. Mathematical Model

Greitzer [4] set up a mathematical model to analyze the characteristics of stall and surge. This model maps out not only the compressor but also the whole compressor system and may be used to predict whether a stall or a surge will occur at stall limit.

A compressor system normally consists of the following components: a compressor, an inlet annular duct, a plenum and a throttle in an exit duct whose diameter is much smaller than that of the plenum. The flow in the compressor system must obey the laws of motions. Based on the Greitzer's B-parameter model, the motion equations can be written as

$$\frac{d\tilde{m}_C}{d\tilde{t}} = B(\tilde{C} - \Delta\tilde{P}) \tag{1a}$$

$$\frac{d\tilde{m}_T}{d\tilde{t}} = \frac{B}{G}(\Delta\tilde{P} - K\tilde{m}^2) \tag{1b}$$

$$\frac{d\Delta\tilde{P}}{d\tilde{t}} = \frac{1}{B}(\tilde{m}_C - \tilde{m}_T) \tag{1c}$$

$$\frac{d\tilde{C}}{d\tilde{t}} = \frac{1}{\tilde{\tau}}(\tilde{C}_{SS} - \tilde{C}) \tag{1d}$$

Three non-dimensional parameters are also included in these equations. They are, $B = \frac{U}{2a}\sqrt{\frac{V_P}{A_C L_C}}$; $G = \frac{L_T A_C}{L_C A_T}$; $K = \frac{A_C^2}{A_T^2}$.

Here $\tilde{\tau}$ is the non-dimensional time lag; ω is the Helmholtz frequency; $\Delta P = P_P - P$ is the pressure difference across the duct; and C is the pressure rise across the compressor. $\dot{m}$ is the mass flow rate. L and A are the length and area. τ and C_{SS} are the compressor flow field time constant and the steady-state measured compressor curve respectively. N is the time for some number of rotor revolutions. The subscripts C, P and T are the compressor duct, plenum and throttle duct respectively. The subscript O is initial nondimensionalized value at surge point (or stall point). The top cap ~ is the non-dimensionalized quantity. The non-dimensional parameters are $\rho U A_C$, the pressure difference variable $0.5\rho U^2$ and the time variable $1/\omega$.

In Greitzer's B-parameter model, three parameters, *B, G* and *K,* together with the characteristic length L_C, will be investigated in a series of combinations to analyse the effects of each parameter on the whole compressor system using the Taguchi method.

3. Results And Discussion

In the current work, the four parameters, *B, G, K* and L_C, are taken into account for comparison. There will be three different values for each parameter in different combinations using the Taguchi method in running the program. Here the values of *G* will be set at three levels with *0.16, 0.36* and *0.56* respectively. In the Greitzer's paper [4], surge will occur when the critical value of *B* is approximately *0.7*. Hence in the current study, the value of *B* used will be kept below *0.7* to avoid the occurrence of surge with the three values being *0.50, 0.55* and *0.60* respectively. The two parameters, *K (5.32, 5.52, 5.72)* and L_C *(1.36,1.46.1.56)*, will be categorized into two separate cases with parameters *B* and *G* as: Case 1; *B, G, K*; and Case 2: *B, G,* L_C. Therefore, each case involves *3* factors and *3* levels.

Upon executing the program, the results are compiled and included/tabulated in Table 1 and Table 2. As shown in the two tables, there are columns for values of *Ave X* and *Ave Y*, which are calculated from the average of the *X* and *Y* coordinates of the cross point *(X, Y)* of the compressor curve when it stabilizes to a stationary point.

Table 1. Case 1: Results for effects of parameters *B*, *G* and *K* on the compressor system

	Ave X	*Ave Y*	*Ave Diff M*	*Ave Diff P*	*R(X)%*	*R(Y)%*	*R(Diff M)%*	*R(Diff P)%*
	0.414529	0.951680	0.042722	0.054561				
B	0.415944	0.954201	0.062836	0.093696	0.142237	0.252039	5.360667	10.329344
	0.415951	0.954045	0.096328	0.157855				
	0.414513	0.951581	0.074464	0.114550				
G	0.415952	0.954147	0.065120	0.098131	0.144583	0.261614	1.216133	2.111956
	0.415959	0.954197	0.062302	0.093431				
	0.426744	0.972833	0.060423	0.095681				
K	0.415472	0.952675	0.065525	0.099169	2.253574	3.841621	1.551422	1.558044
	0.404208	0.934417	0.075937	0.111262				

Referring to Table 1 for the results of the parameters *B, G* and *K*, it can be seen that parameters *G* and *K* affect the coordinates of the cross point *(X, Y)* most with parameter *B* having the least influence. The values of the *x*–coordinate and *y*–coordinate will shift an overall of *2.25%* and *3.84%* respectively when *K* is changed, as indicated in Table 1. On the other hand, the *x*–coordinate and *y*–coordinate will move an overall of *0.145%* and *0.262%* respectively when *G* changes. This implies that *K* has a greater effect on the position of cross point *(X, Y)*.

With a percentage of about *5%* and *10%* influence on the values of *Diff M* and *Diff P*, *B* is considered the deciding factor to the diameter of oscillation in this case. While the effect of *G* upon the diameter of oscillation is much smaller with an average value of *1.22%* and *2.11%* influence on the values of *Diff M* and *Diff P*.

Table 2. Case 2: Results for effects of parameters *B*, *G* and L_C on the compressor system

	Ave X	*Ave Y*	*Ave Diff M*	*Ave Diff P*	*R(X)%*	*R(Y)%*	*R(Diff M)%*	*R(Diff P)%*
	0.415453	0.952757	0.037022	0.045106				
B	0.415459	0.952761	0.063036	0.094409	0.002900	0.034756	5.959733	11.358556
	0.415482	0.952413	0.096619	0.158691				
	0.415422	0.952525	0.068912	0.105351				
G	0.415479	0.952649	0.065314	0.098809	0.007220	0.023289	0.646244	1.130422
	0.415494	0.952757	0.062450	0.094047				
	0.415468	0.952743	0.057156	0.079110				
L_C	0.415472	0.952675	0.065525	0.099164	0.001674	0.022910	1.683922	4.082233
	0.415455	0.952514	0.073996	0.119932				

Similarly analysis is done for the second group of parameters *B*, *G* and L_C. Referring to Table 2, it can be deduced that parameters *B* and *G* are the two prime factors while L_C is the least important one.

The last two columns of Table 2 indicate the percentage values of the diameter of oscillation for each parameter. It can be seen that parameter *B* has the highest value followed by parameter L_C and then parameter *G*. For this case, *G* is kept constant in the analysis of results. It can be seen that *B* parameter actually plays a more important role in the diameters of oscillations thus affecting the time taken for the system to reach a steady state.

5. Conclusion

In this paper, after reviewing the previous work on stall and surge of an axial compressor system and their mathematical models, Taguchi method is used to analyze the extent of the effects that the four parameters *B, G, K* and L_C have on the system.

The analysis on the four parameters, *B, G, K* and L_C has been carried out with satisfying results. It is observed that parameters *B* and *K* are crucial in maintaining the steady state operation of the compressor system. With values of *5%* and *10%* for the changes in *Diff M* and *Diff P*, the range of diameters of oscillation is highly affected by *B*. On the other hand, *K* is the deciding factor in the position of the cross point with percentage values at *2.25%* and *3.84%* in the variation of *x*–coordinate and *y*–coordinate.

In brief, the Taguchi method is successful in analyzing the important parameters that will affect the compressor system. This analysis can be extend further to a more challenged areas, such as where the system may be out of limits of rotating stall or surge, say, involving a greater range of *B*.

References

[1] Kreyszig, E., "*Advance engineering mathematics*", 7th edition, John Wiley & Sons, Inc., New York, (1993).

[2] Taguchi, G., "*Taguchi on robust technology development: Bringing Quality Engineering upstream*", New York: ASME Press, (1993).

[3] Taguchi, G., "*Introduction to quality engineering: designing quality into products and processes*", Japan: Asian Productivity Organization, (1986).

[4] Greitzer, E. M, "Surge and rotating stall in axial flow compressors, part I: theoretical compression system model", *Transactions of the ASME, Journal of Engineering for Power*, Vol. 98, Apr. (1976), pp190-198.