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Management of Discon tinuous Reconstruction in the Ev olution Stage of Kinetic Scheme

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ABSTRACT
A New kinetic scheme for the compressible Na vier-Stokes equations is developed. While the conv entional approach, such as KFVS scheme, employs the splitting algorithm and computes the numerical flux on the basis of the collisionless equation, the present approach employs the splitting algorithm in the evaluation of the numerical flux, where the collision effect is explicitly taken into account. However, the initial condition employed in the computation is slightly different from the conventional Chapman-Enskog NS distribution function. The present study also reveals the background of the existing kinetic schemes, such as the KFVS scheme and Gas-Kinetic BGK scheme.

Key Words: Kinetic Scheme, Boltzmann Equation, Navier-Stokes Equation, Discontinuity, Chapman-Enskog Expansion

1. INTRODUCTION

In the present study, we derive two kinetic schemes for the compressible NS equation. The Gas-Kinetic BGK scheme,[3] which yields successful results in the boundary layer problem, is derived by a simple modification of one of the schemes. The other scheme is based on the splitting algorithm. While Chou-Baganoff KFVS[1] mimics the DSMC method for the Boltzmann equation and handles the relaxation implicitly in each cell, the splitting method is directly applied to the computation of the numerical flux. However, the initial data employed in the scheme is slightly different from the conventional Chapman-Enskog NS distribution function. The modification of the initial data is done under the guideline of the railroad method of Ref. [2]. The present study emphasizes the importance of the inclusion of collision effect and the management of discontinuity in the numerical flux.

2. THEOR Y

In Ref. [2], it is shown that high order kinetic scheme is derived if the numerical flux is computed by using the solution for the modified BGK equation

$$\frac{\partial f}{\partial t} + \zeta_1 \frac{\partial f}{\partial x_1} = \frac{\rho_*}{\epsilon}(f_0 - f), \tag{1}$$

from the Chapman-Enskog NS distribution function

$$f = f_0 + \epsilon f_1, \tag{2}$$

where ρ_* is the density at the cell interface and $t = 0$, ϵ is a constant of the order of the Knudsen number ($\epsilon \ll 1$). f_0 is the local Maxwellian, and f_1 is the second order approximation of f in the Chapman-Enskog expansion.

3. NEW SCHEME

Because of the limited space, in this extended abstract, we introduce only one of the kinetic schemes, which is based on the splitting algorithm.

We solve the above Cauchy problem by the splitting algorithm and compute the numerical flux using the solution. The solution is extended to the case of the reconstruction that allows the discontinuity at cell interfaces. The result of so constructed kinetic scheme for the viscous boundary layer

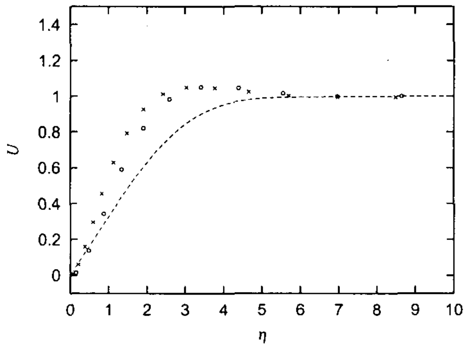

Figure 1: Blasius flow of U velocity.

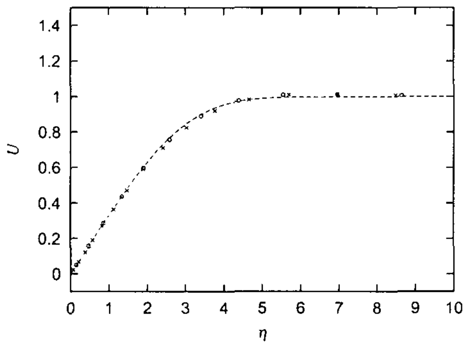

Figure 2: Blasius flow of U velocity. The ϵ in the initial condition is replaced by β.

problem is shown in Fig. 1. Because of the error of the splitting algorithm, the result is poor. This is overcome by replacing ϵ in the initial data with β defined by

$$\beta = \rho_* \Delta t \coth(\frac{\rho_* \Delta t}{2\epsilon}) - \epsilon, \quad (3)$$

where Δt is the time step. The result for the modified initial data is shown in Fig. 2. The modification is done according to the guideline of Ref. [2].

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