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Multibubble Dynamics in an Acoustic Field: Theoretical Study and Direct Numerical Simulation by MTS–DiCUP

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Abstract

This paper presents a theoretical study concerning multibubble dynamics in a sound field and the numerical validation for it by employing our new CFD code MTS–DiCUP. In recent papers, the author has shown theoretically that an unknown characteristic frequency, named "transition frequency," exists in a multibubble system. For a N-bubble case, up to $2N-1$ transition frequencies per bubble have been predicted, only N ones of them correspond to the natural frequencies of the system. The transition frequencies that do not correspond to the natural frequencies give rise to the phase reversal of bubbles' pulsation without resonant response. In this paper, it has been suggested theoretically that those transition frequencies may cause the sign reversal of the secondary Bjerknes force, which is an interaction force acting between acoustically coupled gas bubbles. This theoretical result has been validated by the direct numerical simulation, at least in a qualitative sense.

Keyword: *Gas bubble, Radiative interaction, Secondary Bjerknes force, Natural frequency, DNS*

1. Introduction: Transition frequencies of acoustically coupled gas bubbles

In recent papers [1, 2], we have shown by investigating the phase property of acoustically coupled gas bubbles immersed in a liquid that a multibubble system has an unknown characteristic frequency. This characteristic frequency, named "transition frequency", was found by seeking the driving frequencies that make the phase difference between an external sound and a bubble's pulsation $\pi/2$ (or $3\pi/2$), i.e., roughly speaking, driving frequencies giving rise to the phase reversal of a bubble in the system. In a double-bubble system, a bubble has up to 3 transition frequencies [1]; this number is greater than that of natural frequencies, up to 2 [1, 3]. Moreover, one of the 3 transition frequencies does not cause the resonant response in bubble pulsation. The preliminary discussion performed in Ref. [2] shows that a bubble in a N-bubble system (N denotes an arbitrary positive integer) has up to $2N-1$ transition frequencies, N ones of which correspond to the natural frequency. This result means that in multibubble cases the number of transition frequencies may be, in general, greater than that of the natural frequencies, and that observing the transition frequencies can provide detailed insights of phase properties of acoustically coupled bubbles.

In this paper, we perform further theoretical studies concerning multibubble dynamics in order to show that the transition frequencies not corresponding to the natural frequency play an important role in determining the sign of the secondary Bjerknes force [4], which is an interaction force that pulsating bubbles undergo (For this force, see, e.g., Ref. [5]). Subsequently, this theoretical result is verified by the direct numerical simulation using a CFD code that the author has developed recently [6, 7, 8].

2. Theories

2.1 Secondary Bjerknes force

The secondary Bjerknes force acting between two gas bubbles for sufficiently weak driving is expressed with $\boldsymbol{F} \propto \langle \dot{V}_1 \dot{V}_2 \rangle (\boldsymbol{r}_2 - \boldsymbol{r}_1)/\|\boldsymbol{r}_2 - \boldsymbol{r}_1\|^3 \propto K_1 K_2 \cos(\phi_1 - \phi_2)(\boldsymbol{r}_2 - \boldsymbol{r}_1)/\|\boldsymbol{r}_2 - \boldsymbol{r}_1\|^3$, where V_j ($j = 1, 2$), $\boldsymbol{r}_j$, and K_j (> 0), respectively, are the volume, the position vector, and the pulsation amplitude of bubble j, and ϕ_j is the phase delay of bubble-j's volume oscillation measured from the phase of an external sound. The sign of this force changes only when the sign of $\cos(\phi_1 - \phi_2)$ inverts, revealing that the phase property of bubbles has a significant influence on this sign. If the phase shifts resulting from the radiative interaction between bubbles are neglected, this force is repulsive when the driving frequency stays between the monopole natural frequencies of the bubbles, and is attractive otherwise [5]. For more general cases, see Ref. [4] and references therein.

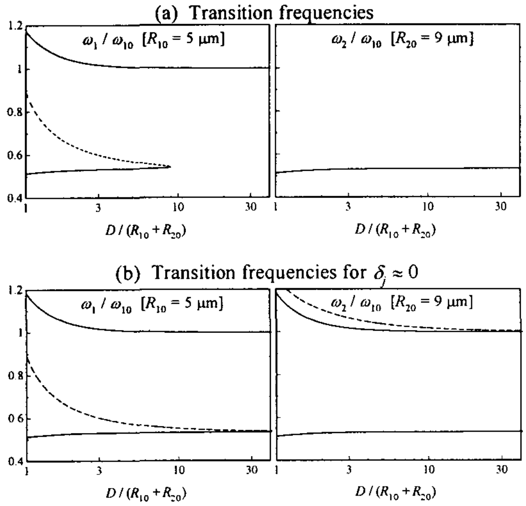

Fig. 1. Transition frequencies in a double-bubble case.

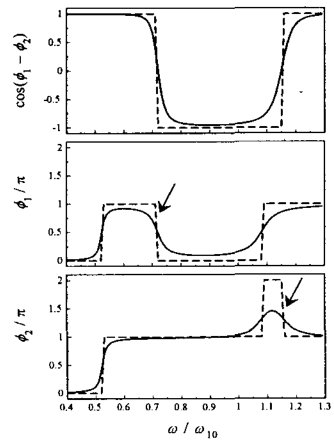

Fig. 2. Sign of the force and phase delays. The arrows indicate the phase reversal due to the transition frequencies not corresponding to the natural frequency.

2.2 Relationship between the force and the transition frequencies

Figure 1 shows the transition frequencies of the bubbles whose equilibrium radii are $R_{10} = 5$ μm and $R_{20} = 9$ μm, as functions of $D/(R_{10} + R_{20})$, where D is the distance between the centers of the bubbles. These results are obtained by using the linear coupled oscillator model [1] and by assuming that the gas inside the bubbles is an ideal gas with a polytropic exponent of $\gamma = 1.33$ and the liquid surrounding the bubbles is water at room temperature. The thermal conduction is neglected. In this figure, δ_j denotes the damping coefficient of bubble j determined by the sum of viscous and radiation damping, and the lower graphs (Fig. 1(b)) display the transition frequencies in the idealized case of $\delta_j \to 0$. We can observe, as mentioned, up to 3 transition frequencies per bubble varying with D and also depending on δ_j (Detailed discussions regarding the dependency of transition frequencies on the damping effects are given in Ref. [1]). The dashed lines represent the transition frequencies that do not cause the resonant response, and the solid lines shown in Fig. 1(b) correspond to the natural frequencies that can be determined by the classical theory [3, 1]. It is known that in the case of $\delta_j \to 0$ both the bubbles have the same two natural frequencies [1]; namely, the solid lines displayed in the left and the right graphs of Fig. 1(b) just correspond to each other, meaning that simultaneous phase reversals of the bubbles should occur at these frequencies.

Figure 2 shows $\cos(\phi_1 - \phi_2)$ and the phase delays as functions of the driving angular frequency, ω, normalized by the monopole natural angular frequency of bubble 1, ω_{10}, where the dashed lines indicate the result for $\delta_j \to 0$. Note that the sign reversal of the force (i.e., that of $\cos(\phi_1 - \phi_2)$) takes place at two frequencies where only one bubble undergoes phase reversal. This reveals that the sign reversal is caused by the transition frequencies other than the natural frequencies, because, as mentioned, the transition frequencies that give rise to the (approximately) simultaneous phase reversals (but not to the sign reversal) correspond to the natural frequencies.

3. Direct numerical simulation and discussion

Here we perform a numerical experiment by employing our CFD code MTS–DiCUP (Multi-Time-Step, Discontinuous-density-Interface algorithm based on Cip–cUP) [6, 8], which allows us to simulate dynamics of acoustically coupled deformable bubbles translating through a viscous liquid [7, 8]. The initial distance between the centers of the bubbles is set to $D(t=0)=20$ μm, and other physical parameters are the same as used already. The computational space is axisymmetric, and the grid width is $5/20$ μm. Figure 3 shows a typical result given by the CFD code with $\omega=\omega_{20}$, in which we can observe not only the volume oscillation but also the attraction and resulting coalescence of the bubbles.

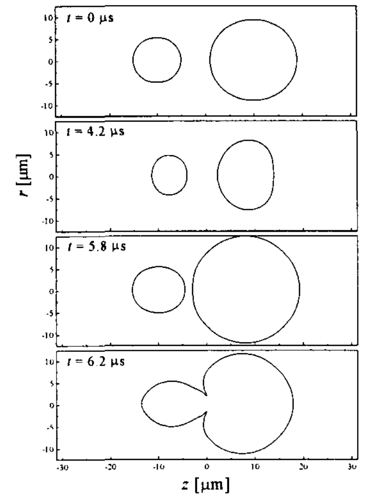

Fig. 3. A typical DNS result for $\omega = \omega_{20}$; $0.56\omega_{10}$.

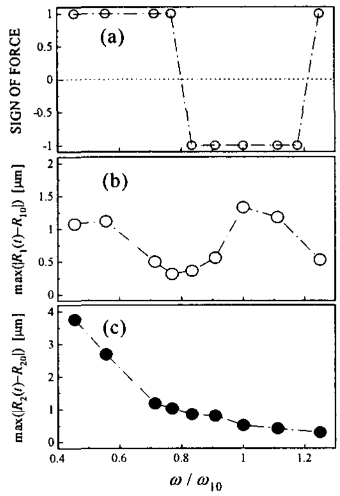

Fig. 4. Sign of the force and pulsation amplitudes obtained by DNS.

Figure 4(a) shows the sign of the (time-averaged) interaction force determined by observing the translational motion of the bubbles. This numerical result indicates that two sign reversals can take place in the frequency range considered, agreeing well with the theoretical prediction provided in the previous section, at least in a qualitative sense. Figures 4(b) and 4(c) display the pulsation amplitudes of the bubbles, given numerically. Obviously, the sign reversals take place at different frequencies from those resulting in these resonant responses. Because these resonances should appear at (or, more correctly, near) the natural frequencies, we may be able to conclude as that these sign reversals are not due to the natural frequencies. This result roughly validates our theoretical suggestion.

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