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An efficient and higher order accurate entropy variables based upwind method for inviscid flow computations

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ABSTRACT

1. Introduction

Considerable efforts are being made recently to develop new upwind schemes to improve accuracy and robustness of the flow solvers. Some of these schemes either fail or give undesirable results for problems involving strong shocks or rarefactions. This may be probably due to lack of positivity preserving property. In this paper a new higher order PVU (Peculiar Velocity Upwind) scheme called *q*-PVU scheme is presented which employs reconstruction of entropy variables *q*. The q variables were used [1] while casting Euler equations in symmetric hyperbolic form.

2. Basic theory of q-PVU Method

The Kinetic theory based upwind schemes [2,3,4] obtain solutions for Euler equations governing the dynamics of inviscid compressible flows starting from Boltzmann equation of the kinetic theory of gases. KFVS (Kinetic Flux Vector Splitting) and PVU are one such class of schemes under this category. *q*-KFVS scheme is nearly 3 times faster than higher order KFVS apart from giving smooth solutions [5]. It has been shown earlier that the PVU method is computationally more efficient compared to KFVS at least by a factor of 3 in case of 2D problems [4] and by a factor of 2.5 in case of 3D problems [6]. It is therefore desirable to improve further the efficiency and accuracy of the PVU scheme by employing entropy based *q* variables. The *q* variables are related to the usual flow variables by the following relation [1]

$$q = \left[ln\,\rho \;+\; ln\,\beta/(\gamma\; -\; 1) - \beta\left(u_1^2 + u_2^2\right),\, 2u_1\beta\;,\, 2u_2\beta\;,\, -\;2\beta\;\right]^T \qquad (1)$$

Where ρ is the fluid density, u_1 and u_2 are the fluid velocities, $\beta = 1/(2RT)$, R is the gas constant.

For a 2-D finite volume scheme, the state update formula at the Boltzmann level is given by

$$F^{n+1} = F^n \;-\; \frac{\Delta t}{A}\sum_{k\;1}^{K}|S_k|\;\left(\bar{c}.\bar{n}_k\,F_{sk}\right) \;-\; \frac{\Delta t}{A}\sum_{k\;1}^{K}|S_k|\;\left(\bar{u}.\bar{n}_k\,F_{sk}\right) \qquad (2)$$

Where A is the area of the cell of K sides, S_k is the length of the cell face k with normal $\bar{n}_k$, and F_{sk} is the distribution function at the cell face k. F_{sk} is the combination of two half Maxwellians. In the present case, this distribution function is obtained by interpolating the q variables from cell centers. The principle of least squares is employed for linear reconstruction. The distribution function at a cell face is thus nonlinearly dependent on F at neighboring points, and is always positive. The q-PVU scheme is then obtained by employing the moment method strategy and is thus expected to be positivity preserving.

3. Results and Discussions

The q-PVU scheme has been successfully applied to various standard test cases (including transonic and supersonic cases). Pressure contours for the standard shock reflection problem (180 x 60 grid) and for a ramp in a channel (150 x 50 grid) are shown in figs.1 and 2 respectively. Transonic flow past NACA 0012 airfoil for M=0.85 and $\alpha = 1°$ has been computed. A grid of 200 x 150 has been employed. The pressure contours plot is shown in figs. 3 and C_p distribution is shown in the Fig.4. The use of q-variables gives computationally efficient and smooth solution.

4. Conclusions

The PVU scheme which is faster since it does not contain error functions and exponentials in the expressions for fluxes when extended to q-variables has been further shown to be computationally efficient and gives smooth solution for various test cases. The use of conserved variables or primitive variables results in undesirable oscillations in the solution. q variables based method results in a positivity preserving scheme.

References

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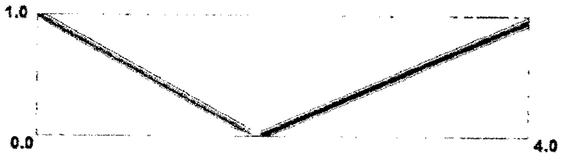

Fig. 1

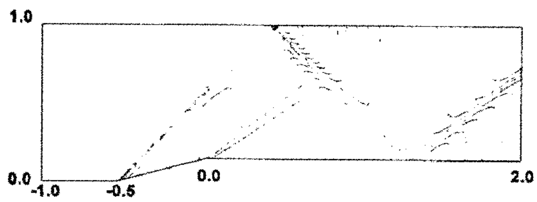

Fig. 2

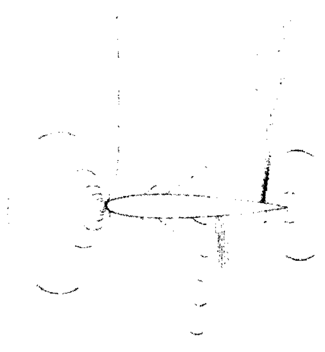

Fig. 3

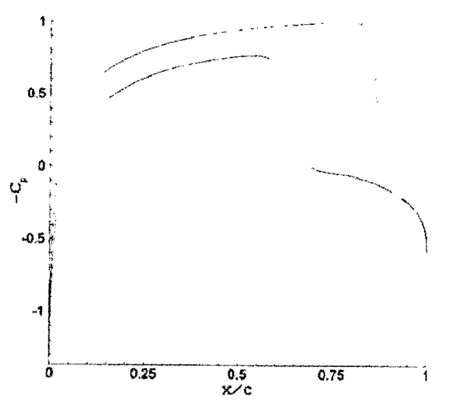

Fig. 4