Influence of non-uniform distribution of reflection coefficient phase on statistical characteristics of waves in layered medium with random and regular inhomogeneities

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Extended abstract

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Investigation of propagation of different waves in 3D random layered medium is one of the central problems in the general theory of the wave processes. In many cases these processes are usually described on the foundation of the Helmholtz equation with boundary conditions. This problem is reduced to the 1D one for the spectral components after carrying out the Fourier transformation. By analyzing the Fourier components for different values of spectral parameter q we can investigate statistical characteristics of the wavefield. In particular, we obtain the average wavefield from the average amplitude of the spectral component by means of the inverse Fourier transformation. In order to obtain the average intensity we must consider the correlator of the spectral components. We shall illustrate this idea for a random layered medium occupying the region $[L_0, L]$ of a homogeneous space with a point source inside it (see, e.g.[1]). Its wavefield $G(x,x_0,\vec{\rho}-\vec{\rho}_0)$ is described by the Helmholtz equation:

$$\left[\frac{\partial^2}{\partial x^2}+\Delta_\perp+k^2[1+\varepsilon(x)+i\gamma]\right]G(x,x_0,\vec{\rho}-\vec{\rho}_0)=\delta(x-x_0)\delta(,\vec{\rho}-\vec{\rho}_0),$$

where $\vec{\rho}=(x,y)$, $\Delta_\perp=\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$, k is a wavenumber, function $\varepsilon(x)$ describes random inhomogeneities, γ is the absorption parameter. By applying the Fourier transformation with respect to $\vec{\rho}-\vec{\rho}_0$ we obtain the 1D Helmholtz equation for the spectral components:

$$\left[\frac{\partial^2}{\partial x^2}+p^2\left[1+\frac{k}{p^2}^2(1+\varepsilon(x)+i\gamma)\right]\right]G(x,x_0,\vec{q})=2i\pi\,\delta(x-x_0)\ ,\ p=\sqrt{k^2-q^2}\ ,$$

$$G(x,x_0,\vec{\rho}-\vec{\rho}_0)=(2\pi)^{-2}\int d\vec{q}\ \exp[-i\vec{q}(\vec{\rho}-\vec{\rho}_0)]\ G(x,x_0,\vec{q}_0)/(2\pi i)\,.$$

For simplicity of further consideration we suppose that the source is placed between random and homogeneous media, and random media occupies the half-space. Then the inside wavefield characteristics depend on $\xi=L-x$. The standard approach for investigation of 1D stochastic Helmholtz equation is to distinguish processes with different scales. The value p defines the first space scale, $l_1=p^{-1}$, of the wave problem. The second space scale is $l_2=p\gamma^{-1}$. The third space scale l_3 is linked with fluctuations, and is determined by the diffusion coefficient D. The quantity l_3 is given by $l_3=1/D$. We shall scale l_1 and l_2 relative to l_3. Then we have dimensionless parameters $\beta_1=l_3/l_1=p/D$, $\beta_2=l_3/l_2=p\gamma/D$. Usually,

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it is assumed that $\beta_2 << \beta_1$ and $1 << \beta_1$. These inequalities are the conditions for applicability of the average method.

We introduce the reflection coefficient $R_j = R_j(L, q_j)$ as follows: $G(0, q_j) = 1 + R_j(L, q_j)$. Statistical characteristics of the reflection coefficient R_L define the statistical behavior of the wavefield. When we are interested in the reflection coefficient R_L, it is necessary to write the Fokker-Plank equation for the joint distribution probability of the phase φ and the modulus ρ. This equation has the large parameter β_1, and we can present a solution in the form of a series with respect to $1/\beta_1 << 1$. A leading term of the series doesn't contain φ and it is calculated from the shortcut equation, which follows from the Fokker-Plank equation by integrating (averaging) over φ. In this case the phase has the uniform distribution, i.e. the steady-state probability distribution of the reflection coefficient phase $P(\varphi) = 1/2\pi$. If the absorption is absent, that is $\beta_2 \to 0$, the half-space of the randomly inhomogeneous medium reflects the incident wave: $|R_L|^2 = 1$ with probability unit. To calculate the statistical characteristics of the wavefield we integrate the corresponding characteristics of the spectral components for all the values of q in the inverse Fourier transformation. When the values of q tend to k the conditions for applicability of the average method are violated. Then we supposed that the steady-state probability distribution of the reflection coefficient phase isn't the uniform one.

In this report we show how one can calculate the steady-state probability distribution of the reflection coefficient phase $P(\varphi)$ for arbitrary values of β_1 at $\beta_2 \to 0$.

We extended our approach for calculation of correlation between different spectral components $\langle G(L-x, q_1) G^*(L-x, q_2) \rangle$ where the brackets $\langle\ \rangle$ mean $\varepsilon(x)$ ensemble averaging. The spectral components $G(L-x, q)$ with real values of p $(q < k)$ characterise propagating waves whereas the spectral components with complex of p $(q > k)$ correspond to so-called evanescent waves. We analyzed contribution of the propagating waves to the field intensity $\langle I(L-x, \rho) \rangle$. The value $\langle I(0, \rho) \rangle$ was calculated analytically and numerically. In the last case numerical results were obtained with the help of the method of the statistical simulation. Comparison of analytical results with numerical ones showed that they coincided with accuracy about 5%.

Then we considered the problem about joint influence of regular and random inhomogeneities on the wavefield characteristics. Principal difficulty of constructing a solution is linked with the fact that its statistical behavior depends on a solution of the problem with regular inhomogeneities. In this report we introduce a linear regular profile. From the mathematical point it means that we must replace $\varepsilon(x) \to \varepsilon(x) + \alpha x$. It is supposed that a parameter $\sigma = \alpha / p$ is a small one. Then its influence on the spectral components is taken into account by means of the perturbation theory, and a stochastic equation for the reflection coefficient phase is written in a shortcut form. The statistical analysis of this equation is carried out in the framework of the approach formulated for the statistical problem without a regular inhomogeneity. Then the steady-state probability density P(z) of this value has the form

To describe physical results about the wavefield behavior in the medium with regular and random inhomogeneities we use the numerical calculations of the quantity $GG = < G(0, \rho) > / < G_0(0, \rho) >$ where $< G_0(0, \rho) >$ is the is the Green function in the homogeneous space. When there are no inhomogeneities inside the medium the quantity GG is equal to unity. Calculations show that inhomogeneities result in appearance of periodical peaks in GG. These peaks exist for all the values of σ which were used for the calculations, and their period doesn't depend on σ. Hence appearance of periodical peaks is a statistical effect. The numerical analysis show that the regular inhomogeneities influence on the amplitude of the peaks: the amplitude rises while the stratification parameter increases.

References

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