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Calculation of Cavity Flow with FEM & Finite Spectral Method

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Abstract

The streamfunction-vorticity equations for two-dimentional cavity flow are solved by a new finite element method which uses finite spectral basis functions as interpolation functions for rectangular elements. Results for several cases with different Renold's number are compared with benchmark solutions and found to be in well agreement.

Keyword: *driven cavity flow, finite spectral method, finite element method*

1. Introduction

The finite spectral method based on non-periodic Fourier integral has succeeded in dealing with spectral methods pointwise [1,2]. It is characterized by local property, non-periodicity, orthogonal relation, efficiency and simplicity. This makes it possible to apply finite spectral method to finite element method's calculation by using basis functions of finite spectral method as interpolation functions.

2. Basis functions of finite spectral method

The N -th order basis function of finite spectral method, $W_N(x)$ is represented as

$$W_N(x) = \frac{1}{2N}\sum_{n=-N}^{N} C_n \exp(\frac{i\pi nx}{l}) = \frac{1}{2N}\sum_{n=-N}^{N} C_n \cos(\frac{\pi nx}{l}) \tag{1}$$

where $-l \le x \le l$, and C_n takes 0.5 for $n = \pm N$ but 1 for others. Basis functions of finite spectral method are defined in a piece-wise fashion and are guaranteed orthogonal. Hence they can be used as FEM basis functions.

3. Calculation of Cavity Flow

Using finite element method with finite spectral basis functions, the streamfunction-vorticity equations for two-dimentional cavity flow are uncoupled and solved in sequence. The discrete elements are 4-node rectangular elements. Results for $\mathrm{Re} = 100, 400, 1000$ are obtained. The figures of Contours of streamfunction and Contours of vorticity for $\mathrm{Re} = 400$ that show as Fig. 1 and Fig. 2 are similar to previous work [3]. Comparison between current results and benchmark solutions [4] are found in well agreement..

3.3 Finite element formulation

Throughout the solution domain, the unknown variables can be approximated by means of the standard expansions

$$\psi \approx \sum_{i=1}^{n} N_i \psi_i \qquad (2)$$

$$\omega \approx \sum_{i=1}^{n} N_i \omega_i \qquad (3)$$

where N_i are two-dimensional finite spectral basic function, ψ_i and ω_i are nodal values of ψ and ω respectively and n is the total number of nodes in the discretization. Following the Bubnov-Galerkin method, equations (1) and (2) can be writern in matrix form as

$$A_{ij}^{(e)}\psi_j - B_{ij}^{(e)}\omega_j = C_i^{(e)} \qquad (4)$$

$$D_{ij}^{(e)}\omega_{j,t} + E_{ijk}^{(e)}\psi_j\omega_k + F_{ij}^{(e)}\omega_j = G_i^{(e)} \qquad (5)$$

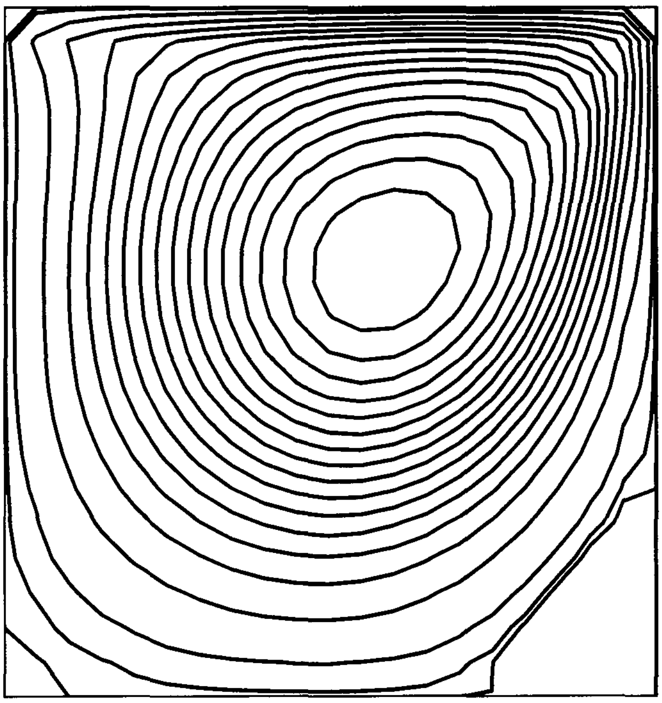

Fig. 1. Contours of streamfunction

Fig. 2. Contours of vorticity

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