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A New Wall-Distance Free One-Equation Turbulence Model

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Abstract

We propose a wall distance free one-equation turbulence model. The model is organized in an extremely simple form. Only a few model constants were introduced into the model. The model is numerically tough and easy-of-use. The model also demonstrated the ability to simulate the laminar to turbulent flow transition. The model has been applied to the channel flow, the plane jet, the backward facing step flow, the flat plate boundary layer, as well as the flow around the 2D airfoil at large angles of attack, which obtained satisfactory results.

Keyword: *One-equation turbulence model, RANS, Wall-distance free turbulence model, Transition*

1. Introduction

In the last decade, several one-equation turbulence models solving for the eddy viscosity[1-3] were proposed and have been applied to a number of engineering flow problems. These models are cheaper, easier of use than the two-equation models, and have been shown to provide results as accurate as two-equation models in most cases. The general form of the one-equation turbulence model for the eddy viscosity (ν_t) may be written as

$$\frac{D\nu_t}{Dt} = [\text{production}] + [\text{destruction}] + [\text{diffusion}] \tag{1}$$

To construct the destruction term requires the explicit or implicit specification of the turbulence length scale. The Spalart and Allmaras (SA) model[1] uses the distance from the wall for this purpose, while the Baldwin-Barth (BB) model[2] employs the gradient of the eddy viscosity. In the outer part of a boundary layer, however, the latter does not reflect the physical nature of the destruction of the eddy viscosity. The BB model additionally introduced the wall distance to calculate the dumping function of the viscous sub-layer. Computing the distance from wall boundary is rather complex for engineering problems having multiple wall boundaries.

The objective of the present research is to build an entirely wall-distance free one-equation turbulence model for the eddy viscosity. This model must be numerically stable, easy-of-use and competitive to models directly employing the wall distance.

2. The New Model

The proposed model is written as:

$$\frac{D\hat{\nu}_t}{Dt} = \underbrace{c_0\nu_t|\hat{\omega}|}_{\text{Production}} + \underbrace{c_1 \min\left(0, \frac{\nu_t}{|\hat{\omega}|}\nabla\cdot|\hat{\omega}|\nabla\hat{\nu}_t\right)}_{\text{Destruction}} + \underbrace{\nabla\cdot\left(\nu_0 + \frac{1}{\sigma}\nu_t\right)\nabla\hat{\nu}_t}_{\text{Diffusion}} \tag{2}$$

$$\nu_t = \hat{\nu}_t f_\nu,\ f_\nu = 1 - \frac{1}{1+\left(u^+/A_u^+\right)^2}\exp\left[-\left(\hat{\nu}_t/A^+\kappa\nu\right)^3\right] \tag{3}$$

$$u^+ = \frac{|u|}{u_\tau},\ u_\tau = \left(\nu_0 + \nu_t\right)|\omega| \tag{4}$$

$$\kappa = 0.41,\ c_0 = 0.21,\ \frac{1}{\sigma} = 1.0,\ c_1 = \frac{c_0}{\kappa^2} + \frac{1}{\sigma},\ A^+ = 33,\ A_u^+ = 40 \tag{5}$$

where $|\hat{\omega}|$ and $|u|$ are the vorticity and velocity of the averaged flow field, respectively. ν_0 and ν_t are the molecular viscosity and the eddy viscosity, respectively. f_ν is the dumping function. κ is the Karmann constant, and $c_0, c_1, \sigma, A^+, A_u^+$ are model constants. To avoid zero division in Eq.(2), a tiny constant 10^{-6} is added to the vorticity (i.e., $|\hat{\omega}| \Leftarrow |\hat{\omega}| + 10^{-6}$).

The second term in the right-hand side of Eq.(2) is the original destruction term proposed here. By assuming the log-law distribution of the velocity for the turbulence flow in a 2D channel, an analytic expression of the eddy viscosity is derived.

$$\nu_t = \kappa u_\tau \left(y - \frac{y^2}{2y_0} \right) \qquad (6)$$

where $2y_0$ is the width of the channel.

The eddy viscosity agrees with the Prandtl's mixing-length theory in the inner log-law layer and reaches a plateau at the center line of the channel.

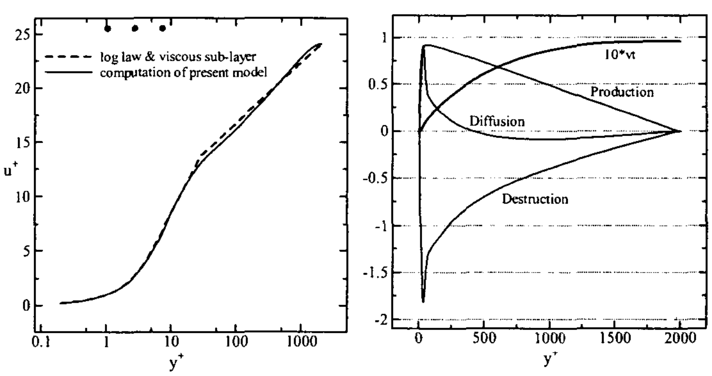
$\nabla \cdot |\hat{\omega}| \nabla \hat{\nu}_t$ in Eq.(2) possesses a negative value in almost the whole computational region. However, it may become positive near the out edge of a boundary layer flow. The minimum function is employed in Eq.(2) to avoid this deficiency.

The dumping function of Eq.(3) approaches $(\hat{\nu}_t / A^+ \kappa \nu)^3$ as $y^+ \to 0$. $\left[1 + (u^+ / A_u^+)^2\right]^{-1}$ is introduced to prevent the artificial dumping at the out edge of the boundary layer. Incorporation of this term also enables the model to simulate the laminar to turbulent flow transition.

Since Eq.(2) allows zero solution ($\hat{\nu}_t = 0$) in the whole computational region, a non-zero initial value of $\hat{\nu}_t > 2(A^+ \kappa \nu)$ should be given to start the computation.

3. Numerical Results

Figure 1 shows the numerical results at $Re_\tau = u_\tau y_0 / \nu = 2000$. Figure 2 shows the computed velocity profiles of the plane jet at $Re = 10^6$. Figure 3 shows the computed velocity profiles of the backward-facing step flow [4] at Re=5541. Figure 4 shows the results of flat plate boundary layer flow. Figure 5 shows the computed surface pressure distribution for NACA63$_3$-018 airfoil at $Re = 5.8 \times 10^6$. Satisfactory results were obtained. Discussion on the numerical results is provided in the fully length paper.



(a) Velocity distribution (b) Eddy viscosity and its budget

Figure 1 Numerical results of 2D channel flow at $Re_\tau = 2000$.

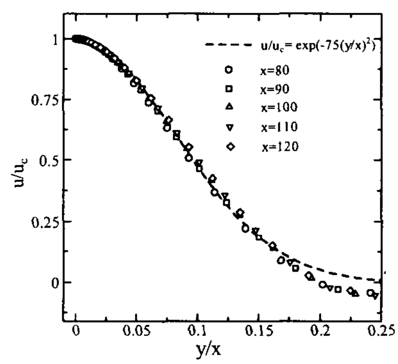


Figure 2 Computed results of a plane jet.

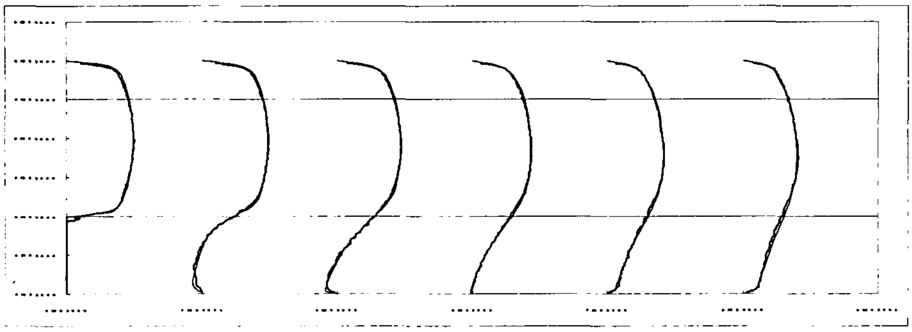
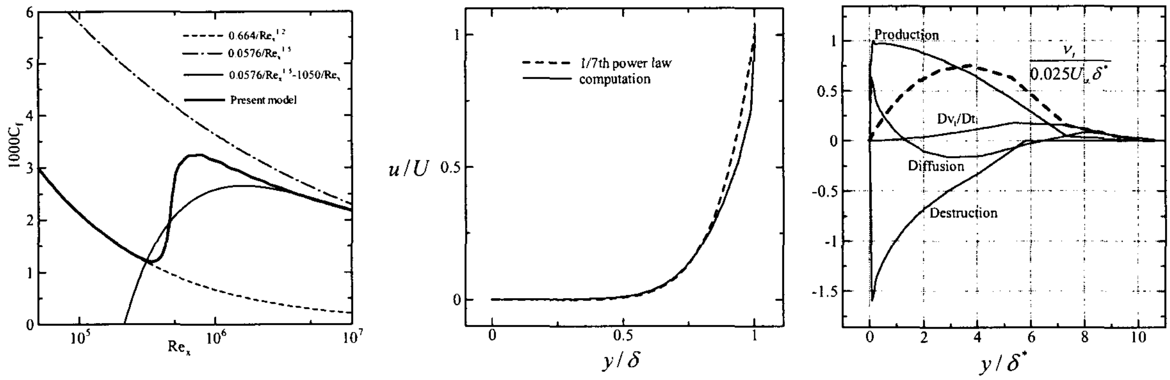
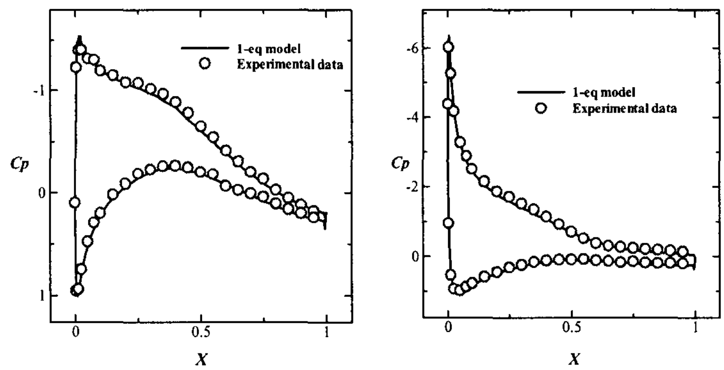


Figure 3 Comparison of computed velocity profiles (solid lines) and data of measurements (Bold lines) of backward-facing step flow.



(a) Local skin-friction coefficient (b) Velocity profile (c) Eddy viscosity and its budget

Figure 4 Boundary layer flow over a flat plate



(a) $\alpha = 5.26°$ (b) $\alpha = 13.6°$

Figure 5 Surface pressure of NACA63_3-018

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