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On the Spectral Eddy Viscosity in Isotropic Turbulence

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Abstract

The spectral eddy viscosity model is investigated through the large eddy simulation of the decaying and forced isotropic turbulence. It is shown that the widely accepted 'plateau and cusp' model overpredicts resolved kinetic energy due to the amplification of energy at intermediate wavenumbers. Whereas, the simple plateau model reproduces a correct energy spectrum. This result overshadows *a priori* tests based on the filtered DNS or experimental data. An alternative method for the validation of subgrid-scale model is discussed.

Keyword: *spectral eddy viscosity, isotropic turbulence, large eddy simulation*

1. Introduction

In spite of very low tensor-level correlation with true subgrid-scale (SGS) stress, the eddy-viscosity type models are still widely used in large eddy simulation (LES). They generally produce the proper amount of SGS dissipation especially for isotropic turbulence, where the forward scatter is dominant.

One can assume that the eddy-viscosity may vary with the flow scales rather than being uniform. This idea has been realized by the 'spectral' eddy-viscosity in the wavenumber space. Kraichnan [1] used the Test-Field Model (TFM) to calculate the spectral eddy-viscosity for high-Reynolds number isotropic turbulence. The result consisted of a plateau in the inertial range terminating with a sharp upward cusp near the cut-off wavenumber, which takes the form

$$\nu_t(k \mid k_c) = \{A + B\exp(-Ck_c / k)\}[E(k_c)/k_c]^{1/2}, \quad (1)$$

where A, B, and C are adjustable constants and k_c is the cut-off wavenumber. Similar calculations and results were later obtained based on the Eddy Damped Quasi-Normal Markovian (EDQNM) closure [2], for spectral cutoff filter. Tests using low Reynolds number DNS [3] and the experiment [4] have in general confirmed these trends, which directly come from the truncation of triadic wavenumber interactions. As the fictitious filtering hinders the energy transfer across k_c, the transfer function of the resolved nonlinear term has the cusp toward k_c, and so does the spectral eddy-viscosity.

However, it should be noted that the spectral eddy-viscosity has been derived only from *a priori* tests based on fully resolved energy spectrum and/or velocity field. Thus, it is quite questionable whether such a 'plateau and cusp' model would give a proper SGS dissipation for the actual LES, or the time-dependent simulation with already filtered field.

The objective of the present study is to examine the validity of the spectral eddy viscosity model through the LES using both the filtered Navier-Stokes equation and EDQNM closure.

2. Results and Discussion

Fig. 1(a) shows the energy spectra computed by LES of the decaying isotropic turbulence of Comte-Bellot and Corrsin [5]. Positive, zero, and negative values of B in Eq. (1) are considered, which correspond to the 'upward cusp', 'plateau', and 'downward cusp' model, respectively. The 'upward cusp' model shows a rapid fall-off near the cut-off, while large excess of energy is observed in the intermediate wavenumbers. Overall, it somewhat over-predicts the resolved kinetic energy. We found that the presence of the cusp is responsible for this behavior because a small $E(k_c)$ gives an insufficient

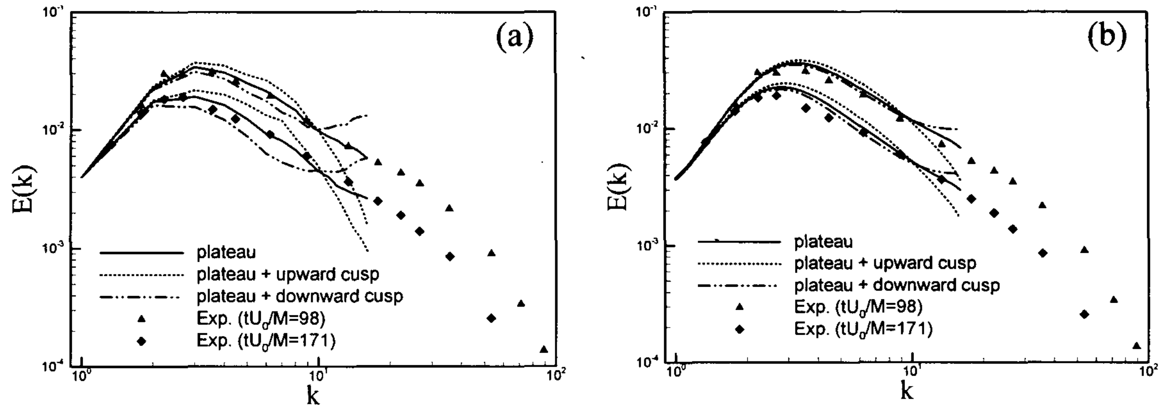

Fig. 1 Time evolution of 3-D energy spectra for decaying, isotropic turbulence: (a) LES, (b) EDQNM-LES.

SGS dissipation as indicated by Eq.(1).

On the other hand, a correct –5/3 inertial range spectrum is recovered by the plateau model. At first glance, it seems that the plateau model decreases the SGS dissipation because the cusp has been flattened. Its dynamic effect, however, increases the eddy viscosity and the SGS dissipation because of increased $E(k_c)$. This feature cannot be captured by the static *a priori* test. Therefore, we conclude that the spectral eddy-viscosity cannot be the function of wavenumber, otherwise the prescribed behavior of the eddy-viscosity leads to an opposite results to our intention. This conclusion is supported by the results from 'downward cusp' model, which shows the largest level of SGS dissipation at intermediate wavenumbers.

It may be argued that the origin of this anomaly is the fact that the spectral eddy-viscosity has been derived from a closure theory based on the energy equation, not on the momentum equation. In this regard, we performed a filtered EDQNM simulation, or EDQNM-LES:

$$\left[\frac{\partial}{\partial t} + 2\left(\nu + \nu_t(k \mid k_c)\right)k^2\right]\bar{E}(k,t) = \iint_{\Delta_k}^{p,q\le k_c} dpdqS(k,p,q)\,, \tag{2}$$

where the term on the right-hand side is the resolved nonlinear term with EDQNM closure [6]. The result from EDQNM-LES (Fig. 1(b)) shows a qualitative similarity to those from actual LES, which verifies that this phenomenon is not due to different context of the theory and the simulation.

Also, similar trends are observed from the LES of the forced isotropic turbulence. This result contrasts to *a priori* tests based on the filtered DNS or experimental data, and requires an alternative diagnostics for the validation of SGS models.

We proposed a 'dynamic *a priori*' test to find out the optimal spectral eddy-viscosity, which fully accounts for time-stepping effect. It is shown that the optimal eddy-viscosity is very close to the plateau model, which confirms the finding from numerical tests.

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