Higher Order Wall Boundary Conditions for Incompressible Flow Simulations

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Abstract

In this paper, the new higher order wall boundary conditions are proposed for solving the incompressible flows. The square driven cavity flows are simulated by using the variable order method of lines with the present wall boundary conditions. The variable order method of lines is constructed by the spatial discretization, i.e., the variable order proper convective scheme for convective terms and the modified differential quadrature method for diffusive terms, and time integration. The 2nd, 4th, and 6th order solutions are presented and these results show this higher order boundary conditions are very promising for the incompressible flow simulations.

Keyword: *Wall boundary condition, higher order solution, variable order method of lines*

1. Introduction

In the practical flow simulations, the enormous computational time and memory are necessary in order to obtain the reliable solution. To relax the restriction on the limit of grid resolution, the higher order numerical simulation is one of the means of solving. However, the boundary conditions, especially wall boundary conditions, with higher order of spatial accuracy is the more important problem.

In this paper, the higher order wall boundary conditions are proposed. The square driven cavity flows are considered and the present boundary conditions are verified.

2. Computational Method

2.1 Higher order flow solver

The variable order method of lines [1,2] is adopted for solving the incompressible Navier-Stokes equations. The variable order method of lines is constructed by two steps, i.e., the spatial discretization and the time integration. The spatial derivatives are discretized by the variable order proper convective scheme for convective terms and the modified differential quadrature method for diffusive terms. Then the partial differential equations in space and time are reduced to the system of ordinary differential equations (ODEs) in time. The resulting ODEs in time are integrated by the appropriate time integration scheme, e.g., the Runge-Kutta type scheme. In this paper, the collocated grid system which all variables are defined at cell center is adopted. The fractional step technique [3] is used for the solution procedure. The resulting pressure equation is solved by the variable order SOR method.

2.2 Higher order wall boundary conditions

In the collocated grid system, the wall boundary location is different from the grid point. Then, in order to estimate the values at virtual grid point inside the wall, it is necessary that the interpolated values are equivalent to the wall (boundary) values. Generally, the interpolation is defined by the linear combination of the simple averages and coefficients.

$$f_i = \sum_{\ell=1}^{M/2} c_{\ell'} \overline{f_i}^{\ell'} \quad , \quad \overline{f_i}^{\ell'} = \frac{1}{2}(f_{i-\ell'/2} + f_{i+\ell'/2}) \quad , \qquad (1)$$

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where M is the order of accuracy and $\ell' = 2\ell - 1$. The coefficients $c_{\ell'}$ can be obtained automatically by using the modified differential quadrature method. The coefficients satisfy the relation $\sum_{\ell=1}^{M/2} c_{\ell'} = 1$. Then, the values at frictional grid points can be determined by $\overline{f_{wall}}^{\ell'} = f_{wall}$.

On the other hand, the first derivative of pressure at the boundary point is necessary to solve the pressure equation. The first derivative can be estimated by the linear combination of the second order finite difference with different grid spacings and coefficients.

$$\frac{\partial f}{\partial x_i} = \sum_{\ell=1}^{M/2} c_{\ell'} \frac{\delta_{\ell'} f}{\delta_{\ell'} x_i} \quad , \quad \frac{\delta_{\ell'} f}{\delta_{\ell'} x_i} = \frac{1}{\ell' \Delta x_i}\left(-f_{x_i - \ell'/2} + f_{x_i + \ell'/2}\right) \quad , \tag{2}$$

where Δx_i is the grid spacing in the x_i direction. Then, the each second order finite difference is equal to the boundary value, so that the global Neumann condition can be satisfied.

3. Computational Results

In this paper, the square driven cavity flow problem is considered. The computational conditions are that the number of grid points is *41x41*, Reynolds number *Re=1000*, and the convergence criteria is *L2-residual*$<10^{-6}$ for the Navier-Stokes and pressure equations.

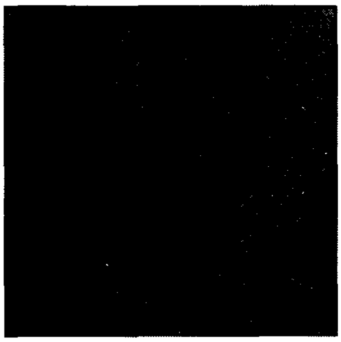

Fig. 1. 2nd order solution

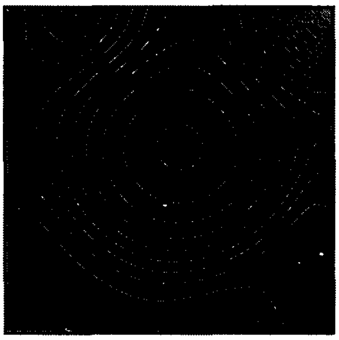

Fig. 2. 4th order solution

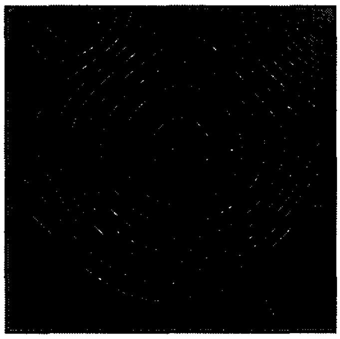

Fig. 3. 6th order solution

Figures 1-3 show the pressure coefficient distributions with the 2nd, 4th, and 6th order of spatial accuracy, respectively. Figure 4 shows the velocity profiles along the center lines. These results show that as the order of spatial accuracy becomes higher, the primary vortex becomes stronger. And the velocity components near the boundaries are clearly improved.

Then, it is concluded that the present higher order wall boundary conditions are more available for the incompressible flow simulations.

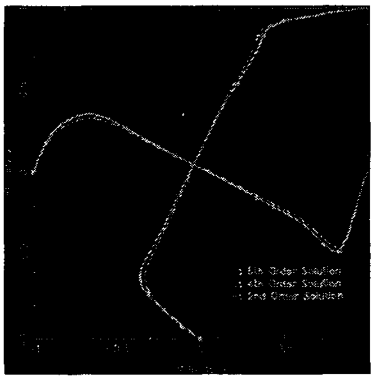

Fig. 4. Velocity profiles along the center lines

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