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# Numerical Analysis of Laminar Natural Convection Heat Transfer around Two Vertical Fins by a Spectral Finite Difference Method

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**Abstract:** A numerical solution is presented for the natural convection heat transfer from two vertical fins using a spectral finite difference method. Virtual distant boundary conditions for two bodies that are compatible with plume behavior and with an overall continuity condition are introduced. A boundary-fitted coordinate system is formed. Streamlines, isotherms, mean Nusselt numbers and drag & lift coefficients are presented for a variety of dimensionless parameters such as a Grashof number and a Prandtl number at a steady-state. Extensive effectiveness of a spectral finite difference method was established.

***Keyword**: natural convection, vertical fin, spectral finite difference method, virtual distant boundary.*

## 1. Introduction

Laminar natural convection heat transfer from vertical fins is one of the major concerns on heat exchangers. There are many articles reported on natural convection heat transfer for one body accounting plume behavior. Mochimaru introduced virtual distant boundary conditions that are compatible with plume behavior above uniformly heated body and with an overall continuity condition. An analytical object is two vertical fins located in a row. The surfaces of two fins are assumed to be maintained at a uniform high temperature. At this stage, it is necessary to introduce virtual distant boundary conditions for two bodies.

## 2. Analysis

Fluids are assumed to be Newtonian. The flow and temperature field is assumed to be two-dimensional and laminar. Under a Boussinesq approximation, the governing equation can be written in non-dimensional forms as follows:

$$J\frac{\partial \varsigma}{\partial t}+\frac{\partial(\varsigma,\psi)}{\partial(\alpha,\beta)}=\frac{1}{\sqrt{Gr}}\left\{\frac{\partial^2\varsigma}{\partial\alpha^2}+\frac{\partial^2\varsigma}{\partial\beta^2}\right\}+\frac{\partial(T,y)}{\partial(\alpha,\beta)},\quad J\varsigma+\frac{\partial^2\psi}{\partial\alpha^2}+\frac{\partial^2\psi}{\partial\beta^2}=0$$

$$J\frac{\partial T}{\partial t}+\frac{\partial(T,\psi)}{\partial(\alpha,\beta)}=\frac{1}{Pr\sqrt{Gr}}\left\{\frac{\partial^2 T}{\partial\alpha^2}+\frac{\partial^2 T}{\partial\beta^2}\right\},\quad J\equiv\frac{\partial(x,y)}{\partial(\alpha,\beta)}$$

where $J$ is Jacobian; $Gr$, a Grashof number; $Pr$, a Prandtl number; $T$, a temperature; $\psi$, a stream function; $\zeta$, a vorticity. Equations are considered conformal mapping from the physical domain to the analytical domain.

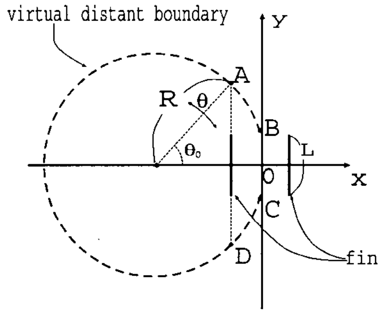


**Fig. 1. Geometry and configuration**

The mapping function is used

$$x+iy=\frac{K'}{\pi}\left(Z+\frac{\mathrm{dn}(\xi+i\eta,k)\mathrm{cn}(\xi+i\eta,k)}{\mathrm{sn}(\xi+i\eta,k)}+\frac{\pi(\xi+i\eta)}{2K'K}\right),\quad \frac{\pi}{K'}(\xi+i\eta)=\alpha+i\beta$$

where $k$=0.1434, $K$ and $K'$ are complete elliptic integrals of the first kind with the complementary parameters $\sqrt{(1-k^2)}$ and $\sqrt{(1-k'^2)}$ respectively. $Z$ is a zeta function of Jacobi.

Paper No. : 1-2C-2

Boundary conditions at the surface of the left fin are specified as

$$\psi(\alpha_0,\beta)=\frac{\partial}{\partial\alpha}\psi(\alpha_0,\beta)=0\,,\ T(\alpha_0,\beta)=1\,.$$

The coordinates and the geometry are defined as shown in Fig. 1. Virtual distant boundary conditions under a steady-state are introduced. Since the excess of a vorticity and temperature is remarkable in the plume, the behavior of vorticity and temperature, stream function can be expressed as

$$\xi=Gr^{1/5}R\,|\cos\theta_0-\cos\theta\,|\,(R\sin\theta_0+\delta)^{-2/5},\quad T_\infty=Gr^{-1/5}(R\sin\theta_0+\delta)^{-3/5}A^4h(A\xi)\,,$$

$$\psi_\infty=Gr^{-3/10}(R\sin\theta_0+\delta)^{3/5}\sin\theta_0Af(A\xi)H(\theta)\ \ ,\ \ \varsigma_\infty=-Gr^{1/10}(R\sin\theta_0+\delta)^{-1/5}A^3f''(A\xi)\,,$$

$$H(\theta)=\frac{\cos1.4\theta_0+\cot1.2\pi\sin1.4\theta_0}{\sin2\theta_0}\sin0.6\theta+\frac{\cos0.6\theta_0-\cot1.2\pi\sin0.6\theta_0}{\sin2\theta_0}\sin1.4\theta\,,$$

where $\delta$ is a suitable parameter to be determined, and $f$, $h$ are given by

$$f'''+\frac{3}{5}ff''-\frac{1}{5}f'^2+h=0\ \ ,\qquad h'+\frac{3}{5}Prfh=0$$

with $h(0)=1, f(0)=f''(0)=0, h(\infty)=f'(\infty)=0$. The prime denotes differentiation with respect to $A\xi$.

At the upper right side of the left fin (from the point A to the point B in Fig. 1.), $H$ is determined so as to be a unity. And, the lower right side of the left one (from the point C to the point D) is expressed through supposition of a uniform flow. The stream function near to the y-axis (from the point B to the point C) is constant considering existence of the right side fin. On this virtual boundary, heat flux is specified to be zero, and the vorticity is vanished.

**3. Results**

Steady-state streamlines and isotherms at $Pr$ =0.7, $Gr$ =1000 are shown in Fig. 2. Lift coefficients and drag coefficients, mean Nusselt numbers for various Grashof numbers and Prandtl numbers are presented in Fig. 3.

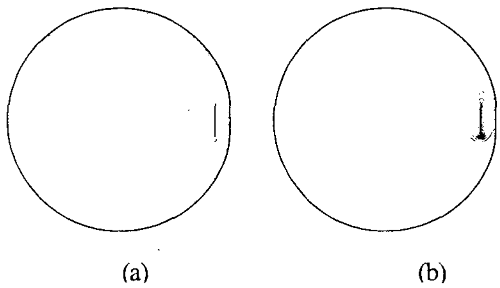


**Fig. 2. (a) Steady-state streamlines; $\Delta\psi$ =0.07, (b) isotherms; $\Delta T$ =0.1 at $Pr$ =0.7, $Gr$ =1000**

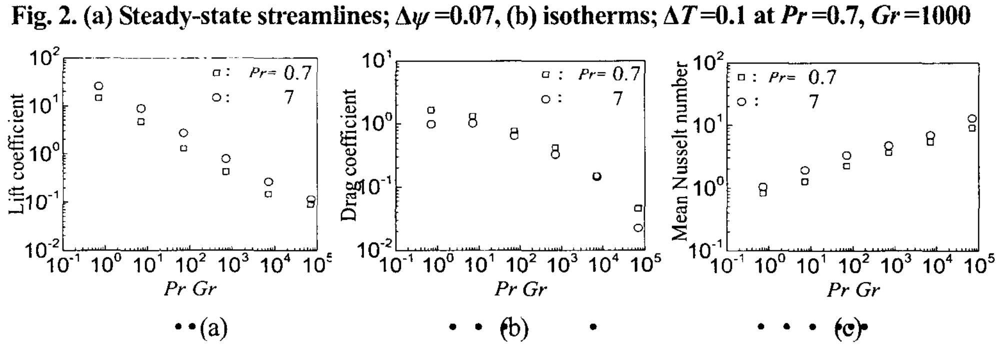


**Fig. 3. (a) A lift coefficient, (b) a drag coefficient, (c) a mean Nusselt number against *PrGr***

**4. Conclusion**

Steady-state laminar natural convection heat transfer around two vertical fins was studied numerically for various Grashof numbers and Prandtl numbers. Extensive effectiveness of a spectral finite difference method was established.

**References**

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