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Computing Fluid Flow without Grid Generation

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Abstract

It is shown that using the unified coordiantes of Hui *et al.* [1 – 4], one can now compute fluid flow without prior grid generation. This represents a great saving of computing time.

1. Introduction

After decades of intensive research on the shock-capturing method using Eulerian coordiantes, two drawbacks remains: (i) slip lines are poorly resolved, and (ii) prior to flow field computation, it is necessary to generate a computation grid, which can be very time consuming.

Recently, a unified coordinate system was introduced by Hui *et al.* [1 – 4] which moves with velocity $\boldsymbol{hq}$, $\boldsymbol{q}$ being the fluid velocity. By choosing h to preserve grid orhtongonality, it resolves slip lines sharply, get the computatioin does not break down as the Lagrangian system does.

The fact that the unified coordiante system moves with velocity $\boldsymbol{hq}$ can be utilized to generate computational grid while the flow computation goes on. This is in direct contrast to the Eulerian computation, where a grid must be generated prior to flow computation, and this represents a great saving of computing time. It is noted that in typical aeronautical applications, computing the grid takes much longer time than computing the flow field itself.

2.The Unified Coordinates

Starting from Cartesian coordiantes (x, y, z) and time t in the Eulerian description, we make a transformation to the unified coordinates $(\lambda, \xi, \eta, \zeta)$,

$$\begin{cases} dt = d\lambda & \text{(1a)} \\ dx = hud\lambda + Ad\xi + Ld\eta + Pd\zeta & \text{(1b)} \\ dy = hvd\lambda + Bd\xi + Md\eta + Qd\zeta & \text{(1c)} \\ dz = hwd\lambda + Cd\xi + Nd\eta + Rd\zeta, & \text{(1d)} \end{cases}$$

where u, v, and w are the x, y and z components of fluid velocity $\boldsymbol{q}$, respectively. Let

$$\frac{D_h}{Dt} \equiv \frac{\partial}{\partial t} + hu\frac{\partial}{\partial x} + hv\frac{\partial}{\partial y} + hw\frac{\partial}{\partial z} \tag{2}$$

denote the material derivative following the *pseudo-particle*, whose velocity is $\boldsymbol{hq}$. Then, it is easy to show that

$$\frac{D_h\xi}{Dt} = 0, \quad \frac{D_h\eta}{Dt} = 0, \quad \frac{D_h\zeta}{Dt} = 0; \tag{3}$$

that is, coordinates (ξ, η, ζ) are material functions of the pseudo-particles. Accordingly, *computational cells move and deform with pseudo-particles*, rather than with fluid particles as in Lagrangian coordinates. As a special case when h=1, the coordinates (λ, ξ, η) move with fluid velocity $\boldsymbol{q}$ and are thus Lagrangian coordiantes. On the other hand when h=0, (λ, ξ, η) do not move and are thus are Eulerian coordinates.

Consider two-dimensional unsteady flow for which the Euler equations are

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ \rho e\end{pmatrix}+\frac{\partial}{\partial x}\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ \rho u\left(e+\frac{p}{\rho}\right)\end{pmatrix}+\frac{\partial}{\partial y}\begin{pmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ \rho v\left(e+\frac{p}{\rho}\right)\end{pmatrix}=0, \tag{4}$$

where p and ρ are pressure and density, respectively, and

$$e=\frac{1}{2}(u^2+v^2)+\frac{1}{\gamma-1}\frac{p}{\rho}.$$

Under the transformation (1) we get

$$\frac{\partial \mathbf{E}}{\partial \lambda}+\frac{\partial \mathbf{F}}{\partial \xi}+\frac{\partial \mathbf{G}}{\partial \eta}=0, \tag{5a}$$

where

$$\mathbf{E}=\begin{pmatrix}\rho\Delta\\ \rho\Delta u\\ \rho\Delta v\\ \rho\Delta e\\ A\\ B\\ L\\ M\end{pmatrix},\quad \mathbf{F}=\begin{pmatrix}\rho(1-h)I\\ \rho(1-h)Iu+pM\\ \rho(1-h)Iv-pL\\ \rho(1-h)Ie+pI\\ -hu\\ -hv\\ 0\\ 0\end{pmatrix},\quad \mathbf{G}=\begin{pmatrix}\rho(1-h)J\\ \rho(1-h)Ju-pB\\ \rho(1-h)Jv+pA\\ \rho(1-h)Je+pJ\\ 0\\ 0\\ -hu\\ -hv\end{pmatrix}, \tag{5b}$$

with

$$\Delta=AM-BL,\qquad I=uM-vL,\qquad J=Av-Bu. \tag{6}$$

We note that the Euler equations (5) written in the unified coordinates are in conservation form, so any standard shock-capturing method, e.g. Godunov method, can be used to compute their solutions. The free function h is best chosen to preserve grid angles, hence grid orthogonality [1].

3. Computaiton Procedure

To compute a steady uniform flow past a given body,

(1) Begin with a column of uniform orthogonal cells of the width Δx, where the uniform flow is given.

(2) Compute the solution to Eq. (5) by marching in λ, using the Godunov/MUSCL scheme. After one time step $\Delta\lambda$, the initial column of cells move to the right by $h\mathbf{q}_\infty\Delta\lambda$ where $\mathbf{q}_\infty$ is the velocity of the uniform free stream.

(3) After several time steps when the initial column of cells has moved to the right by Δx, add one new column of cells on the left that is identical to the initial column.

(4) Repeat the above process, until the columns of cells representing the given uniform flow meet a solid boundary, impose the boundary condition there.

(5) Continue this process until the columns of cells cover the whole body surface, then we simultaneously add a new column of cells on the left while, at the same time, delete the right-most column of cells from our computation; this gives us a fixed window to watch the flow.

(6) Continue this process until a steady state is judged to have been reached.

4. An Example

An example of supersonic flow M=1.8, in a channel is shown in Fig. 1. It represents a Mach reflection on the upper boundary.

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References

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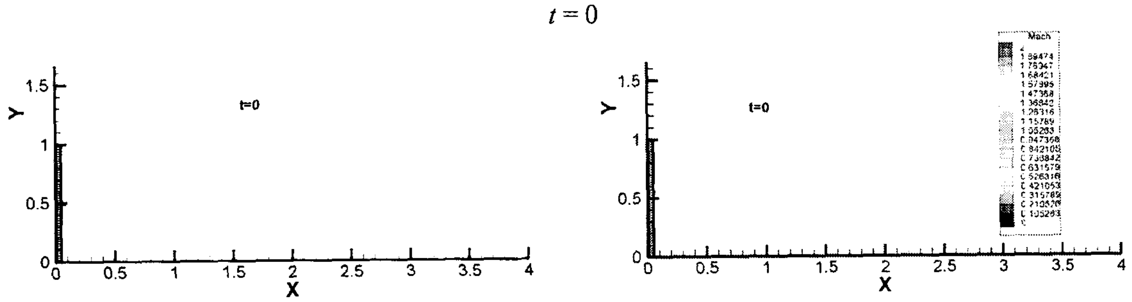
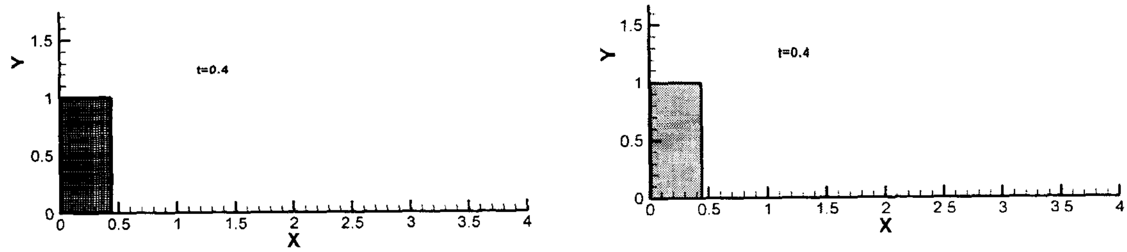
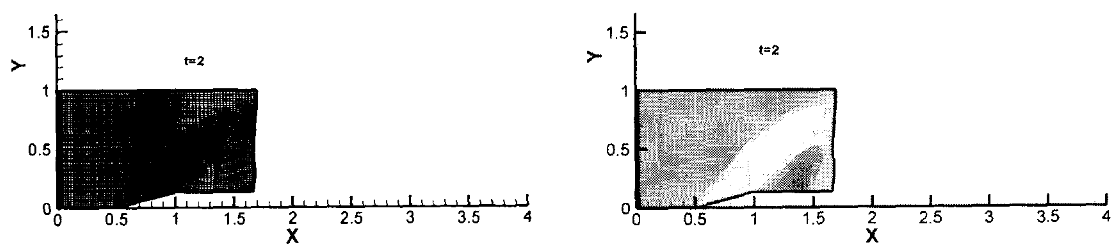
$t = 0$



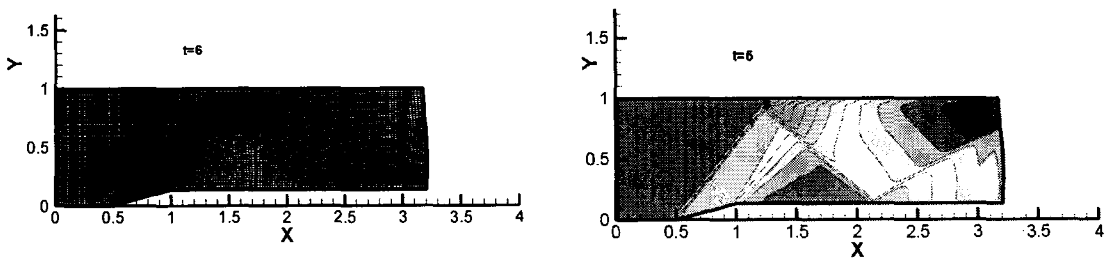
$t = 0.4$



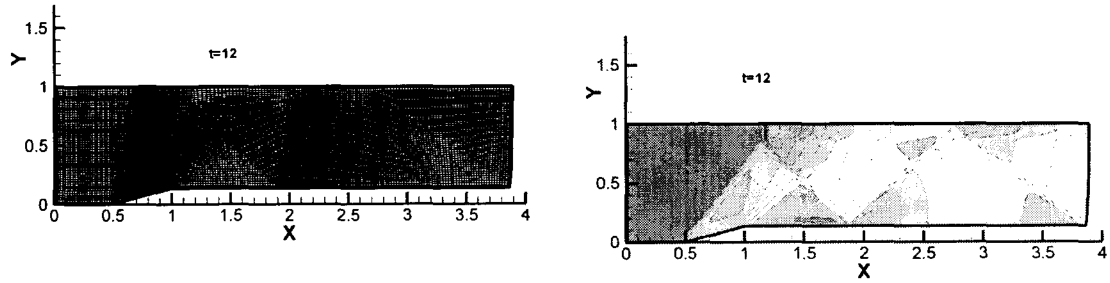
$t = 2.0$



$t = 6.0$



$t = 12$



$t = 17$

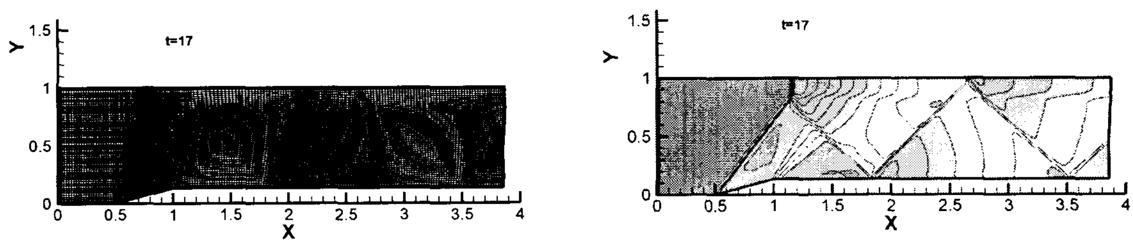


Fig. 1 Supersonic flow in a channel showing Mach reflection, $M_\infty = 1.8$.