BAYESIAN INFERENCE FOR MTAR MODEL WITH INCOMPLETE DATA

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Abstract

A momentum threshold autoregressive (MTAR) model, a nonlinear autoregressive model, is analyzed in a Bayesian framework. Parameter estimation in the presence of missing data is done by using Markov chain Monte Carlo methods. We also propose simple Bayesian test procedures for asymmetry and unit roots. The proposed method is applied to a set of Korea unemployment rate data and reveals evidence for asymmetry and a unit root.

Keywords. Asymmetry; Markov chain Monte Carlo; Unit root.

1 Introduction

One important research area in modern time series analysis is the class of nonlinear models which is useful for representing several important aspects of time series data. One of such nonlinear aspect is dynamic asymmetry which states different dynamic properties depending on status of the process. Such asymmetry is well represented by MTAR models for which Enders and Granger(1998), Caner and Hansen(2001), and Shin and Lee(2001) considered classical regression approaches and Koop and Potter(1999) considered a Bayesian scheme. These MTAR models consist of two different autoregressive regimes depending on whether the time series process is increasing or decreasing at each time. The MTAR models prove to be useful in explaining asymmetries and nonstationarities of many economic and finance time series such as unemployment rates, interest rates, GNP, productions, etc. See the above mentioned works and references therein.

The methods developed by the above authors are based on complete data sets. However, it is very common that time series data are not complete. There are many sources which induce incomplete time series data such as missing, outlier removal, and change of sampling scheme. For example, it is very common that a quarterly sampling scheme is enhanced to a monthly scheme so that the initial quarterly data set contains unobserved data if the data set is regarded as a monthly data set.

Now, it would be important to develop a statistical method for MTAR models which admit incomplete data. We are specially interested in developing inference methods for the major issues of tests for symmetry and tests for unit roots. There are two strategies for handling incomplete data in linear ARMA time series models, that is, classical methods and Bayesian methods. Various classical methods were proposed by Ansley and Kohn(1983), Dunsmuir and Robinson(1981), Wincek and Reinsel(1986), Shin and Sarkar(1995), and many others. Some Bayesian methods were considered by McCulloch and Tsay(1994). Compared to the classical methods, the Bayesian methods have simpler distributional properties of test procedures. For analyzing MTAR models,

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the Bayesian methods seem to be more promising because the classical methods would no longer retain computational simplicity due to nonlinearity.

In this article, we propose a Bayesian approach to parameter estimation in the MTAR model with incomplete data by using MCMC. We recast tests of asymmetry and unit roots as Bayesian model selection problems and propose a simple and efficient method to compute Bayes factors by using the outputs from MCMC. The proposed method is applied to a set of Korea unemployment rate. The study reveals dynamic asymmetry and a unit root of the data generating process.

The remainder of the paper is organized as follows. The model and prior specifications are described in Section 2. A Bayesian estimation scheme and tests of hypotheses for asymmetry and unit roots are developed in Section 3. A practical example is illustrated in Section 4.

2 Model and Prior Specification

We consider an MTAR model defined as

$$\Delta y_t = \begin{cases} \rho_1(y_{t-1}-\mu_{1t}) + \alpha_{11}\Delta y_{t-1} + \cdots + \alpha_{1p}\Delta y_{t-p} + a_t \text{ if } I_t = 1 \\ \rho_2(y_{t-1}-\mu_{2t}) + \alpha_{21}\Delta y_{t-1} + \cdots + \alpha_{2p}\Delta y_{t-p} + a_t \text{ if } I_t = 0, \end{cases}$$

where $\Delta y_t = y_t - y_{t-1}$, $I_t = I[\Delta y_{t-1} > 0]$ is the indicator function for the event $\Delta y_{t-1} > 0$, $(\rho_1, \rho_2, \alpha_1, \alpha_2, \mu_1, \mu_2, \sigma^2)$ are unknown model parameters, $\alpha_i = (\alpha_{i1}, \cdots, \alpha_{ip})', i = 1, 2$, a_t is the sequence of independent normal errors having zero mean and variance σ^2, and μ_{it} is a mean function. Observations y_t are made for the time span $t = 1, 2, ..., T$. Let $\mathcal{M}$ be a subset of $\{1, 2, ..., T\}$ on which y_t is unobserved so that the observed data set is $\{y_t, t = 1, ..., T, t \notin \mathcal{M}\}$. In case of missing data, we assume missing completely at random (MCAR) or missing at random (MAR) in the sense of Rubin(1976) for the missing mechanism. We develop methods assuming simple means $\mu_{it} = \mu_i$. However, given the methods, an extension to the trend model with $\mu_{it} = \beta_{i0} + \beta_{i1}t$ would be straightforward.

Let $\eta_i = -\rho_i\mu_i, i = 1, 2$. For implementing a Bayesian method, we use independent conjugate priors for all unknown model parameters so that $(\rho_1, \rho_2, \alpha_1, \alpha_2, \eta_1, \eta_2, \sigma^2)$ are independently distributed as

$$\rho_i \sim N(\rho_i^0, \sigma_{\rho_i}^2),\ \alpha_i \sim N_p(\alpha_i^0, \Sigma_{\alpha_i}),\ \eta_i \sim N(\eta_i^0, \sigma_{\eta_i}^2), i = 1, 2,\ \sigma^2 \sim IG(\gamma, \delta) \qquad (1)$$

for some hyperparameters $\rho_i^0, \sigma_{\rho_i}^2, \alpha_i^0, \Sigma_{\alpha_i}, \eta_i^0, \sigma_{\eta_i}^2, \gamma, \delta$, where N, N_p, and IG denote normal, p-dimensional normal, and inverse gamma distributions, respectively.

We adopt the empirical Bayes approach for the hyperparameters so that they are determined from the data. A simple initial value can be obtained from the symmetric model, i.e., the AR model. We set $\mu_1^0 = \mu_2^0 = \bar{y}$, where $\bar{y}$ is the sample mean of y_t. We set $(\rho_1^0, \alpha_1^0) = (\rho_2^0, \alpha_2^0)$ to the estimators of $(\rho_1, \alpha_1) = (\rho_2, \alpha_2)$ in the OLS-fitting to the AR model, i.e., in the regression Δy_t on $(y_{t-d} - \bar{y}), \Delta y_{t-1}, \ldots, \Delta y_{t-p}$ for all t such that all terms $\Delta y_t, y_{t-1}, \Delta y_{t-1}, \ldots, \Delta y_{t-p}$ are available. The variance parameters $\sigma_{\rho i}^2, \Sigma_{\alpha i}$ are set to those obtained from the estimated variances of the OLSE ρ_i^0, α_i^0 multiplied by a positive constant τ. If we want vague priors, we choose τ greater than one. Let σ^{02} be the error variance estimator. We let $\gamma = 3/2, \delta = \sigma^{02}/2$ so that the prior mean of σ^2 is σ^{02}. We finally let $\eta_i^0 = -\rho_i^0\mu_i^0$. In order to initiate the Gibbs sampler, we need to specify initial values of the unobserved values $y_t, t \in \mathcal{M}$, for which we use the simple average of two adjacent available observations.

3 A Bayesian Method

We first describe the full conditional distribution of each of the unknown parameters as well as the unobserved values. We later provide a complete implementation of a Gibbs sampler(Gelfand and Smith,1990) for parameter estimation, and posterior probabilities of hypotheses for asymmetry and unit roots.

Let $Z_t = \Delta y_t - X_{1t}I_{1t} - X_{2t}I_{2t}$ where $I_{1t} = I_t, I_{2t} = 1 - I_t$, and $X_{it} = \rho_i(y_{t-1} - \mu_i) + \alpha_{i1}\Delta y_{t-1} + \cdots + \alpha_{ip}\Delta y_{t-p}$, $i = 1, 2$. Let $Y = (y_1, ..., y_T)'$ and $\theta = (\rho_1, \rho_2, \alpha_1, \alpha_2, \eta_1, \eta_2, \sigma^2)'$. Let t_0 be the first t such that both Δy_{t-1} and Δy_{t-p} are available. The joint density of θ and Y is proportional to $[\Pi_{i=1}^2 \pi(\rho_i)\pi(\alpha_i)\pi(\eta_i)]\pi(\sigma^2)exp(-\sum_{t=t_0}^T Z_t^2/(2\sigma^2))$ where $\pi(\rho_i), \pi(\alpha_i), \pi(\eta_i)$, and $\pi(\sigma^2)$ are the prior deisnities defined in (1).

Note that, as a function of ρ_i, $\sum_{t=t_0}^T Z_t^2 = \sum_{t=t_0}^T(\rho_i^2 y_{t-1}^2 I_{it} - 2\rho_i y_{t-1}e_{\rho_{it}}I_{it}) + constant$, where $e_{\rho_{it}} = \Delta y_t - (\eta_i + \sum_{j=1}^p \alpha_{ij}\Delta y_{t-j})$. Hence, investigating the joint density of θ and Y as a function of ρ_i, we find that the full conditional distribution of ρ_i given all the other elements of θ and Y is

$$(\rho_i \mid all\ others) \sim N(\mu_{\pi\rho_i}, \sigma^2_{\pi\rho_i})$$

where $\mu_{\pi\rho_i} = \sigma^2_{\pi\rho_i}(\frac{1}{\sigma^2}\sum_{t=t_0}^T y_{t-1}e_{\rho_{it}}I_{it} + \frac{\rho_i^0}{\sigma^2_{\rho_i}})$, $\sigma^2_{\pi\rho_i} = (\frac{1}{\sigma^2}\sum_{t=t_0}^T y_{t-1}^2 I_{it} + \frac{1}{\sigma^2_{\rho_i}})^{-1}$. As a function of α_i, $\sum_{t=t_0}^T Z_t^2 = \alpha_i'(\sum_{t=t_0}^T Y_tY_t'I_{it})\alpha_i - 2\alpha_i'\sum_{t=t_0}^T Y_t e_{\alpha_{it}}I_{it} + constant$ where $Y_t = (\Delta y_{t-1}, ..., \Delta y_{t-p})'$, $e_{\alpha_{it}} = \Delta y_t - (\rho_i y_{t-1} + \eta_i)$. Therefore, the full conditional distribution of α_i is given by

$$(\alpha_i \mid all\ others) \sim N_p(\mu_{\pi\alpha_i}, \Sigma_{\pi\alpha_i}),$$

where $\mu_{\pi\alpha_i} = \Sigma_{\pi\alpha_i}\{\frac{1}{\sigma^2}\sum_{t=t_0}^T Y_t e_{\alpha_{it}}I_{it} + \Sigma_{\alpha_i}^{-1}\alpha_i^0\}$, $\Sigma_{\pi\alpha_i} = \{\frac{1}{\sigma^2}\sum_{t=t_0}^T Y_tY_t'I_{it} + \Sigma_{\alpha_i}^{-1}\}^{-1}$. Also, as a function of η_i, $\sum_{t=t_0}^T Z_t^2 = \sum_{t=t_0}^T(\eta_i^2 I_{it} - 2\eta_i e_{\eta_{it}}I_{it}) + constant$, where $e_{\eta_{it}} = \Delta y_t - (\rho_i y_{t-1} + \sum_{j=1}^p \alpha_{ij}\Delta y_{t-j})$, hence we have

$$(\eta_i \mid all\ others) \sim N(\mu_{\pi\eta_i}, \sigma^2_{\pi\eta_i}),$$

where $\mu_{\pi\eta_i} = \sigma^2_{\pi\eta_i}(\frac{1}{\sigma^2}\sum_{t=t_0}^T e_{\eta_{it}}I_{it} + \frac{\eta_i^0}{\sigma^2_{\eta_i}})$, $\sigma^2_{\pi\eta_i} = (\frac{1}{\sigma^2}\sum_{t=t_0}^T I_{it} + \frac{1}{\sigma^2_{\eta_i}})^{-1}$. Finally, the full conditional distribution of σ^2 is specified by

$$(\sigma^2 \mid all\ others) \sim IG(\tfrac{1}{2}(T - t_0 + 1) + \gamma, \tfrac{1}{2}\sum_{t=t_0}^T Z_t^2 + \delta).$$

In order to characterize the full conditional distribution of $y_t, t \in \mathcal{M}$, we investigate $\sum_{s=t_0}^T Z_s^2$ as a function of y_t. Some algebra yields that, as a function of y_t,

$$\sum_{s=t_0}^T Z_s^2 = \sum_{i=1}^2[\{1 + (1 + \rho_i + \alpha_{i1})^2 + \sum_{k=2}^p(\alpha_{ik} - \alpha_{i,k-1})^2 + \alpha_{ip}^2\}y_t^2 I_{it} - 2\{g_{i0t} + (1 + \rho_i + \alpha_{i1})g_{i1t} + \sum_{k=2}^p(\alpha_{ik} - \alpha_{i,k-1})g_{ikt} - \alpha_{ip}g_{i,p+1,t}\}y_t I_{it}] + constant, \quad (2)$$

where $g_{i0t} = (1 + \rho_i)y_{t-1} + \eta_i + \sum_{k=1}^p \alpha_{ik}\Delta y_{t-k}$, $g_{i1t} = y_{t+1} - \{\eta_i + \sum_{k=1}^p \alpha_{ik}\Delta y_{t+1-k} - \alpha_{i1}y_t\}$, $g_{ikt} = y_{t+k} - \{(1 + \rho_i)y_{t+k-1} + \eta_i + \sum_{j=1}^p \alpha_{ij}\Delta y_{t+k-j} - (\alpha_{ik} - \alpha_{i,k-1})y_t\}, k = 2, ..., p$, $g_{i,p+1,t} = y_{t+p+1} - \{(1+\rho_i)y_{t+p} + \eta_i + \sum_{k=1}^p \alpha_{ik}\Delta y_{t+p+1-k} + \alpha_{ip}y_t\}$. Note that, owning to the term I_{it}, (2) cannot be given in a form $a(y_t - b)^2 + constant$ for a, b free from t. Therefore, the full conditional density of y_t is not given in a simple known density form. Hence, we use a Metropolis-Hastings algorithm(Hastings,1970) for sample generation of y_t. For each $t \in \mathcal{M}$, we generate a y_t from the full conditional distribution of y_t given all others according to the following steps:

Step 1. Start with an initial value of y_t.

Step 2. Generate y_t^* from a density function $r(\cdot)$.

Step 3. Replace y_t by y_t^* with probability $min\left[\frac{f(y_t^*)/f(y_t)}{r(y_t^*)/r(y_t)}, 1\right]$, where $f(\cdot)$ is the full conditional density of y_t given all others.

Step 4. Repeat Step 2 - Step 3 until convergence.

Note that $f(y_t) \propto exp(-\sum_{s=t_0}^{T} Z_s^2/(2\sigma^2))$. Therefore, $f(y_t^*)/f(y_t)$ can be easily computed from (2) because the multiplicative constant for $f(\cdot)$ needs not be specified.

Even though $r(\cdot)$ in Step 2 can be any density function, it should be convenient for simple random generation of y_t^* and should be close to the real conditional density of y_t for efficiency of the algorithm. A good compromise between simplicity and efficiency would be the posterior density of y_t under the assumption of no threshold effect, i.e., under $\rho_1 = \rho_2 = \rho$, say, $\alpha_{1k} = \alpha_{2k} = \psi_k$, say, $k = 1, ..., p$, under which $r(\cdot)$ is a simple normal density. Investigating (2) under this symmetry condition renders us to choose $r(\cdot)$ as the density of $N(\mu_{\pi m}, \sigma_{\pi m}^2)$, where $\mu_{\pi m} = \sigma_{\pi m}^2[(\sigma^2)^{-1}\{g_{0t}+(1+\rho+\psi_1)g_{1t}+\sum_{k=2}^{p}(\psi_k-\psi_{k-1})g_{kt}-\psi_p g_{p+1,t}\}]$, $\sigma_{\pi m}^2 = [(\sigma^2)^{-1}\{(1+(1+\rho+\psi_1)^2+\sum_{k=2}^{p}(\psi_k-\psi_{k-1})^2+\psi_p^2)\}]^{-1}$, and g_{kt} is the common $g_{1kt} = g_{2kt}$ under symmetry.

Now the Gibbs sampler generates a sequence of random sample of each component of θ and $y_t, t \in \mathcal{M}$, according to the full conditional distributions and the Metropolis-Hastings algorithm stated above. The Gibbs sampler first performs temporary burn-in iterations until the generated samples achieve stationarity. Later it is iterated $N \times \kappa$ times, where N and κ are positive integers. In order to obtain independence of the generated Gibbs samples, every κ-th, $\kappa > 1$, sample is taken to produce Gibbs samples $\hat{\theta}_\ell$ and $\hat{y}_{t\ell}, \ell = 1, 2, ..., N$ which form a basis for our Bayesian analysis. The Bayesian estimate of θ is $\hat{\theta} = N^{-1}\sum_{\ell=1}^{N}\theta_\ell$ and the Bayesian prediction of the unobserved value y_t is $\hat{y}_t = N^{-1}\sum_{\ell=1}^{N} y_{t\ell}, t \in \mathcal{M}$. Bayesian standard errors of $\hat{\theta}$ and $\hat{y}_t$ are the standard deviations of the Gibbs samples $\{\theta_\ell, \ell = 1, ..., N\}$ and $\{y_{t\ell}, \ell = 1, ..., N\}$, respectively.

Moreover, posterior probabilities of some important hypotheses can be computed from the outputs of MCMC as described in more detail below.

The hypotheses related with asymmetry are $H_0 : \rho_1 \neq \rho_2, \alpha_1 \neq \alpha_2, H_1 : \rho_1 = \rho_2, \alpha_1 \neq \alpha_2, H_2 : \rho_1 \neq \rho_2, \alpha_1 = \alpha_2, H_3 : \rho_1 = \rho_2, \alpha_1 = \alpha_2$. According to Oh(1999), we can compute the posterior probability of each hypothesis in the following way.

$P(H_0|Y) = (1 + \sum_{i=1}^{3} p_i)^{-1}, \;\; P(H_i|Y) = P(H_0|Y)p_i \;, \; i = 1, 2, 3,$

where $p_1 = \pi(\rho_1{=}\rho_2|Y)/\pi(\rho_1{=}\rho_2)$, $p_2 = \pi(\alpha_1{=}\alpha_2|Y)/\pi(\alpha_1{=}\alpha_2)$, $p_3 = \pi(\rho_1{=}\rho_2, \alpha_1{=}\alpha_2|Y)/\pi(\rho_1{=}\rho_2, \alpha_1{=}\alpha_2)$. Values of the posterior densities are computed from the full conditional densities as

$\pi(\rho_1 = \rho_2|Y) = \frac{1}{N}\sum_{\ell=1}^{N}\pi_{\rho_1}^{\ell}(\rho_{2\ell}), \;\; \pi(\alpha_1 = \alpha_2|Y) = \frac{1}{N}\sum_{\ell=1}^{N}\pi_{\alpha_1}^{\ell}(\alpha_{2\ell}),$

$\pi(\rho_1 = \rho_2, \alpha_1 = \alpha_2|Y) = \frac{1}{N}\sum_{\ell=1}^{N}\pi_{\rho_1}^{\ell}(\rho_{2\ell})\pi_{\alpha_1}^{\ell}(\alpha_{2\ell}|\rho_1 = \rho_{2\ell}),$

where $\pi_{\rho_1}^{\ell}(\rho_{2\ell})$ is the value of the full conditional density of ρ_1 evaluated at the ℓ−th MCMC sample $\theta_\ell = (\rho_{1\ell}, \rho_{2\ell}, \alpha_{1\ell}, \alpha_{2\ell}, \eta_{1\ell}, \eta_{2\ell}, \sigma_\ell^2)'$ with $\rho_{1\ell}$ replaced by $\rho_{2\ell}$, $\pi_{\alpha_1}^{\ell}(\alpha_{2\ell})$ is the value of the full conditional density of α_1 evaluated at θ_ℓ with $\alpha_{1\ell}$ replaced by $\alpha_{2\ell}$, and $\pi_{\alpha_1}^{\ell}(\alpha_{2\ell}|\rho_1 = \rho_{2\ell})$ is the value of the full conditional density of α_1 evaluated at θ_ℓ with $(\rho_{1\ell}, \alpha_{1\ell})$ replaced by $(\rho_{2\ell}, \alpha_{2\ell})$. Values of the prior densities $\pi(\rho_1 = \rho_2), \pi(\alpha_1 = \alpha_2), \pi(\rho_1 = \rho_2, \alpha_1 = \alpha_2)$ are computed in the same way as those for the posterior densities if the posterior means and variances are replaced by the corresponding prior means and variances.

For the unit root hypotheses $H_0' : \rho_1 \neq 0, \rho_2 \neq 0, H_1' : \rho_1 = 0, \rho_2 \neq 0, H_2' : \rho_1 \neq 0, \rho_2 = 0, H_3' : \rho_1 = 0, \rho_2 = 0$, the posteror probabilities $P(H_0'|Y), P(H_1'|Y), P(H_2'|Y), P(H_3'|Y)$ are computed in the same way as those $P(H_0|Y), P(H_1|Y), P(H_2|Y), P(H_3|Y)$ stated above if densities related with $(\rho_1 = \rho_2), (\alpha_1 = \alpha_2), (\rho_1 = \rho_2, \alpha_1 = \alpha_2)$ are replaced by those related with $(\rho_1 = 0), (\rho_2 = 0), (\rho_1 = 0, \rho_2 = 0)$, respectively. For example, $\pi(\rho_1 = \rho_2|Y)$ is replaced by $\pi(\rho_1 = 0|Y) = \frac{1}{N}\sum_{\ell=1}^{N}\pi_{\rho_1}^{\ell}(0)$.

Table 1: Information Criteria

Dataset	p	0	1	2	3	4	5	6	7	8
Full	AIC	-1280.3	-1286.3	-1263.8	-1282.3	-1267.2	-1282.3	-1277.9	-1278.4	-1272.4
	BIC	-1272.4	-1270.5	-1240.2	-1250.8	-1227.9	-1235.1	-1222.9	-1215.6	-1201.8
Quarterly	AIC	-252.3	-263.9	-257.7	-254.7	-261.0	-261.4	-254.8	-248.3	-257.0
	BIC	-246.6	-252.6	-240.8	-232.2	-232.9	-227.8	-215.7	-203.9	-207.1

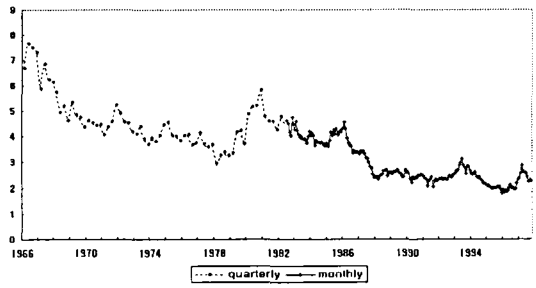
4 An Example

We apply the proposed Bayesian method for analyzing a series of seasonally adjusted Korea unemployment rate for the period of Jan. 1966 to Oct. 1997 depicted in Figure 1. Until Jun. 1982, observations are obtained quarterly and, from that time on, observations are made monthly. If the series is regarded as a monthly data set, unobserved values are present for the period of Jan. 1966 -Jun. 1982. The ratio of the total number of unobserved points relative to the total number of observations is 132/382=0.35. For comparative purposes, we also analyze the quarterly data set for the whole period of 1966 - 1997.

Table 2: Bayesian estimates and their standard errors

Parameter	Full	Quarterly
ρ_1	-0.009(0.005)	-0.077(0.017)
ρ_2	-0.022(0.005)	-0.083(0.016)
α_{11}	-0.102(0.048)	-0.168(0.075)
α_{21}	-0.109(0.048)	-0.261(0.073)
σ^2	0.033(0.003)	0.114(0.015)
η_1	0.054(0.015)	0.251(0.064)
η_2	0.047(0.014)	0.244(0.083)

Figure 1: Korea Unemployment Rate



The autoregressive order p of the MATR model is chosen by investigating the AIC and the BIC defined by AIC=$n\log(\hat{\sigma}^2)+4(p+1)$ and BIC=$n\log(\hat{\sigma}^2)+2(p+1)\log(n)$, where $\hat{\sigma}^2$ is the Bayesian estimate obtained by the method in Section 3 and n is the effective number of observations. For the quarterly data set, $n = 127 - 2 = 125$ and for the monthly data set $n = 382 - 2 = 380$. Table 1 presents values of the AIC and the BIC for $p = 0, ..., 8$. For the full data set, we select $p = 1$ because the AIC takes minimum value and the BIC takes nearly minimum value at $p = 1$. For the quarterly data set, we choose also $p = 1$ because both the AIC and the BIC take minimum values at $p = 1$.

The parameters are estimated from a set of MCMC samples obtained from $N \times \kappa = 30,000 \times 3 = 90,000$ iterations after 5,000 iterations of burn-in. The normal and inverse gamma random variables are generated by FORTRAN subroutines RNNOR and RNGAM from IMSL(1989).

Figure 2 shows sample path of generated parameter samples $\{\theta_\ell, \ell = 1, ..., N\}$, which indicate convergence has been achieved. Parameter estimates are given in Table 2. Looking into the standard errors in the parentheses, we see that those of parameter estimates obtained from the full data set are substantially smaller than those from the quarterly data set. This implies that, compared to

Figure 2: Time sequence plot of parameter estimates(full data)

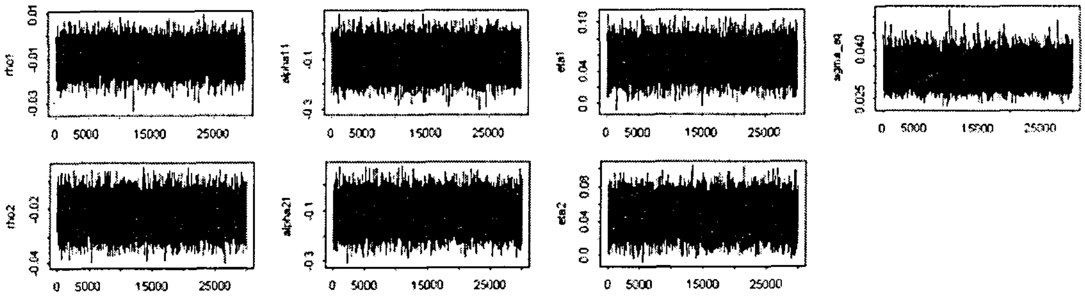


Table 3: Posterior probabilities of hypotheses of asymmetry and unit roots

Hypothesis		Full	Quarterly	Hypothesis		Full	Quarterly
$H_0 : \rho_1 \neq \rho_2$ & $\alpha_1 \neq \alpha_2$	:	0.350	0.233	$H_0' : \rho_1 \neq 0$ & $\rho_2 \neq 0$	:	0.382	0.993
$H_1 : \rho_1 = \rho_2$ & $\alpha_1 \neq \alpha_2$	:	0.133	0.305	$H_1' : \rho_1 = 0$ & $\rho_2 \neq 0$	:	0.618	0.007
$H_2 : \rho_1 \neq \rho_2$ & $\alpha_1 = \alpha_2$	:	0.371	0.193	$H_2' : \rho_1 \neq 0$ & $\rho_2 = 0$	:	0.000	0.000
$H_3 : \rho_1 = \rho_2$ & $\alpha_1 = \alpha_2$	:	0.145	0.270	$H_3' : \rho_1 = 0$ & $\rho_2 = 0$	:	0.000	0.000

the quarterly data set, the extra data other than the quarterly data points in the full data set provide us much more information, yielding much more precise parameter estimates.

The two data sets show somewhat different shapes in asymmetry features. For the full data set, asymmetry is mainly represented in different values of (ρ_1, ρ_2) and (μ_1, μ_2). On the other hand, for the quarterly data set, asymmetry is mainly presented in different values of $(\alpha_{11}, \alpha_{21})$. For the full data set, estimates of (ρ_1, ρ_2) are more close to zero and more different from each other and $\hat{\mu}_1 = 0.054/0.009 = 6.000$ and $\hat{\mu}_2 = 0.047/0.022 = 2.136$ are more different from each other than $\hat{\mu}_1 = 3.260$ and $\hat{\mu}_2 = 2.940$ for the quarterly data set,.

These asymmetry features are investigated in more formal manner as hypothesis testing. The hypothesis $H_3 : \rho_1 = \rho_2$ & $\alpha_1 = \alpha_2$ in Table 3 corresponds to the symmetric model and the others corresponds to the asymmetric models. We see that the full data set gives a stronger evidence for asymmetry in (ρ_1, α_{11}) and (ρ_2, α_{21}) because the posterior probability of symmetry are 0.145 and 0.270 for the full data set and quarterly data set, respectively. The analysis with both full and quarterly data sets reveal some evidences for asymmetry. However, those evidences come from different hypotheses. For the full data set, the hypotheses $H_0 : \rho_1 \neq \rho_2$ & $\alpha_1 \neq \alpha_2$ and $H_2 : \rho_1 \neq \rho_2$ & $\alpha_1 = \alpha_2$ have probabilities 0.350 and 0.371 respectively, summing up to 0.721, the posterior probability of $\rho_1 \neq \rho_2$. On the other hands, for the quarterly data set, the hypotheses $H_0 : \rho_1 \neq \rho_2$ & $\alpha_1 \neq \alpha_2$ and $H_1 : \rho_1 = \rho_2$ & $\alpha_1 \neq \alpha_2$ have probabilities 0.233 and 0.305 respectively, summing up to 0.538, the posterior probability of $\alpha_{11} \neq \alpha_{21}$.

We next investigate results for unit roots hypotheses. According to Table 3, when we use the full data set, the probability supporting the hypothesis of no unit roots is just 0.382. However,when we use quarterly data, the posterior probability for H_0' is 0.993, much larger than 0.382. Therefore, the full data set provides stronger evidence for unit roots than the quarterly data set. According to the analysis of the full data set, the model with partial unit root, i.e. $(\rho_1 = 0, \rho_2 \neq 0)$ is the most probable one.

REFERENCES

Ansley, C. F. and Kohn, R. (1983), Exact likelihood of vector autoregressive-moving average process with missing or aggregated data, *Biometrika*, 70, 275-278.

Caner, M. and Hansen, B. E. (2001), Threshold autoregression with a unit root, *Econometrica*, 69, 1555-1596.

Dunsmuir, W. and Robinson, P. M. (1981), Estimation of time series models in the presence of missing data, *Journal of the American Statistical Association*, 76, 560-568.

Enders, W. and Granger, C. W. J. (1998), Unit root tests and asymmetric adjustment with an example using the term structure of interest rates, *Journal of Business & Economic Statistics*, 16, 304-311

Gelfand, A. E. and Smith, A. F. M. (1990), Sampling-based approaches to calculating marginal densities, *Journal of the American Statistical Association*, 85, 398-409

Hastings, W. K. (1970), Monte Carlo sampling methods using Markov chains and their applications, *Biometrika*, 57, 97-109.

IMSL (1989). User's Manual, IMSL, Houston, Texas.

Koop, G. and Potter, S. M. (1999), Dynamic asymmetries in U.S. unemployment, *Journal of Business & Economic Statistics*, 17, 298-312.

McCulloch, R. E. and Tsay, R. S. (1994), Bayesian analysis of autoregressive time series via the Gibbs sampler, *Journal of Time Series Analysis*, 15, 235-250.

Oh, M. S. (1999), Estimation of posterior density functions from a posterior sample, *Computational Statistics & Data Analysis*, 29, 411-427.

Rubin, D. B. (1976), Inference and missing data, *Biometrika*, 63, 581-592.

Shin, D. W. and Sarkar, S. (1995), Estimation for stationary AR(1) models with nonconsecutively observed samples, *Sankhya: the Indian Journal of Statistics. Series A* , 57, 287-298.

Shin, D. W. and Lee, O. S. (2001), Tests for asymmetry in possibly nonstationary time series data, *Journal of Business & Economic Statistics*, 19, 233-244.

Wincek, M. A. and Reinsel, G. C. (1986), An exact maximum likelihood estimation procedure for regression ARMA time series models with possibly nonconsecutive data. *Journal of Royal Statistical Society*, 48, 303-313.