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High Resolution Frequency Estimation of Real Sinusoids

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Abstract - In this paper, we propose a new high resolution frequency estimator for real sinusoids by using short time data and the AWLS/MFT (Adaptive Weighted Least Squares/ Modulation Function Technique) algorithm. Monte-Carlo simulations verify better performances of the proposed frequency estimator and demonstrate that the proposed AWLS sinusoidal estimator is a high resolution estimator.

1. Introduction

High resolution estimation has been investigated in many ways during the last three decades both in parametric and nonparametric algorithms. In the literature, the high resolution frequency estimator is defined by its ability to resolve two closely located frequency components, i.e., two frequencies whose separation (Δf) is less than 1/T [14] [2]. Some approaches, like the maximum likelihood estimation, nonlinear optimization algorithm, etc., work well in the low SNR situation [14] [15] [16]. However, they require either long data lengths or sophisticated computing and time costly algorithms. The high-order Yule-Walker (HOYW) algorithm was proposed for improving the frequency resolution [1]. But the problem is that sometimes it is difficult to separate "spurious zeros"from the signal frequencies. Recently, several adaptive algorithms have been developed for frequency estimation. Etter and Hush [8] have proposed a computationally efficient algorithm for nonstationary frequency estimation. The concept involves maximizing the mean square difference between the input sequence and its delayed version using the adaptive time delay estimator [9]. An improvement upon this method that provides more accurate frequency estimates has been suggested [10] with the use of Lagrange interpolation. Another adaptive algorithm (Direct Frequency Estimator) has been proposed for tracking the frequency of a real sinusoid embedded in white noise with known variance [12]. Using a least mean square (LMS) style algorithm [11], the frequency was adjusted directly on a sample by sample basis.

Meanwhile, the least squares (LS/MFT) algorithm in continuous time using a Shinbrot-type Fourier modulation function was devised by Pan [4] and applied for the frequency estimation of real sinusoids in white noise. Compared with the HOYW equation, LS/MFT gave better performance, especially for short time observation and low SNR data. For a more stochastic signal model with additive measurement noise, the adaptive weighted least squares (AWLS/MFT) [5] in continuous time was devised which shows better performance in system identification compared with LS/MFT and prediction error method (PEM).

Here, we devise a high resolution frequency estimator in continuous time employing the AWLS algorithm which includes the DC component in the covariance matrix for better performance. We will compare the estimation performance of the AWLS frequency estimator and the LS frequency estimator [4].

The paper is organized as follows. First the problem is stated. Then the AWLS sinusoidal estimator for the frequency estimation of real sinusoids is derived in Section III. Section IV presents simulation results. Finally, concluding remarks are given in Section V.

2. Problem Formulation

We focus on the problem of estimating the angular frequencies $\omega_1, \omega_2, \cdots, \omega_K$ for a given superimposed real sinusoidal signal over a finite time interval [0, T]:

$$y(t) = x(t) + v(t) \tag{1}$$

where $$x(t) = \sum_{i=1}^{K} A_i \cos(\omega_i t + \varphi_i) \tag{2}$$

In equations (1)-(2), $A_i > 0$, $\omega_i \in (0,\pi)$, and $\omega_i \neq \omega_j$. K is the number of superimposed sinusoids and is assumed to be known a priori. A_i and φ_i are unknown amplitudes and phases of the signal $x(t)$. The measurement signal $y(t)$ is corrupted with noise $v(t)$, which is a stationary white Gaussian noise and has zero mean and variance σ^2. It is assumed that $x(t)$ and $v(s)$ are uncorrelated for all t and s.

To introduce the MFT, define a set of the nth order complex Fourier type modulating function [6]:

$$\phi_m(t) = \frac{1}{\sqrt{T}} e^{-im\omega_0 t}(e^{-i\omega_0 t} - 1)^n, \quad m = 0, 1, \cdots, M, \quad 0 \le t \le T \tag{3}$$

where ω_0 is the resolving frequency defined as $\omega_0 = 2\pi / T$, T is the time interval of the data block, and M is an integer for controlling the highest frequency and number of algebraic equations. Each $\phi_m(t)$ satisfies the end point conditions: $p^k \phi_m(t)|_{t=0} = 0$,

$$p^k \phi_m(t)|_{t=T} = 0, \quad k = 0, 1, \cdots, (n-1), \quad p = \tfrac{d}{dt} \tag{4}$$

Using the binomial expansion, $\phi_m(t)$ can be written as:

$$\phi_m(t)=\frac{1}{\sqrt{T}}\sum_{k=0}^{n}c_k e^{-i(m+k)\omega_0 t}, \quad c_k=(-1)^{n-k}\binom{n}{k} \tag{5}$$

Then define a Shinbrot-type moment functional [3] of order n given $x(t)$ on [0, T]:

$$f_m(x)=\int_0^T \phi_m(t)x(t)dt=\sum_{k=0}^{n}c_k X[m+k] \tag{6}$$

where $X[k]=\frac{1}{\sqrt{T}}\int_0^T x(t)e^{-ik\omega_0 t}dt$ is the Fourier coefficient of $x(t)$ at frequency $k\omega_0$.

If $P(p)$ is any polynomial of degree n (or less) in the differential operator $p=d/dt$ and if $x(t)$ is any n-times differentiable function on [0, T] or n-times mean-square differentiable in the case of stochastic signals, then as stated in [3]:

$$f_m(P(p)x)=\sum_{k=0}^{n}c_k P[m+k]X[m+k] \tag{7}$$

where $P[k]=P(ik\omega_0)$.

One of main advantages of using the MFT is that we can handle continuous time models directly, which avoids the potentially significant errors in approximating derivatives from noisy input and output signals. Another advantage of using the MFT is that it is not necessary to deal with the complicated initial value problem as explained above.

III. AWLS Sinusoidal Estimator

It is well known that a real signal $x(t)=\sum_{i=1}^{K}A_i\cos(\omega_i t+\varphi_i)$ obeys a homogeneous continuous time autoregressive (AR) differential equation of order 2K [4]. The coefficients of this model only depend on the angular frequencies and neither on the amplitudes nor phases. The noise model can be expressed by

$$E\{v(t)\}=0, \qquad E\{v(t)v(t+\tau)\}=\sigma^2\delta(\tau) \tag{8}$$

where E denotes the statistical expected value operator and $\delta(\tau)$ is the Dirac delta function. We note that the signal $x(t)$ satisfies a linear, homogeneous differential equation with constant coefficients.

Define:

$$\Pi(p)=\prod_{i=1}^{K}(p^2+\omega_i^2)=p^{2K}+a_{K-1}p^{2(K-1)}+\cdots+a_1p^2+a_0 \tag{9}$$

where $p=d/dt$ is the differential operator, and

$$a_{K-1}=\sum_{j=1}^{K}\omega_j^2, \quad a_{K-2}=\sum_{j\neq k}\omega_j^2\omega_k^2, \quad \cdots, \quad a_0=\prod_{j=1}^{K}\omega_j^2 \tag{10}$$

Note that the values for a_k are real coefficients.

On a fixed time interval $[0,T]$, the signal $x(t)$ and $y(t)$ satisfy the following differential equation models:

$$\Pi(p)x(t)=0 \tag{11}$$

$$\Pi(p)y(t)=p^{2K}y(t)+a_{K-1}p^{2(K-1)}y(t)+\cdots+a_1p^2y(t)+a_0y(t)=\varepsilon(t) \tag{12}$$

where $\varepsilon(t)=\Pi(p)v(t)$.

The above model (12) is equivalent to

$$p^{2K}y(t)=[Q(p)y(t)]\cdot[-\theta_a]+\Pi(p)v(t) \tag{13}$$

where the following definitions and vectors apply:

$$Q(p)=(p^{2(K-1)},p^{2(K-2)},\ldots,p^2,1), \tag{14}$$

$$\theta_a=(a_{K-1},a_{K-2},\ldots,a_1,a_0)',$$

$$\Pi(p)=\sum_{j=0}^{K}a_jp^{2j}=a_Kp^{2K}+Q(p)\theta_a, \tag{15}$$

where $a_K=1$. Note that vector θ is a real parameter vector and $Q(p)$ is a vector with an even order of p. Now, the frequency estimation problem is changed into a parameter estimation of the linear differential system of order 2K.

Applying $f_m(x)$ in the equation (7) to both sides of (13), we get the complex-valued regression model:

$$c'\Xi_m=c'\Gamma_m\theta+c'E_m \tag{16}$$

where the following definitions apply:

$$c=(c_0,c_1,\ldots,c_{2K})' \quad \text{and} \quad \theta=-\theta_a$$

$$\Xi_m=\begin{bmatrix}\Xi[m]\\ \Xi[m+1]\\ \vdots\\ \Xi[m+2K]\end{bmatrix}, \quad \Gamma_m=\begin{bmatrix}\Gamma[m]\\ \Gamma[m+1]\\ \vdots\\ \Gamma[m+2K]\end{bmatrix}, \text{and } E_m=\begin{bmatrix}E[m]\\ E[m+1]\\ \vdots\\ E[m+2K]\end{bmatrix} \tag{17}$$

$$\Xi[k]=(ik\omega_0)^{2K}Y[k], \quad \Gamma[k]=[Q[k]Y[k]], \quad E[k]=\Pi[k]V[k],$$

$$Y[k]=\frac{1}{\sqrt{T}}\int_0^T y(t)e^{-ik\omega_0 t}dt, \quad V[k]=\frac{1}{\sqrt{T}}\int_0^T v(t)e^{-ik\omega_0 t}dt,$$

$$Q[k]=Q[ik\omega_0] \text{ and } \Pi[k]=\Pi(ik\omega_0)$$

The m is the frequency index including D.C, i.e., $m=0,1,\cdots,M$. $Y[k]$ and $V[k]$ are the discrete Fourier transforms of the measurement signal $y(t)$ and noise $v(t)$. Note that $\Pi[k]$ is real because $\Pi(p)$ is a polynomial of degree 2K which has an even order in p.

To change the complex-valued regression model into a real-valued column vector linear regression model, let M be auser chosen frequency index which represents the maximum value to be used in constructing the least squares estimate, and define combined constituents by

$$\xi=\begin{bmatrix}c'\Xi_0\\ c'\Xi_1\\ \vdots\\ c'\Xi_M\end{bmatrix}, \quad \Phi=\begin{bmatrix}c'\Gamma_0\\ c'\Gamma_1\\ \vdots\\ c'\Gamma_M\end{bmatrix}, \quad \varepsilon=\begin{bmatrix}c'E_0\\ c'E_1\\ \vdots\\ c'E_M\end{bmatrix} \tag{18}$$

$$\xi_C=\begin{bmatrix}\mathrm{Re}\,\xi\\ \mathrm{Im}\,\xi\end{bmatrix}, \quad \Phi_C=\begin{bmatrix}\mathrm{Re}\,\Phi\\ \mathrm{Im}\,\Phi\end{bmatrix}, \quad \varepsilon_C=\begin{bmatrix}\mathrm{Re}\,\varepsilon\\ \mathrm{Im}\,\varepsilon\end{bmatrix}$$

Note that the size of the real-valued composite regressand, ξ_C, is $2(M+1)\times 1$; the size of the real-valued composite regressor matrix, Φ_C, is $2(M+1)\times K$ and the size of the real-valued composite residual vector, ε_C, is $2(M+1)\times 1$.

The model within a stochastic framework can be represented by a real-valued column vector linear regression model:

$$\xi_C=\Phi_C\theta+\varepsilon_C \tag{19}$$

Now, we can estimate the coefficients parameters $\hat{a}_{K-1},\hat{a}_{K-2},\ldots,\hat{a}_1,\hat{a}_0$ using LS.

$$\hat{\theta}=(\Phi_C'\Phi_C)^{-1}\Phi_C'\xi_C \tag{20}$$

assuming $\Phi_C'\Phi_C$ is nonsingular.

The covariance matrix for the $2(M+1)$ dimensional residual vector ε_C has the following structure:

$$W=E\{\varepsilon_C\varepsilon_C'\}=\begin{bmatrix}W_R & \Theta\\ \Theta & W_I\end{bmatrix} \tag{21}$$

where the functional relationship between the W matrix and the parameter vector θ can be shown to be:

$$W_R(\theta)=\frac{\sigma^2}{2}\left[CPP'C'+c_0^2(\Pi[0])^2e_1e_1'\right] \tag{22}$$

$$W_I(\theta)=\frac{\sigma^2}{2}\left[CPP'C'-c_0^2(\Pi[0])^2e_1e_1'\right] \tag{23}$$

where prime denotes a transpose of vector/matrix, $c_0=1$, $\Pi[0]=a_0$; the $(M+1)\times(M+2K+1)$ real matrix C and diagonal matrix PP' and unit column vector e_1 are given $C=\begin{bmatrix} c_0 & c_1 & \cdots & c_{2K} & 0 & \cdots & 0 \\ 0 & c_0 & c_1 & \cdots & c_{2K} & 0 & \vdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & c_0 & c_1 & \cdots & c_{2K} \end{bmatrix}$, $e_1=\begin{bmatrix} 1 \\ \Theta_{(M+2K)\times 1} \end{bmatrix}$

$$PP'=diag\{(\Pi[0])^2,(\Pi[1])^2,(\Pi[2])^2,\ldots,(\Pi[M+2K])^2\} \quad (24)$$

$\Theta_{(M+2K)\times 1}$ denotes a $(M+2K)\times 1$ null column vector. Note that the diagonal matrix PP' is the same as PP^H because $\Pi[k]$'s are real. This covariance matrix W is almost the same as the one in [3] except that ours include the DC component. The covariance matrix W is expressed by using a banded real constant matrix C and a diagonal complex matrix P, which is a function of unknown parameters. The advantage of this covariance matrix form is that it is easy to program. This is the basis for the adaptive least squares algorithm developed in Pearson et al[6]. Thus, the covariance matrix is banded by the system order 2K in this frequency domain model.

If we denote

$$W^{-1}=\begin{bmatrix} W_R^{-1} & \Theta \\ \Theta & W_I^{-1} \end{bmatrix} \quad (25)$$

then using the covariance matrix W as a weighting matrix, the Weighted LS estimate of θ can be written as

$$\hat{\theta}=\left(\Phi'_C W^{-1}\Phi_C\right)^{-1}\Phi'_C W^{-1}\xi_C \quad (26)$$

Moreover, $\hat{\theta}$ can be estimated by the AWLS in [5], which can be expressed by the following equation:

$$\hat{\theta}_k=\left(\Phi'_C W_{k-1}^{-1}\Phi_C\right)^{-1}\Phi'_C W_{k-1}^{-1}\xi_C,\quad k=1,2,\cdots \quad (27)$$

where W_{k-1} denotes the covariance matrix of the residual vector shown in equation (21), as a function of unknown parameter θ and evaluated at the previous iterate θ_{k-1}. Thus $W_{k-1}=W(\theta_{k-1})$. The initial weighting matrix W_0 is taken as the identity matrix.

Using the AWLS/MFT, we can estimates the unknown coefficients $\hat{\theta}=[\hat{a}_{K-1},\hat{a}_{K-2},\ldots,\hat{a}_0]$ for the autoregression model (12). Here, estimated $\hat{a}_k$'s are real and a function of ω's. We can easily obtain the frequency estimates using the following procedure: Substitute estimated coefficients $\hat{\theta}$ into differential equation model (11) and construct the polynomial equation:

$$x^{2K}+\hat{a}_{K-1}x^{2(K-1)}+\ldots+\hat{a}_1x^2+\hat{a}_0=0 \quad (28)$$

Solve this equation to get K pairs of complex conjugate roots, say $x_1,x_2,\cdots,x_{2K}$. Then we can obtain the frequency estimates by taking only the positive imaginary part of each x_k, which has the form of $x_k=\pm i\omega_k$:

$$\omega_k=|\mathrm{Im}(x_k)|,\quad k=1,2,\cdots,K \quad (29)$$

IV. Numerical Example

In this section, we present a numerical example which illustrate the performance of the AWLS Sinusoidal Estimator for real sinusoidal estimation and compare it with that of the LS Sinusoidal Estimator in [4].

Example Here we give an example for the estimation of frequency from real data which compares the algorithm with the Least Squares method in continuous time domain. This example is adapted from [4] to compare the estimation performance. In this example, the analog angular frequencies, amplitudes and phases are given by: $\omega_1=50$, $\omega_2=55$, $\omega_3=100$; $A_1=A_2=A_3=6$; $\phi_1=0.123$, $\phi_2=0.541$, $\phi_3=0$. The measurement signal is $y(t)=6\sin(50t+0.123)+6\sin(55t+0.541)+6\sin(100t)+v(t)$ where $v(t)$ is a stationary white Gaussian noise with variance σ^2.

The sampling frequency F_s is 100 Hz and the total observation time $T=0.64$ seconds. The resolving frequency ω_0 is 9.82 rad/sec, The frequency separation between ω_1 and ω_2 is 5 rad/sec, which is almost a half of ω_0. ω_3 is four times the frequency of ω_1. We have a total of $N=64$ points. Angular frequencies normalized by ω_3 are $\overline{\omega}_1=0.5$, $\overline{\omega}_2=0.55$, $\overline{\omega}_3=1$, and the corresponding normalized frequencies are $f_1=0.079$ Hz, $f_2=0.0875$ Hz, $f_3=0.1592$ Hz, respectively. The signal-to-ratio SNR_i is defined for each frequency component as $SNR_i=10\log_{10}\left(A_i^2/2\sigma^2\right)$ to produce different noise variance σ^2. Because the example in [4] used the parabolic rule for the approximation of the Fourier transform integrals, we also use the same rule shown in [19]. Using the parabolic rule, FFT of observed signal $y_i=y(iT_s)$, $i=0,1,\cdots,N-1$, is

$$Y=\frac{T_s}{3}FFT[(y_0+y_N),4y_1,2y_2,\cdots,4y_{N-1}] \quad (30)$$

where T_s is the sample interval and N is the number of data samples.

Considering some trade off among 3 frequencies, we selected $M=6$ for optimal maximum frequency index like Pan [4] did with the LS sinusoidal estimator.

Fig. 1(a)-(c) present the normalized bias of $\overline{\omega}_1$, $\overline{\omega}_2$, $\overline{\omega}_3$ using the proposed AWLS sinusoidal estimator and Pan's LS sinusoidal estimator based upon the total of 50 Monte-Carlo runs. From Fig. 1, we can notice that the biases of $\overline{\omega}_1$ and $\overline{\omega}_2$ for the AWLS sinusoidal estimator are smaller than the biases for the LS sinusoidal estimator when SNR is positive but the biases of $\overline{\omega}_3$ for both algorithms are almost the same and very small over all SNR's , i.e., even the LS provides accurate estimate for a well-separated frequency with high SNR. The biases of the AWLS sinusoidal estimator at negative SNR are not so much better than those of the LS sinusoidal estimator for the three frequency estimation.

From this simulation example, we can see that the AWLS sinusoidal estimator shows the characteristics of a high resolution estimator even for a three frequency estimation problem with short data length even though the performance is somewhat deteriorated compared with two frequencies estimation problem, i.e., it shows good performance only for positive SNR cases. Also, we can say that the breakthrough of the AWLS sinusoidal estimator for three frequencies occurs when SNR is 0 dB, where the performance decreases sharply.

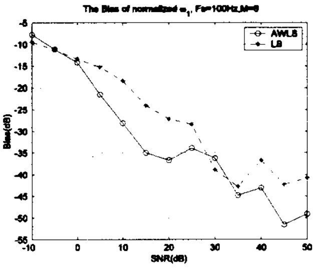

Fig. 1(a): The estimation bias(dB) for ω_1

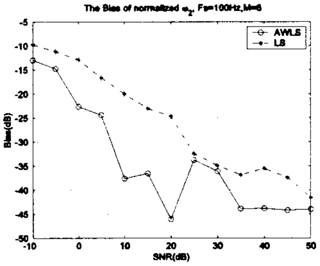

Fig. 1(b): The estimation bias(dB) for ω_2

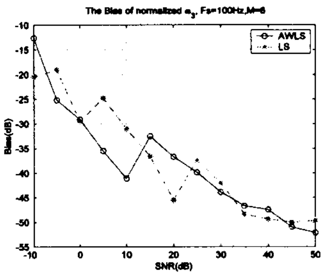

Fig. 1(c): The estimation bias(dB) versus ω_3

V. Conclusion

A high resolution frequency estimator, AWLS Sinusoidal Estimator, has been proposed for estimating real sinusoids and their frequencies in the presence of white noise. From the comparison studies, the AWLS Sinusoidal Estimator has improved the previous LS Sinusoidal Estimator [4] in resolution band and proved that the AWLS Sinusoidal Estimator is a high resolution frequency estimator. For a fixed resolution, the AWLS Sinusoidal Estimator permits the use of a shorter length of data, allows the use of much lower SNR and gives higher resolution for very closely spaced frequencies.

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