전력계통의 Chaos 위상학적 특성 해석

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Topological analysis of Chaos Characteristics in A Power System

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Abstract - This paper proposes a totally new method in the chaos characteristics analysis of power systems, the introduction of topological invariants. Using a return histogram the bifurcation graph was drawn, the periodic orbits and topological invariants the local crossing number, relative rotation rates, and linking number during the process of period-doubling bifurcation and chaos were extracted. This study also examined the effect on the topological invariants when the sensitive parameters were varied. In addition, the topological invariants of a three-dimensional embedding of the strange attractor was extracted and the result was compared with those obtained from differential equations. This could be a new way for a state detection and fault diagnosis in a dynamical system.
Keywords: chaos, topological invariants, power system

1. Introduction

Loads in a power system have become rather complicated in recent years. It is difficult for the power system to stay in one equilibrium point, and it may sometimes operate beyond the limit of its stability. This situation creates concern regarding the bifurcation and chaotic attractors in practice. This originates from the nonlinear and deterministic structure of the power system itself but not to random load disturbances [1]. Therefore, chaotic analysis can lead to a better understanding of the problem of stability and to become useful techniques for the control and operation of power system. Because the essence of chaos is the strange attractor, it is important to study the characteristics of the strange attractor.

Currently, there are two main approaches for analyzing the chaotic time series in a dynamical system. They are metric [2] and topological approaches [3]-[6]. The metric approach is based on the distance between the points in the attractor [4]. In this approach it is customary to compute the fractal dimension, the Lyapunov exponent, the spectrum singularities, etc. It generally requires a large amount of data and degrades rapidly with additive noise. The topological approach is based on the organization of unstable periodic orbits embedded in the strange attractor [5]. It is responsible for creating a strange attractor using stretching and squeezing mechanisms. The extraction of these quantities from the time series data is robust against noise and is independently verifiable. These quantities are defined as topological invariants that are effective in modeling the dynamic system.

This paper is organized as follows. The next section introduces the topological invariants, the Local crossing number, the Relative Rotation Rates and the Linking numbers, and shows how to extract them from the periodic orbits. In Section 3, the topological invariants in a simple power system are calculated and the stability is analyzed as the parameters are changed.

The topological invariants were also extracted directly from the periodic orbits reconstructed from the time series and the result was to those computed obtained from the equation. The conclusions are reported in Section 4.

2. Extraction of topological invariants

The local crossing number is the number of half-twists of the period-doubled orbit along the tubular neighborhood [6]. It is an important index that describes period-doubled process.

The topological organization of all the unstable periodic orbits extracted from the time series is determined by calculating the relative rotation rates and linking the numbers of all pairs of periodic orbits, the self-relative rotation rates and self-linking number of each individual periodic orbits.

The RRR describe how often one orbit rotates around another on average. The sum of the RRR notes over all the pairs of initial conditions is the linking number that links the two periodic orbits [5].

The topological properties are closely linked to physical processes and are often insensitive to noise changes, which are the advantage in model the system [4].

3. Extraction and analysis of topological invarients in a simple power system

This section presents the topological properties of double-periodic bifurcation and chaos on a detailed model.

3.1 A Simple power system

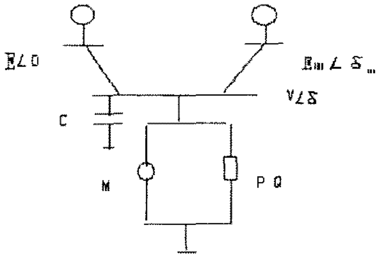

Fig. 1 A simple three-bus system

Consider the power system shown in Fig. 1, which consists of a load that is supplied by two generators

[1]. An induction motor in parallel with a constant PQ load is used to represent the load. The equation of this system consists of four state variables that correspond to the generator angle (δ_m), the generator angular velocity (ω), the angle (δ) and the magnitude (V) of the load voltage.

The load reactive power is chosen as the system parameter, so that increasing Q_1 corresponds to increasing the load reactive power demand. Ref. [1] provides detailed system equations of this model.

Consider the power system shown in figure 2 that cons

$$\dot{\delta}_m = \omega \tag{1}$$

$$+ V_m^2 Y_m \sin\theta_m \tag{2}$$

$$k_{q\omega}\dot{\delta} = -k_{qv}V - k_{qv_2}V^2 + Q - Q_0 - Q_1 \tag{3}$$

$$Tk_{q\omega}k_{pv}\dot{V} = k_{p\omega}k_{qv_2}V^2 + (k_{p\omega}k_{qv} - k_{q\omega}k_{pv})V + k_{p\omega}(Q_0 + Q_1 - Q) - k_{q\omega}(P_0 + P_1 - P) \tag{4}$$

3.2 Topological properties of double period bifurcation

Many studies have found that chaos exists via a period-doubling route in this model (see Ref. [1]). Fig. 2 shows that the power system is stable near Q_1 =11.39.

As Q_1 is slowly decreased, the period 2 orbit grows at Q_1 =11.3885, and then undergoes a sequence of a period-doubling bifurcation, leading to chaos. Fig. 3 shows several stable periodic orbits in the sequence of period doubling discussed above.

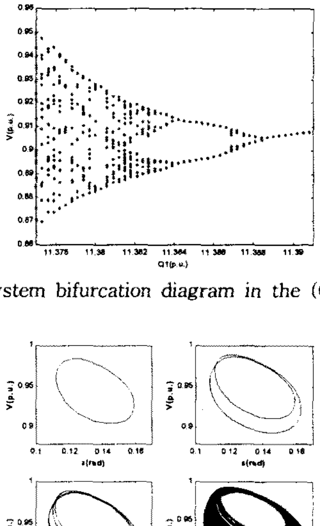

Fig. 2 System bifurcation diagram in the (Q_1,V) plane

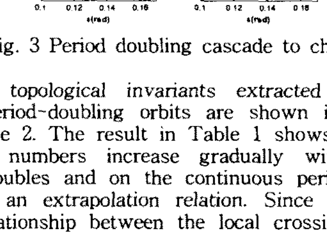

Fig. 3 Period doubling cascade to chaos

The topological invariants extracted from the stable period-doubling orbits are shown in Table 1 and Table 2. The result in Table 1 shows that local crossing numbers increase gradually with as the period doubles and on the continuous periodic orbits there is an extrapolation relation. Since there is a close relationship between the local crossing number and the power spectrum of the orbit, the harmonics components are necessarily increased with periodic doubling.

Table 1. Local crossing number of period-doubling bifurcation

Period	1	2	4	8
L of δ	0	1	3	5
L of V	0	1	3	5

Table 2. Relative rotation rates and linking numbers of period-doubling bifurcation

Period	1	2	4	8
1	0	-1/2	-1/2	-1/2
2		-1/2,0	-1/2,-1/4	-1/2,-1/4
4			(-1/2)2,-1/4,0	(-1/2)2,-1/4,-3/8
8				(-1/2)2,(-1/4)2,-3/8,0

(a) RRR of period-doubling bifurcation

Period	1	2	4	8
1	0	1	2	4
2		1	3	6
4			5	13
8				23

(b) L of period-doubling bifurcation

From the above analysis it can be seen that, the topological properties of period 2^n contain all the topological information in the period $1,2,\cdot,2^{n-1}$ orbits. This means that the orbit structure from period 1 to period 2^n can be determined if period 2^n orbit twist on the period 2^{n-1} orbit is known.

3.3 Topological characteristics in chaos state

For the forth order model chaos is observed in the approximate range Q_1=11.377-11.382 (see Fig. 2). We can obtain the fundamental periodic of the unstable period orbit at Q_1=11.379 by the return histogram, which is approximately 2.1s. According to the relationship between T and ω, $\omega=2\pi/T=2.99$, which is the imaginary part of the complex eigenvalues at Q_1=11.379. The unstable period orbits can then be extracted (see Fig. 4) and the topological invariants can be calculated (see Table 2).

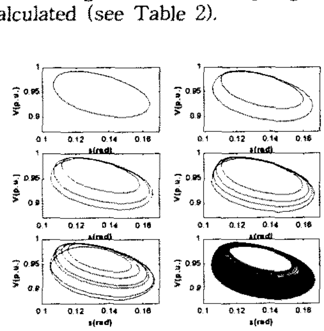

Fig. 4 Extraction of period orbit at Q_1=11.379

Indeed there is another chaos region induced from the period-doubling bifurcation around Q_1=10.89 [1] (fundamental period T=1.67s).

The region corresponding to Q_1=10.89 is referred to as the left chaos; the region corresponding to Q_1=11.379 is known as the right chaos. The topological invariants of the left chaos are shown in Table 4. The results in Table 3 and Table 4 are apparently different, namely, the topological organization of two attractors are different. Ref. [1] have noted that the attractor in the left chaos region will disappear as Q_1 is increased, while the attractor in the right chaos region will lead to a voltage collapse as Q_1 is decreased. On other hand, the results in Table 3 and Table 4 suggests that the topological invariants are sensitive to changes in the parameter and system structure, which will contribute to the system state detection and fault diagnosis.

Table 3. Relative rotation rates at Q_1=10.89

period	1	2	3	4	6
1	0	-1/2	-1/3	-1/4	-1/3
2		-1/2,0	-1/3	-1/4	-1/3,-1/6
3			(-1/3)2,0	-1/4	(-1/3)2,-1/6
4				(-1/4)3,0	-1/4,-1/6
6					(-1/3)2,(-1/6)3,0

Table 4. Relative rotation rates at Q_1=11.379

period	1	2	3	4	6
1	0	-1/2	-1/3	-1/2	-1/3
2		-1/2,0	-1/3	-1/2,-1/4	-1/3,-1/6
3			(-1/3)2,0	-1/3	(-1/3)2,-1/6
4				(-1/ 2)2,-1/4,0	-1/3
6					(-1/3)4,-1/6,0

3.4 Topological characteristics from time series

If a model of the power system is not known precisely, and only the measurement data i.e. time series, is available, then the topological invariants can still be extracted. First, it is postulated that system model is not known and only the time series V(t) of the magnitude of the terminal voltage, which are the simulation results at Q_1=11.379, is available. Because the topological invariants can not be described unless a three-dimensional embedding can be found, a three-dimensional embedding of the strange attractor needs to be constructed. An integral-differential filter is constructed [4], which is easily implemented by an electronic circuit and reduces S/N (Signal/Noise) ratio.

$$Y_1(i)=\sum_{j<i} V(j)\, e^{-(i-j)/\tau} \qquad (5)$$

$$Y_2 = V(i) \qquad (6)$$

$$Y_3(i)= V(i+1) - V(i-1) \qquad (7)$$

The fundamental period of the attractor is obtained as T=2.1s. The attractor is projected on the phase-phase Y2-Y3 and the unstable periodic orbits in it are extracted (see Fig. 5).

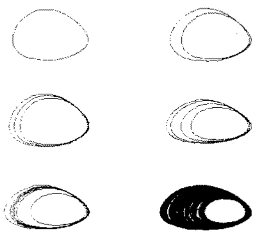

Fig. 5 Extraction of the period orbit at Q_1=11.379

The topological invariants are the same as that shown in Table 4. However, we cannot be sure that the topological organization of the two attractors (one is from the time series and the other from the model) is equivalent. In order to identify whether the two attractors are equivalent, further research using template and symbolic dynamics theory will be needed. The discussion in this section is more suitable for diagnosing a fault in a practical power system.

4. Conclusions

This study analyzed the topological characteristics of a simple power system. The analysis of the topological invariants during period-doubling bifurcation show that the period 2^n orbit contains all the topological information of the previous periodic orbits (period 1, 2, 2^2,...., 2^{n-1} orbits).

The topological invariants can identify the attractor belonging to the different chaos regions and the oscillation period is easy to obtained by a return map and a return histogram.

A three-dimensional embedding was successfully constructed from the time series, and topological invariants of time series were found to be the same as those of the fourth-differential equations. It is useful to analyze the characteristic of a power system from a practical perspective when a fault takes place. The RRR can be taken as a sensitive factor in a fault diagnosis since they indicate whether or not the two dynamic systems are equal. Note that the topological invariants are only sensitive to a parameter variation that leads to qualitative changes in the system, while they are insensitive for the quantitative changes in the system.

In future work, it is hoped that to model the dynamics by a horse-holder template and symbolic dynamics theory based on the topological invariants extracted from the model or the experimental data sets.

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